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Fatigue Evaluation Algorithms in Fiber Reinforced Composites - A Review

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Abstract—The problem of Fatigue life prediction in Fiber reinforced composite materials is being discussed for more than three decades. Number of articles have been published, and experimental data on different materials is available but still specific predictive algorithms is not substantiated, especially for cases containing of complex, variable amplitude stress variables. Several works have attempted a general discussion of the subject. This paper presents a general review of the existing discussion on the fatigue evaluation algorithms and reviews from books on this subject.

1. INTRODUCTION

A number of articles are reviewed recently in [1],[2],[3],[4],[5], on the isotropic materials and anisotropic materials. The modeling of fatigue damage mechanisms in these materials show that it is difficult to predict life in anisotropic materials than isotropic materials. The significant problem that is encountered is in correlating the micro-structural damage to the fatigue performance of Fiber reinforced composite laminates under static and dynamic loading conditions.

2. FATIGUE LIFE MODELS

a. *Review*

First, The empirical models is one of the method used to evaluate damage. In this, no physical quantity is used for interpretation until final failure as a means of accumulating fatigue damage in the composite material. The literatures show many examples of different levels of complexity, one example is the widely used Palmgren-Miner rule [6]. Most of the empirical models are used for metallic materials to predict fatigue damage and life under variable amplitude loading. Some specific parameters such as the loading level, fatigue stress level, etc., are taken into account for non-linear cases of fatigue life.

Second, The Phenomenological models consider a physical measurable quantity for estimation of damage. These models are used to correlate the fatigue damage condition of the material with a physically measurable quantity like residual stiffness or strength. In this, no special attention is given to the actual types of fatigue damage. Here, an average value of the specified mechanical property is obtained to predict and analyze the effects of fatigue damage. Fatigue life prediction is done from the macro-scale to the micro-scale on the basis of hierarchy in these models. The damage is assessed based on the scale, and is pertained as a cause of change of material mechanical properties at the considered scale. These models generally give easy and convenient solutions to the problem as these models give various correlations between fatigue damage mechanisms, and hence phenomenological models appear to be easy and convenient to use.

Third, Mechanistic models are also used for life prediction methods for fiber reinforced composite materials. This model finds use as a link between actual fatigue damage mechanisms and macro-mechanical properties of fiber reinforced composite laminates under arbitrary stress conditions. These models can solve the problem under different scale of consideration moving from the micro-scale to the macro-scale. One limitation, is the complexity involved considering the option of number of mechanisms to arrive at a solution, and which is further complicated by the selection of the chosen parameters, statistical values of the properties of the constituents of the composite materials.

All of the models classified above, require the knowledge of some basic fatigue parameters which are generally divided and according to their type of category, into discreet modules, which constitute the building blocks of a generalized life prediction algorithm under arbitrary fatigue loads (i.e. complex stress levels of variable amplitude).

The Algorithm, which is defined as the kind of input required for any life prediction method under variable amplitude fatigue, is considered as an additional module to analyze irregular load-time variations into a sequence of constant amplitude cycles. This additional function is important for analyzing because, most of the fatigue testing is performed through application of load cycles of sinusoidal wave form. Consequently, an adequate counting method is used in order to correlate the experimentally obtained fatigue characteristics of the material with the variable amplitude load applied, for the loading.

In a review work on life prediction of composite materials [7] Sendeckyj refers, based on the relationship between static strength and fatigue life, he uses the residual strength theories as a means to predict life under constant amplitude fatigue.

Sutherland, Mandell et al [8] proposed a three parameter equation based on the one proposed by Epaarachchi, [9] formulating an S-N curve, which is flat at low cycles, steeper at medium and less steep at high cycles.

Whitney [10] proposed a method for defining the S-N curve at specific reliability levels assuming Weibull probability distribution of fatigue life.

Xiao [11] proposed the power law S-N relation. In this, the temperature dependency is taken into and accounted for by the factor, and which is equal to the ratio of static strength at the considered temperature T, versus the static strength at a reference temperature.

Miyano [12] presented a methodology for deriving S-N curves at arbitrary temperature and loading frequency.

Palmgren-Miner rule [6], establishes a empirical damage accumulation rule which is linearly dependent of fatigue damage D of the material to the residual total life spent during cyclic loading.

Owen & Howe [13], proposed another non-linear damage accumulation rule based on block loading fatigue observations.

Phenomenological damage accumulation models consider residual strength as a property to express damage during fatigue accumulation.

Up to date, residual strength models are widely discussed by Sendeckyj in [14], reviewed in [4], [5] and evaluated. A number of experimental data is available in [15].

Broutman and Sahu [16] has presented one of the first models based on static strength degradation of GFRP composites, and was further developed to a modified Palmgren-Miner rule. This accounts for the load sequence effects.

Chou and Croman [17], proposed, a different wear-out model. In this he included an additional necessary parameter, suitable to define the wear-out. In the sudden death model [18], which he proposed, considers the study of the residual strength as means of prediction. In the sudden death model the residual static strength remains constant, i.e. independent of load cycles, until immediately prior to failure and then drops suddenly.

Reifsnider (e.g. [19]) proposed a *Critical Element Model* theory for modeling the fatigue process in composites. As the laminate is subjected to stress distribution under loading, the critical failure of the element, and the sub-critical failure causing stress redistribution is considered as the critical value of the stress for failure.

Epaarachchi and Clausen [20] proposed a frequency and stress ratio dependent residual strength method as damage accumulation rule. In this, the variable amplitude fatigue is applied stepwise, and the material behaviour is analyzed by deriving the materials S-N curve under different fatigue conditions.

Several researchers proposed phenomenological models relating the observed stiffness degradation during fatigue to fatigue life.

Hwang and Han [21], [22] proposed a model based on the fatigue modulus concept, i.e. the slope between the maximum stress vs strain at an arbitrary cycle and the origin of the stress-strain relationship. This relation expresses the modulus degradation behaviour and residual strains accumulated during loading cycles.

Yang et al. [23] proposed a power function law. He assumed stiffness degradation rate of Graphite/Epoxy laminates under constant amplitude fatigue to be a power function of the number of cycles.

Post et al [24] proposed a stiffness degradation equation based on the model of Reifsnider [19]. He considered statistical life predictive algorithm as a value mechanism. The strain controlled failure criterion assumes failure to occur as soon as fatigue strain equals the failure strain during the static test.

The Mohr-Coulomb or Von-Mises theory uses the failure criteria under complex stress states for prediction in metals. But the same failure theory cannot be applied for composite materials because of its anisotropic nature. Another proposal by Stowell and Liu [25] and Waddoups [26], suggests some arbitrary criteria, at the two principal directions and in shear of a composite laminate, where the maximum stress or maximum strain at the two principal directions are used to derive the failure expressions in the σ_1, σ_2 and τ_{12} directional coordinates. Hill [27] proposed a failure criterion for metals with anisotropic strength properties induced by extreme deformation of their crystalline structure due tovariations in manufacturing process. Thi model is similar to the one proposed later by Azzi and Tsai [28] which suggests a different normal stress correlation parameter.

Norris [29] proposed a quadratic correlation combining the two stresses in the principal coordinate system and in-plane shear, similar to the expression as that of Hill, omitting the normal stress relation term. Later [30] proposed an additional, term, to account for both effects of normal stress distribution and different strength in tension and compression, Hoffman [31] proposed a relation which correlates both tensile and compressive strengths in the symmetry directions for the orthotropic material.

Several other quadratic or cubic failure functions have been proposed, with many amongst them being different adaptations of the general theory proposed by Tsai-Wu [32].

Based on Tsai-Wu theory, a review of existing methodologies and modeling failure in composite materials was done quite recently by Orifici et al. [33]. A great number of reviews of failure predicting models was presented, distinguishing them in terms of the failure mode predicted like fiber failure, matrix failure, shear failure, delamination failure initiation, ply-by-ply failure, etc.

Many efforts are done by taking into account complex stress states in fatigue of polymer composite materials by incorporating and modifying the criteria for static multiaxial stresses: The failure functions are expressed in terms of the fatigue strength of the composite rather than of its static strength. This concept was used by Owen and Griffiths in their critical review of biaxial stress failure criteria [34].

Hashin and Rotem [35] proposed a fatigue failure criterion for unidirectional laminates under constant amplitude fatigue, which distinguishes between fiber failure and matrix failure modes.

The failure criterion for anisotropic materials proposed by Hill was implemented by Philip et al [36] to predict the fatigue behaviour of tubular specimens under combined axial-torsional fatigue stresses.

Ellyin and El-Kadi [37] proposed a different failure criterion based on strain energy density under cyclic loading. According to Ellyin and El-Kadi, through a power type relation, the strain energy density is expressed as a function of the fatigue life under the plane cyclic stresses.

Reifsneider and Gao [38] developed to implement a micro-mechanical methodology for life prediction under complex stresses using the phylosophy of Hashin and Rotem [35]. The stresses used however are calculated using the Mori-Tanaka method [39] dealing with the problem of the stress field around fibers located inside a matrix.

Fawaz and Ellyin [40] proposed a simplified method for determining off-axis S-N curves, based on a reference S-N curve and the off-axis static strength. This was also pointed out by Awerbuch and Hahn [41], in which the off-axis S-N curves follows similar trend when the data are generalized by the corresponding static strength.

Fujii and Lin [42] implemented the failure criterion of Tsai and Wu to model the fatigue failure line under plane stresses of different bi-axial ratios, which are induced through cyclic tests on tubular specimens combining tension and torsion.

Aboudi [43] proposed an extension of a previously presented micro-mechanical model for static strength of composites [44] under plane stresses. This is based on a reference unit cell, representing a simplified model of the fiber and surrounding matrix region. The stress state condition in the fiber and matrix regions are structured based on the properties of the constituents of the composite material and the correlation between the strength quantities of the constituents and the stress concentration matrix.

The fatigue failure model initially proposed by Hashin and Rotem for off-axis laminates [35] and further developed to account for fatigue failure of angle ply laminates [45] was later extended to the general condition of multidirectional laminates [46]. Use of Classical lamination theory is found to derive all in plane stresses in each lamina, and circumferentially rotated to its principal coordinate system. Three failure modes are distinguished: One for fiber failure, one for matrix failure and one for delaminations.

Lawrence Wu [47] proposed a modified version of the Tsai-Hill failure criterion for tri-axial stress states in order to simulate fatigue in CFRP in finite elements.

Fawaz & Ellyin [48], generalized a law for multidirectional laminates under constant amplitude fatigue.

Again, the methodology developed by Jen and Lee [49], [50], is based on Classical lamination theory approach using the Tsai-Hill failure criterion to account for failure under plane fatigue stresses.

Shokrieh and Lessard proposed the 'generalized 'progressive fatigue damage model' for the simulation of the fatigue process in multidirectional laminates under general 3-D loading conditions [51]. He used the properties of the constituents of the Unidirectional ply laminate. The model is based on a previously proposed methodology for Unidirectional laminates under multiaxial fatigue [52].

First, based on CLT approach, stress analysis was performed for the multidirectional laminate under the current cyclic load. Then, after the stresses and stress ratios at each principal direction of each lamina are defined, the bell-shape CLD proposed by Gathercole et al. [53] was used to derive the number of cycles to failure corresponding to each principal stress component. Having calculated each cyclic stress and fatigue life, then calculations for the residual strength corresponding to the fiber, transverse and shear strength of each layer according to the distribution model of Adam et al. [54] was made. The stiffness characteristics of each ply was evaluated and are assumed to change due to fatigue following a similar model as that of strength degradation. Subsequently, Hashin type failure criteria are used in order to distinguish between different modes of failure, In this, the strength is replaced by the residual strength previously calculated.

Tserpes et al. [55] developed another 3-D progressive damage model in a context similar to that of Shokrieh and Lessard [51]. The failure analysis adopted in this case assumes quadratic failure functions for matrix tensile and compressive cracking, fiber-matrix shear and the two modes of delamination (in tension and compression), In this case, the compression failure correlated to the normal stress behaviour component and out-of plane shear stresses. The sudden degradation rules, due to the different failure modes, are applied only on the stiffness properties of the corresponding ply as a means of stress redistribution after failure. This model was opposite to the one and in contrast to the model of Shokrieh [51] which considered zeroing of both strength and elastic properties, only by assuming to degrade according to constant degradation factors.

Dzenis [56] proposed a stochastic meso-mechanics model. In this, each ply of the multidirectional laminate is attributed stochastic (normally distributed) in-plane elastic characteristics. Based on these and on distributed thickness and orientation angle of each ply, the normal effective properties of the laminate were calculated. Then, the externally applied plane stress directions, assumed to be a quasi-stationary cyclic process, can be used to calculate the laminate strains and strain derivatives.

The author [57], experimented fatigue durability analysis of glass fiber reinforced epoxy composites for damage estimation and loading prediction using fatigue damage index criterion. In the other, based on the time-temperature superposition principle, a degradation behaviour using S-N curve was formulated.

b. Counting Algorithm

Cycle counting is the first procedure in any loading range to be analyzed and processed in terms of constant amplitude fatigue cycles. Several methods are proposed for counting the cycles of a spectrum. A review of such methods are found in [58].

Level Crossing Counting: According to this method the load axis is divided according to a suitable scale number of stress levels. One counting is recorded each time the load exceeds or crosses one. Similarly, when all level crossings are recorded, cycles are formed by constructing, first the largest possible cycle, then the second largest, etc until all level crossings are used. This way the spectral loading is transformed into a series of decreasing amplitude cycles. Once this most damaging-counting has been obtained, then the cycles can be rearranged in a particular order and also secondary load effects are induced.

Peak Counting: In this case, the procedure focuses on the identification of peaks and valleys of the considered spectrum. Once all these have been obtained, the first cycle is constructed by combining the highest peak with the lower valley, etc., until all peaks are used. Both peak and level crossing counting methods yield good results, since these methods focus on the construction of the largest cycles obtained from a specific spectrum. Further, they totally neglect the order of occurrence of each loading time and thus load sequence effects, depending on the shape of the spectrum.

Simple Range Counting: This, is the most simple method and considers a range, i.e. the difference between two successive load reversals, to be one half cycle. Here, the order of occurrence of loading cycles is retained during counting. Large cycles produce a greater impact on fatigue analysis may not be recorded by this counting procedure if they include smaller load fluctuations which will divide them into several smaller ranges.

Rainflow Counting: This name represents a family of methodologies developed from the early 1960s in order to analyze a spectrum into loading cycles as accurately as possible.This is based on stress-strain relations called hysteresis loops occurring inside the material during fatigue. This method was introduced almost simultaneously by Matsuishi and Endo [59] (and in English in [60]) and de Jonge [61] the latter called the algorithm range-pair method.

A number of other several alternatives for counting cycles of stress-strain were proposed by scientists. Amongst them, Dowling proposed the Rainflow (or Range-pair method) to give the most accurate cycle content for fatigue analysis [62]. Actually the two algorithms give very similar results in most situations. One notable thing in this is, while the results are identical the sequence can be rearranged starting with the maximum peak or minimum valley.

Downing and Socie [63] proposed two slightly alternate algorithms. The first one, 'Algorithm I', requires rearrangement of the spectrum so that it starts with its maximum or minimum value. 'Algorithm II' counts a number of peaks and valleys, without rearranging them, in sequence as they occur. The closed loops are recorded immediately after completion while the remaining peaks and valleys are processed again.

Glinka & Kam [64] proposed an alternate Rainflow algorithm for counting long time-series by dividing them into smaller parts called blocks. No spectrum rearrangement is required according to this method.

Rychlik [65] proposed an alternative definition of the rainflow counting process based on comparing each peak in the spectrum with all the signal's previous and following down- and up-crossing respectively, resulting in counting full or half cycles depending on some simple rules. This approach was used by many scientists to implement the rainflow algorithm in the frequency domain.

Another alternative algorithm for counting the cycles using rainflow was proposed by Hong [66]. He proposed that the residual can either be treated as half cycles or rearranged to form with its maximum peak.

Amzallag [67], suggested two alternative ways of treating the residual: First, to either add the residual to itself and extract a number of full cycles (decomposition of residue into cycles) or second, to reconstruct the residual as a loading sequence by inserting cycles into it.

Anthes [68], modified the Rainflow counting, and proposed to keep the load sequence by considering the rising half cycles that do not form a hysteresis loop as 'virtual hysteresis loops' that either close later by a decreasing segment or are combined with subsequent virtual loops. The reason was to keep the damage occurances in an order as accurate as possible.

Bannantine and Socie [69] considered the fatigue process in metals to be driven by shear stresses and used and assumed reasonably to reduce the problem to uniaxial rainflow counting of the maximum shear component.

Wang and Brown [70] suggested a critical plane based approach to propose a multi-axial rainflow algorithm for non-proportional loadings.

Chu & Chu CC, [71] proposed to load in incremental steps, thereby, the incremental damages are analyzed, defining the cycles indirectly using two simple rules for assessing the damage rate with respect to the stress amplitude. By this, the closed hysteresis loops are defined and generalized for multi-axial fatigue stresses, by providing an equivalent stress which is assumed to reduce the problem to a uniaxial one. This was proposed as the flow stress, representing the 'equivalent' hysteretic behaviour under such multi-axial loadings.

Langlais et al. [72] used again a critical plane based approach to reduce the order of the problem in the general case of non-proportional cyclic loadings. According to this modification, a counted cycle can be discarded, by considering and choosing the most damaging one.

Beste et al. [73] proposed a different concept, according to which, cycles are counted for every possible linear combination of the plane stress components. Subsequently, the damage is assessed for each combination and the most critical set of parameters is determined. This method is similar to the one defined for the equivalent stress in which the coefficients of the stress components are optimized for maximum fatigue damage, once rainflow counting is performed on each occurring series of the equivalent quantity. This procedure is computationally cumbersome.

c. Frequency Domain

In general, fatigue calculations of composite structures, undergoing random loads is performed in the time domain. The required input is load-time sequence which are produced by applying random loads of specific characteristics (e.g. stochastic wind loads) to the structure, or component using a simulation procedure. In this procedure a number of difficulties are involved: The first is the realistic values of all the random loading characteristics to the time-load sequence generated. But, this requires the simulation of a long time period in order to cover peaks that are more damaging but less frequent.

A second difficulty, is that fatigue calculations in the time domain are time consuming, especially regarding large structures with complicated geometry where the calculation are performed for large number of points. Further, different fatigue loading conditions must be considered each time. This adds to the complexity and the effort and time involved, and the things become even more complicated when accounting for multi-axial fatigue in laminated structures and progressive damage approaches.

To perform fatigue calculations in the frequency domain, a spectral density function can be defined. When the random input signal itself is available in the form of a power spectral density function, significant time is saved if fatigue calculations are performed directly in the frequency domain. PSD is a scalar function that describes how the power of the time signal is distributed among frequencies, This task requires the expression of loading cycles of different materials during fatigue as a function of the imposed loadings, and the statistical values and calculations of the fatigue damage induced by these cycles based on fatigue properties of the material and a cumulative damage law.

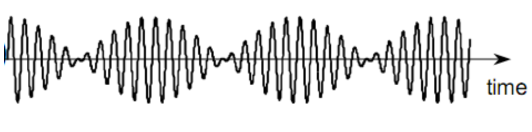
In order to account for the different amount of fatigue damage induced by cycles of different characteristics a damage accumulation rule must be assumed. (e.g. Palmgren-Miner rule).

A new spectral method for deriving the total fatigue damage directly through the signal's PSD, was developed using the probability distribution of the number of cycle ranges, corresponding to the considered spectrum, and subsequently the expected fatigue damage over period T due to the normal loads can be calculated through the S-N curve of the material. The expected rate of stress ranges can be assumed to develop a PSD factor, equal to the rate of peaks in the spectrum. The problem encountered here, is the calculation of the probability distribution of stress ranges corresponding to the signal's PSD.

Even though analytical solutions have been approximated for certain PSD types, the probability distribution of the stress ranges cannot be derived analytically directly from the signal's PSD, especially the ranges counted by Rainflow algorithm. This is the major drawback of spectral methods. Another method is by using the time domain. This is done by simulating long time series based on the stochastic signal and then to perform counting of the ranges obtained in the time domain, using a rainflow algorithm. The result is a simulated PDF for the counted ranges, according to which the fatigue damage can be calculated.

d. Narrow Band Processes

A narrowband process ($a \rightarrow 0$) is characterized by one dominant central frequency. A signal of this type is shown in the following fig. [74], [75].



It can be seen that each peak is followed by a valley of nearly the same amplitude and opposite sign. This implies that the probability density function (PDF) of stress ranges can be approximated by that of peak occurrences, which can be calculated.

Analytically, provided that the input signal is Gaussian and narrow-band, the PDF of stress ranges is proved to follow a Rayleigh distribution depending only on the zero order time [74], [75].

The review of Rayleigh approximation shows that it is formulated and derived under the assumption of Palmgren-Miner rule and log-log S-N curve for composites. The draw back is, the unaccounted sequence effects for different expressions.

Sakrani [76], investigated the effect of S-N curve equations for narrow band stochastic loads in damage accumulation in composites, using the Rayleigh approximation in FRPs. The review showed that, for all S-N curves, predictions were poor compared to the stochastic fatigue tests data, of different types of bolted joints. Regarding the validity of the linear damage accumulation rule, Sakrani et al. [77], investigated the application of different non-linear damage accumulation methods in the time domain, and compared with the validation against the spectral Rayleigh method, showed that the residual strength and stiffness degradation based models were used. Narrow band stochastic stress derivations were simulated, and the predictions appeared to be similar, for all damage rules.

Younesian et al [78], used the Rayleigh approximation method to steel train bogies and the results obtained were compared with a load time series derived through FEM simulation showed that the Rayleigh method results were approximated to the simulation.

When the adjacent peaks are not perfectly correlated the Rayleigh approximation results deviated from the corresponding rainflow count. Yang [79], using the hypergeometric function, generalized the method for non-perfectly correlated signals. Krenk for the same case [80] used a correction factor

e. Broad band processes.

For general cases, for equal peak and cyclic range probability distributions of bimodal or broad-band PSD functions large approximation errors are bound to occur since only one stress range per peak is assumed. This becomes insignificant where a bimodal signal is displayed in the time domain.

Various efforts are put to develop spectral methods correcting this effect for this particular type of PSD in many reviews. Braccesi [82], has recently divided these approaches into three categories. The first, includes the theories that derive coefficients based on spectral moments to generalize the narrow-band approximation to other types of power spectral densities. The second, includes theories which expresses the PDF as a function of basic probability distributions, whose parameters are experimentally found as functions of the spectral moments. Finally, analytical derivations for the PDF for rainflow counted ranges are distinguished.

The proposed methodologies regarding broad-band stochastic methods, primarily distinguishes between (a) those that implement a correction factor to the narrow band approximation, (b) those that use analytical expressions for the PDF of peaks of the counted ranges, (c) combinations of standard distributions and (d) methods for analytically deriving the PDF of the counted cycles.

f. Using a Combination of Distribution

The first category includes semi-empirical methods that approximate the stress range distribution through a combination of basic distributions for the peaks. The most characteristic method in this category is Dirlik's formula [83], used for the purpose, one exponential and two Rayleigh distributions.

Another approximation for the same approach was proposed by Lutes [84]. He approximated the distribution of the peaks of a normal process by a linear combination of a Gaussian and a Rayleigh distribution. This resulted in an incremental fatigue damage.

Sakai and Okamura [85] considered the case of PSD functions with two dominant vibration modes. He used the two Rayleigh distributions to evaluate fatigue life by applying appropriate weighting factors, on the condition that the two frequencies must be well separated.

Fu and Cebon [86] also focused on bimodal PSD functions. Also Sakai et al. assumed two Rayleigh distributions for the ranges corresponding to the two peaks of the PSD.

Zhao and Baker [87], assumed the stress range distribution to be a combination of a Weibull and a Rayleigh distributions. Specific values are given to the Weibull shape and scale parameters depending on the value of the error factor.

g. Counting

Most of the spectral methods are used to solve the problem of deriving the PDF of the stress ranges from a broadband PSD, using different assumptions based on the correlation between the distribution of peaks and the distribution of stress ranges. A number of methods on the other hand, focus directly on the problem of counting the stress ranges from the PSD, for solutions for the theoretical derivation of its PDF. The most adequate solution to the problem incorporates counting of the hysteresis loops applied on the material. This is, possibly, the most accurate way of transforming a random sequence of peaks and valleys to fatigue cycles and is usually performed through rainflow counting. This derivation however is very complex and no analytical solution is proposed.

Another alternative to the rainflow counting is range counting,. This assumes a half cycle count between each peak and valley of the stochastic signal. This half cycle is subsequently paired with a same half cycle of the opposite direction, thus forming one full cycle. In this case, the problem is the derivation of the probability distribution between two adjacent extremes of the spectrum which has been approximated analytically [88], as proposed by Tuna [89]:

Even though many analytical expression are available for the range counting spectrum, as discussed in [90], their result in terms of fatigue damage is inefficient, and leads to non-conservative predictions. This is why special interest is shown in developing techniques to calculate the probability distribution of rainflow-counted stress ranges.

Rychlik [91], proposed a different approach based on this definition of the rainflow process. Rychlik et al. proposed to approximate the probability of each spectrum's peak to satisfy the counting criterion through a Markov chain [92]. Using this assumption one can numerically calculate the conditional probability of the signal crossing a certain level having already crossed it before [93], [94].

Bishop and Sherratt [95] further simplified the process, and proposed to derive the transition probability matrix using the Kowalewski probability density function for the dependence between adjacent peaks, but this was an approximate solution. Olagnon [96], based on the same approach as above, performed computations leading to algorithmic expressions for the elements of the Markov chain transition matrices. This way the probability of each peak satisfying the criteria proposed by Rychlik was calculated based on the construction of a transition (from-to) probability matrix, which was in turn based on the probability of a peak or valley being of a specific level.

3. CONCLUSION

The problem of developing and modeling the damage mechanisms in anisotropic materials during fatigue is much more complicated than for isotropic materials [97]. The correlation of the micro-structural damage and the fatigue performance of general laminates under general loading conditions are now made to be simpler.

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