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Curvelet Transform used Image Denoising

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Abstract- Images are affected by various Noises while receiving, processing and transmitting. Speckle Noise Gaussian Noise, Salt and Pepper Noise are the various noise sources. These Noise are reduced by different Denoising techniques. These Techniques remove the unwanted Noise and preserve useful information of an image. Wavelet Transform is the present Technique to remove the Noise. These techniques have some drawback on edges. So, Curvelet Transform introduced. Curvelet Transform Denoise the image at edges and preserve the information. In this paper edges are improved by curvelet transform by thresholding. The improved curvelet transform has a great advantage to discard the noise in images.

Index Terms- Noise models, Wavelet, Curvelet, Denoising.

1 INTRODUCTION

Reducing noise from the image is a challenge for the researchers in image processing.

Gaussian Noise:

In the signal a gaussian noise is distributed evenly. Probability distribution function is given by

$$F(g)=\frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(g-m)^2}{2\sigma^2}}$$

Where g represents the gray level, m is the mean of the function and σ is standard deviation in the noise.

Speckle Noise:

This type of noise occurs mostly in all coherent imaging systems such as acoustics, laser, acoustics and SAR imagery. Speckle noise follows a gamma distribution as shown below

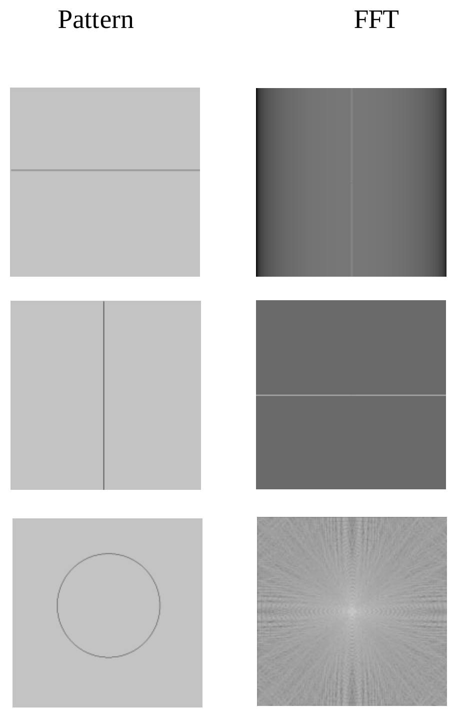
$$F(g)=\frac{g^{\alpha-1}}{(\alpha-1)!\,\alpha^{\alpha}}\, e^{-\frac{g}{a}}$$

where variance is a 2α and g is the gray level.

Environmental conditions during the acquisition, Light levels, Sensor temperature are the sources of noise in digital images arise during image acquisition. There has been various noise removal methods in Digital image processing. In FFT based image de-noising, Low pass, High pass, Band pass filters are used to reduce the noise by frequency filtering and frequency smoothing in the image[1].

In Low pass filter, attenuate high frequency (sharp intensity) and retains low frequency (smoothing).In High pass filter, attenuate low frequency (smoothing) and retains high frequency (sharpening). In Band pass filter, enhance edges (suppressing low frequencies)while reducing the noise at the same time (attenuating high frequencies).

Common patterns and their corresponding Fourier transforms are:



It is different from smoothing as smoothing only removes high frequency while retaining the lower ones[2].

The mother wavelet is shifted or translated by a factor of b and dilated or scaled by a factor of a and to give:

$$\varphi_{a,b}(t) = \left(\frac{1}{\sqrt{|a|}}\right) * \varphi\left(\frac{t-b}{a}\right)$$

where 'a' and 'b' that represent the dilations and translations parameters are two arbitrary real numbers respectively in the time axis.

The transform function known as the mother wavelet is modified by translations and dilations. In order to be set as a wavelet, the analyzing function must satisfy the following admissibility condition [2].

$$C(\varphi) = \int_{-\infty}^{\infty} \frac{|\varphi(\omega)^2|}{|\omega|} d\omega < |\infty$$

With this function, a family of functions can be derived, which are the actual wavelet according to the following equation.

$$f(t) \in V_j \Rightarrow f(dt)| \in V_{j-1}$$

The wavelet transform theory can be generalized to any desired dimensionality.

Where the transform is hierarchy of embedded subspaces such that none of the subspaces intersect is known as two dimensional multi resolution analysis, and each function relates to the following condition. This can be represented as

N=D.u

This procedure is performed successively on the approximation signal to the required number of levels.

The basic method of wavelet based image processing is to:

1. Compute the two-dimensional wavelet transform of an image.
2. Alter the transform coefficients.
3. Compute the inverse transform.

The horizontal edges are present in the horizontal detail coefficients of the upper-right quadrant of the original image. Calculate the inverse of it and compute the absolute value.
Discrete Wavelet Transform decomposes an image into different subband images, as Low-Low (LL), Low- High (LH), High-Low (HL), and High-High (HH). The subband (LL) is the low resolution residual. A specific threshold value is calculated to decompose the original image. Finally, the filter is applied for enhancement of image [3].

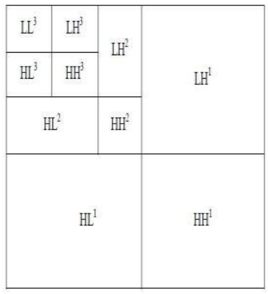


Figure.1: Multilevel Wavelet Decomposition of an Image

The first step is to choose a wavelet type, and a level N of decomposition. In this case bi orthogonal 3.5 wavelets were chosen with a level N of 10. Using this wavelets a wavelet transformation is performed on the two dimensional image. The next step is to determine threshold values for each level from 1 to N. This process individual thresholds are made for N = 10 levels [4]. The final step is to reconstruct the image from the modified levels. Curvelet transform overcome the limitations of Wavelet transform. The curvelet methods have better de-noising capability over wavelets. Curvelet transform is a new multi scale transform used as an effective tool in image denoising [5]. The curvelet transform is better than wavelet in the form of image edges, like geometry features of curve that give good result in image denosing [6].

In this paper, The proposed method used for denoising is explained in Section 2. Section 3 consists explanation about the curvelet transform. In Section 4 results of the proposed method are presented. Section 5 gives about the conclusion and future scope.

II PROPOSED METHOD

The Block diagram of Curvelet Transform is shown in Figure2.

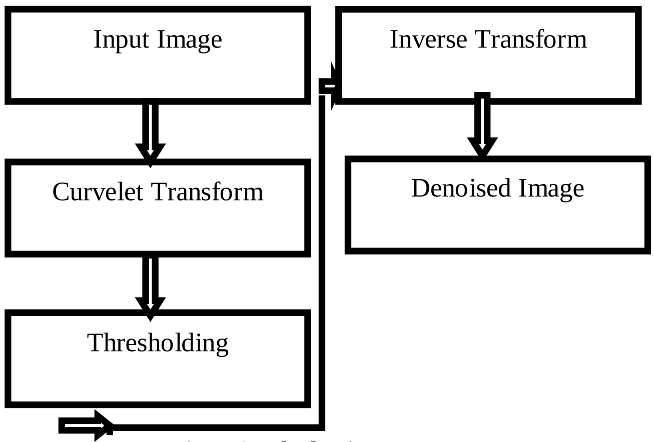


Figure2: Block Diagram

The noisy image is transformed to the curvelet domain by applying forward curvelet transform to the image. The threshold is performed on basis of statistical properties of coefficients[7].

Algorithm
1. Apply the Forward Curvelet transform to the noisy image.
2. Threshold the Curvelet co-efficients to get rid of some insignificant Curvelet co-efficients by employing athresholding operate within the Curvelet domain.
3. Inverse Curvelet transform of the threshold co-efficients to reconstruct a function.

(i) Continuous Ridgelet Transform:
The continuous ridgelet transform (CRT) in R^2can be outlined as,

$$\int \frac{|\psi\hat{}(v)|^2}{|v|^2}dv < \infty,$$

A smooth univariate function $\psi: R \rightarrow R$ with sufficient decay. Where ψ is a vanishing mean $\int \psi(t)dt = 0$ and obey admissibility condition. We define the bivariate ridgelet
$\psi_{\ a,b,c}(X) = \psi_{a,b,c}(x1,x2) = a^{-1/2}.\psi((x_1 cos\theta + x_2 sin\theta - b)/a)$ ridgelet is constant along lines x1 cos θ + x2 sin θ = const. Integral bivariate function f(x), we define its ridgelet coefficients by

$$R_f(a,b,c) := \langle f, |\psi_{a,b,c}\rangle = \int_{R^2} f(X)\overline{|\psi_{a,b,c}}(x)dX.$$

Reconstruction formula is

$$f(x)=\int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\theta)\psi_{a,b,\theta}(X)\frac{da}{a^3}db\frac{d\theta}{4\pi}$$

valid for functions both Integral and square Integral function. This formula is stable and can prove Parseval relation:

$$\int |f(x)|^2 dx$$
$$= \int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{\infty} R_f(a,b,\theta)\psi_{a,b,\theta}(x)\frac{da}{a^2}db\frac{d\theta}{4\pi}$$

(ii) Radon transform:

In 2D, line and point singularities related to radon transform so wavelet and ridgelet transform are linked to radon transform
$Rf(\theta,t)=\int_{R^2} f(x_1,x_2)\delta(x_1 cos\theta + x_2 sin\theta - t)dx_1 dx_2$,
Where δ is daric function. The coefficient of ridgelet transform of an object f are taken by applying 1D wavelet transform to the pieces the Radon transform as:

$$R_f(a,b,\theta) = \int Rf(\theta,t)a^{-0.5}\psi(\frac{t-b}{a})dt$$

(iii) Digital ridgelet transform:

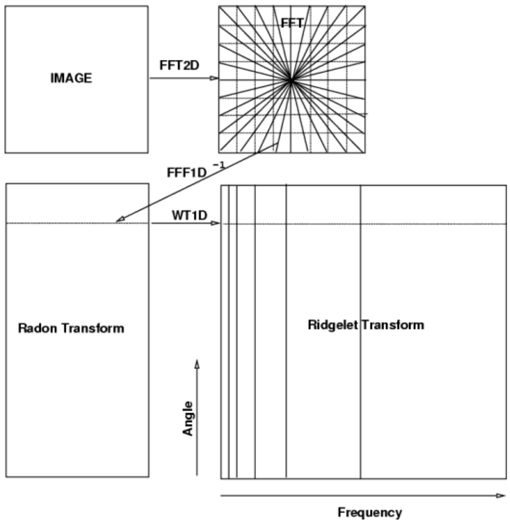


Figure3: Ridgelet transform flow graph

Ridgelet transform is first to measure Radon transform Rf(t; θ) and second, to apply a one-dimensional wavelet transform to the slices Rf(¢; θ). The 2-D continuous ridgelet transform in R2 can be defined as follows[8]. First define a smooth wavelet function :

R R satisfying the admissibility condition given by

$$\int_{-\infty}^{+\infty} \frac{|\widehat{\Psi}(\mathcal{E})|^2}{(\mathcal{E})^2}\, d\mathcal{E} < \infty$$

where $\widehat{\Psi}$ is the Fourier transform of Ψ. Equation (Candes et al 2002) holds if has a vanishing mean, i.e.,

$$\int_{-\infty}^{+\infty} \Psi(t)dt = 0.$$

The bivariate ridgelet Ψ a,b,θ : R2 R2 is defined by Ψ a,b,θ(x,y) = $a^{-\frac{1}{2}}\Psi(\frac{xcos\,(\theta)\,+ysin(\theta)-b}{a})$
and the function is constant along the lines x cos(θ) + y sin(θ) = const. The

ridgelet values for continuous image f(x, y) is given by

Rf (a,b, θ) =∬ f (x, y) Ψ a,b,θ (x, y)dx dy

The Radon transform in continuous terms is defined by

Rf (t,θ) = ∫ f (x, y)δ(t -cos(θ) -ysin(θ)) dx dy
In the on top of equation δ denotes the dirac delta-function. In words, Rf is that the integral of the continual image f, over the road Lt,θ outlined by t = x cos(θ)+y sin(θ). Parameters t and θ are represented.

$$Rf(t,\theta) = \int Rf(t, \theta) a^{-1/2}\, \Psi(\tfrac{t-b}{a})dt$$

In the above equation Ψ a, b(t) = $a^{-1/2}\, \Psi(\frac{t-b}{a})$dt is a 1-D wavelet.

III. CURVELET TRANSFORM

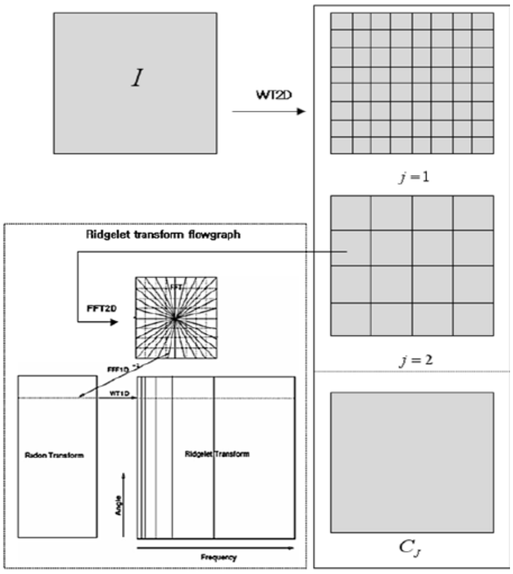


Figure4: Curvelet transform flow graph

The curvelet decomposition can be stated in the following 4steps:

1. Subband Decomposition: Where the object I is filtered into subbands

$$F \rightarrow (P0f, \Delta_1 f, \Delta_2 f. \ldots)$$

2. Smooth Partitioning: Each Subband is smoothly partioning into squares

$$\Delta_s f \mapsto (\omega Q \Delta_s f), Q \in Q_s$$

3 .Renormalization: Each square is renormalized to unit square

$$gQ = (TQ)^{-1}(\omega Q \Delta_s f),\ Q \in Q_s$$

4. Ridgelet Analysis: Each square is analyzed via the discrete ridgelet transform.

Subband Filtering:

Figure shows the the relationship between subbands and the frequency domain where 's' = 0 indicates the basis subband, 's' = 1 indicates the bandpass subband, and 's' = 2 indicates the highpass subband[9].
As an example, the Lena image I is split in the basis subband P_0I, bandpass subband Δ_1I and high pass subband Δ_2I as shown in Figure. Note that, the relationship between the Lena image I and its subbands is given by I = p_0I + Δ_1I +Δ_2I

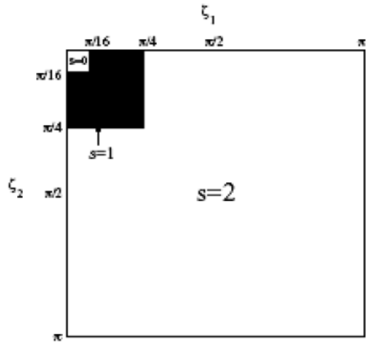


Fig:5: Subband division for a 256 * 256 image

Tiling:

This is how the subbands Δ1 and Δ2 are covered as a result of by zooming well enough into the image, some curving edges can become linear singularities[10].

Rotation Angle: θ_1(J) = 2π $*$ 2^1 (-J) * L

Parameter: $K_\delta = (k_1 * \delta_1, k_2 * \delta_2)$

Where δ_1 and δ_2 are normalizing constants
Curvelet Basis Function:

$\gamma(X_1, X_2) = \Psi(X_1)\ \Psi(X_2)$

(X_1)= Gabore(X1) and $\Psi(X_2)$=Gaussian (X2)

Finally, the curvelet parameterized by (J, K, L) can be defined as
$\Gamma_{(j,l,k)}(X_1, X_2) = 2^{2j/2} * \Gamma(D_j * R_{\theta J} * (X_1, X_2) - K_{\delta})$

The algorithm of the Curvelet transform of an image f can be summarized in the following steps.

The following gives the various steps of curvelet denoising algorithm

1. Apply the 'a trous' algorithm with J scales,

2. Set B1 = Bmin,
3. for j = 1to J do,
   - partition the subband wj with a block size Bj and apply the digital ridgelet transform to each block,
   - if j modulo 2 = 1 then B j+1 = 2Bj ,
   - else Bj+1 = Bj.

The side length of the localizing windows is doubled at every other dyadic subband.

Hence maintaining the fundamental property of the curvelet transform which says that elements of length about 2j=2 serve for the analysis and synthesis of the j-th subband (2j; 2j+1).

4. Perform hard thresholding.
5. Take Inverse Curvelet transform.

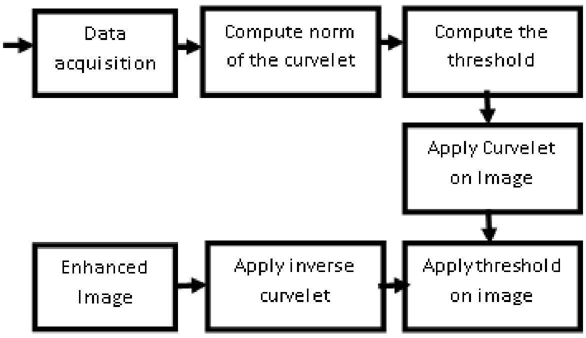


Fig 6: Block Diagram of Curvelet Denoising Algorithm

IV RESULTS:

Noised Images — Curvelet Denoise image

Dataset 1


Dataset 2


Dataset 3


Data	MSE	PSNR
Data set1	11.6700	37.4607
Data set2	17.0607	35.8108
Data set3	9.3775	38.4099

Table1: Quality measurement for different Data sets

The Mean Square Error is:

$$MSE = \sqrt{\frac{\sum_{x=1}^{M}\sum_{y=1}^{N} ||f(x,y)-g(x,y)||^2}{MN}}$$

The peak signal to noise ratio is:

$$PSNR = 20log_{10}\frac{512}{MSE}.$$

V. CONCLUSION AND FUTURE SCOPE

In this work, the curvelet primarily based denoising for various dataset area unit developed. The curvelet wayshave higher de-noising capability over wavelets. This methodology is illustrated by taking pictures and also the results are analyzed for the noise removal and knowledge protective ability. The Curvelet images area unit compared by quality parameters PSNR and MSE. to enhance the curvelet denoising within the swish areas, Saevarsson et al given a brand new technique, adaptational combined technique (ACM), which mixes the curvelet rework and also the wavelet rework. The quality was tested using PSNR and MSE measure for different datasets. Let f(x,y) and g(x,y) are the original and denoise images[11].

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