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Geodetic polynomial and Detour geodetic polynomial of Bistar Graphs

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Abstract: Let $\mathcal{G}$ (G,i) be the family of geodetic sets of the graph G with cardinality i and let $g_e\,(G,i) = \,|\,\mathcal{G}\,(G,i)|$**.** Then the geodetic polynomial $\mathcal{G}\,(G,x)$ of G is defined as $\mathcal{G}\,(G,x) = \sum_{i=\,g(G)}^{|V(G)|} \; g_e\,(G,i)\,x^i$ where $g(G)$ is the geodetic number of G. A graph G for which two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G. In This paper we have determined the radial graph of bistar graphs. The geodetic polynomials and detour geodetic polynomials of bistar graphs are derived and some important results are proved.

Keywords: Distance, Detour geodetic polynomial, Geodetic polynomial, Bistar graph, Radial graph.
AMS Classification: 05C12, 05C60, 05C75

1. INTRODUCTION

In this paper we discuss only finite simple and connected graph. For basic graph theoretical terminology we refer [1]. In [5] the concept of radial graph R(G) is introduced and the characterization for R(G) is proved . The concept of Geodetic polynomials of a graph using Geodetic sets of a graph are introduced in [8]. Geodetic polynomial, Detour geodetic polynomial of some radial graphs are discussed in [6]. Here we have derived some results, on radial graph of bistar graphs and geodetic polynomial, detour geodetic polynomial of bistar graphs .

1.1. Preliminaries

For a graph G, the distance d(u,v) between a pair of vertices u and v is the length of a shortest path joining them. The eccentricity e(u) of a vertex u is the distance to a vertex farthest from u. The radius r(G) of G is defined as the minimum eccentricity of all the vertices of G and the diameter d(G) of G is defined as the maximum eccentricity of all the vertices of G.

A graph G for which r(G) = d(G) is called a self centred graph. Two vertices of a graph are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by R(G), has the vertex set as in G and then two vertices are adjacent in R(G) if and only if they are radial to each other in G.

2. RADIAL GRAPH OF BISTAR GRAPHS

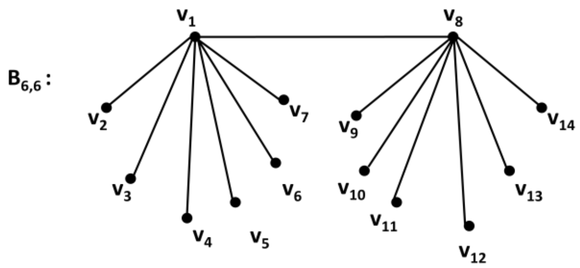
In this section we discuss radial graph of bistar graphs and proved some theorems for finding the radial graphs of bistar graphs **.**

2.1 ***Definition***

The **Bistar graph $B_{n,n}$** graph with 2n+2 vertices obtained by joining the centre(apex) vertices of two copies of complete bipartite graph $K_{1,n}$ by an edge.

2.2 ***Example***

The following is the example for bistar graph $B_{6,6}$.



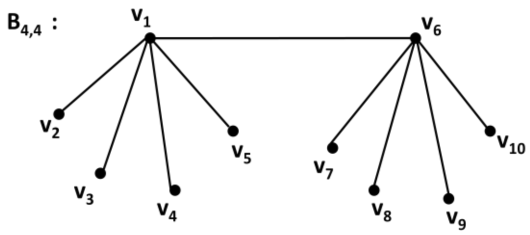
Theorem 2.3

Let $B_{n,n}$ be bistar graph with 2n+2 vertices . Then the radial graph of bistar graph $B_{n,n}$ is $K_{n+1} \cup K_{n+1}$ degree n-2 $n \geq 3$.(i.e) $R(B_{n,n}) = K_{n+1} \cup K_{n+1}$

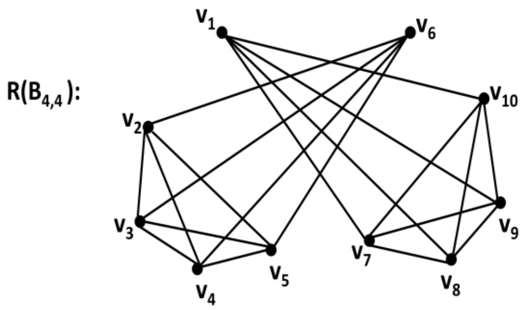
Proof:

Let us prove the theorem by induction on the number of vertices of $B_{n,n}$.

Case (i) Let n=4 then $B_{4,4}$ is a bistar graph with 10 vertices and it will be of the form,

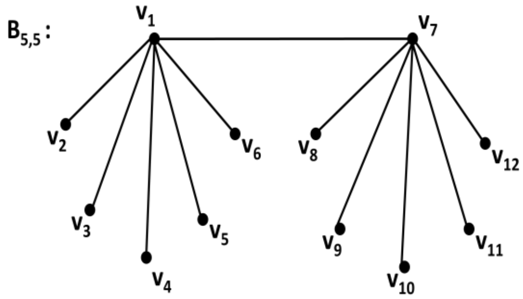


The radial graph of the bistar graph $B_{4,4}$ is

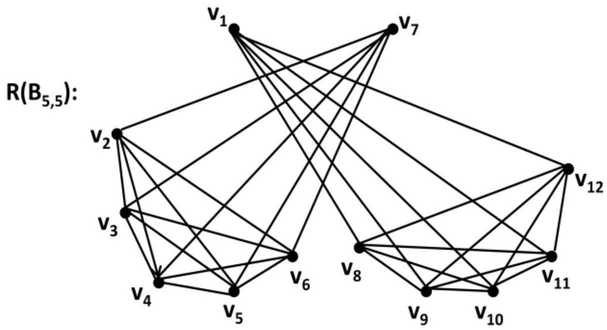


The radial graph of bistar graph contains two copies of complete graph with 4 vertices. Hence the radial graph of the bistar graph is $K_5 \cup K_5$.

Let n=5 then $B_{5,5}$ is a bistar graph with 12 vertices and it will be of the form



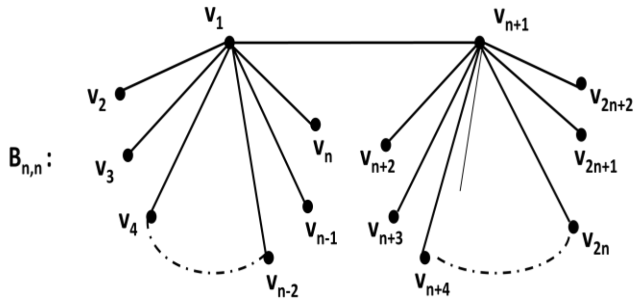
The radial graph of the bistar graph $B_{4,4}$ is



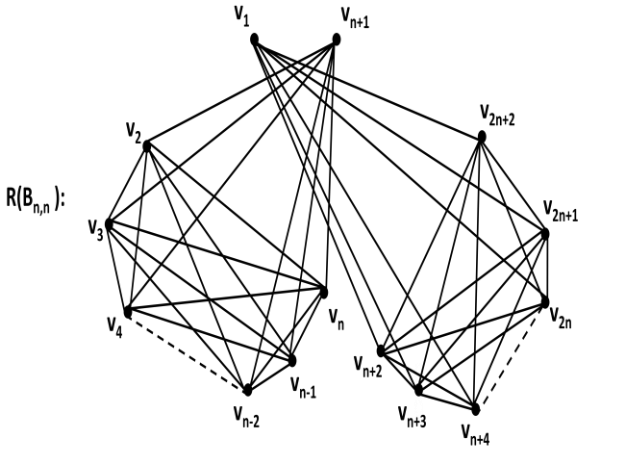
From the above radial graph of bistar graph is the union of two copies of complete graph with 6 vertices. Hence the radial graph of bistar graph is $K_6 \cup K_6$.The theorem is true for n = 4 and n = 5.

Let us assume that the theorem is true for all bistar $B_{n-1,n-1}$ graph with 2n vertices . (i.e) The radial graph of the bistar graph $B_{n-1,n-1}$ is $K_n \cup K_n$.Now we prove the theorem for bistar graph with n vertices.

Let $B_{n,n}$ is the bistar graph with 2n + 2 vertices,it is of the form



The radial graph of the bistar graph $B_{n,n}$ is nn



From the above radial graph . The vertices of the radial graph of bistar graph is splitted into two sets of vertices $\{v_2,v_3,v_4,.......v_{n+1}\}$ and $\{v_1,v_{n+2},v_{n+3},.....,v_{2n+2}\}$.In the first set ,all the vertices are adjacent to other vertices of the set. They make the complete graph with n+1 vertices. In the second set all the vertices is adjacent to other vertices.So they also make a complete graph with n+1 vertices.Hence the radial graph of the bistar graph $B_{n,n}$ is $K_{n+1} \cup K_{n+1}$.

3. GEODETIC POLYNOMIAL OF BISTAR GRAPHS

In this section we discuss geodetic polynomial of bistar graphs

3.1 *Definiton*

Let $\mathcal{G}$ (G,i) be the family of geodetic sets of the graph G with cardinality i and let $g_e\,(G,i) = |\,\mathcal{G}\,(G,i)|$. Then the geodetic polynomial $\mathcal{G}\,(G,x)$ of G is defined as $\mathcal{G}\,(G,x) = \sum_{i=\,g(G)}^{|V(G)|}\, g_e\,(G,i)\,x^i$ where $g(G)$ is the geodetic number of G.

Theorem 3.2

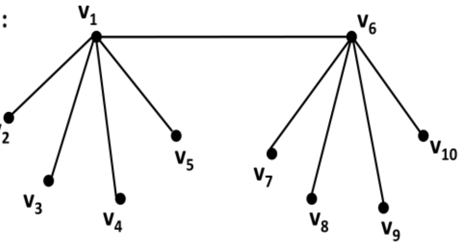
The geodetic polynomial of $B_{n,n}$ is $\mathcal{G}$ ($B_{n,n}$,x) $= x^{2n}\,(x+1)^2$.

Proof.

Let $B_{n,n}$ be a bistar graph with 2n+2 vertices .Without loss of generality we choose $n \geq 3$.
Let $X = \{v_1, v_2, v_3, \ldots\ldots\ldots v_{2n+2}\}$. In the bistar graph there are 2n pendent vertices occur. The only geodetic set with minimum cardinality is 2n in X. Therefore $g_e\left(B_{n,n}, 2n\right) = 1$.The geodetic set with cardinality 2n+1 in X is are $g_e\left(B_{n,n}, 2n+1\right) = 2C_{\,1}$. $g_e\left(B_{n,n}, 2n+2\right) = 2C_{\,2}$.
Therefore
$\mathcal{G}\,(B_{n,n}, x) = x^{2n} + 2nC_1\, x^{2n+1} + 2nC_2\, x^{2n+2}$.
$\mathcal{G}\,(B_{n,n}, x) = x^{2n}\ (x+1)^2$.

3.3 *Example*
Let $B_{4,4}$ is the bistar graph with 10 vertices**.**

$B_{4,4}$:



$g_e\left(B_{4,4}, 8\right) = \{\ v_2\ v_3\ v_4\ v_5\ v_7\ v_8\ v_9\ v_{10}\ \}$
$g_e\left(B_{4,4}, 9\right) = \{\ (v_1\ v_2\ v_3\ v_4\ v_5\ v_7\ v_8\ v_9\ v_{10}),$
$(v_2\ v_3\ v_4\ v_5\ v_6 v_7\ v_8\ v_9\ v_{10})\ \}$
$g_e\left(B_{4,4}, 10\right) = \{\ (v_1\ v_2\ v_3\ v_4\ v_5\ v_6\ v_7\ v_8\ v_9\ v_{10})\ \}$

$$\mathcal{G}\,(B_{4,4}, x) = \sum_{i=\ g(G)}^{|V(G)|} g_e\left(B_{4,4}, i\right) x^i$$

$\mathcal{G}\left(B_{4,4}, x\right) = x^{12} + 2x^{13} + x^{14}$.
The geodetic polynomial of $B_{4,4}$ is
$\mathcal{G}\left(B_{4,4}, x\right) = x^{12} + 2x^{13} + x^{14}$.

Theorem 3.4
The geodetic polynomial of radial graph of bistar graphs is x^{2n+2}.
(i.e) $\mathcal{G}\left(R(B_{n,n}), x\right) = x^{2n+2}$.
Proof
Let $B_{n,n}$ be a bistar graph with 2n+2 vertices.
since $R(B_{n,n}) = K_{n+1} \cup K_{n+1}$,and
$\mathcal{G}\,(K_{n+1}, x) = x^{n+1}$.
$\mathcal{G}\left(R(B_{n,n}), x\right) = \mathcal{G}\,(K_{n+1}, x)\ .\ \mathcal{G}\,(K_{n+1}, x)$
$= x^{n+1}.\,x^{n+1}$
$= x^{2n+2}$.
$\mathcal{G}\left(R(B_{n,n}), x\right) = x^{2n+2}$.

4. DETOUR GEODETIC POLYNOMIAL OF BISTAR GRAPHS

In this section we find detour geodetic polynomial of bistar graphs

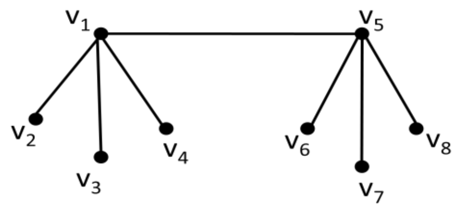
4.1 *Defintion*
Let D $\mathcal{G}$ (G,i) be the family of detour Geodetic sets of the graph G with cardinality i and let $Dg_e\,(G,i) = |D\mathcal{G}\,(G,i)|$. Then the Detour geodetic polynomial $D\mathcal{G}\,(G,x)$ of G is defined as $D\mathcal{G}\,(G,x) = \sum_{i=\ dg(G)}^{dg+(G)} Dg_e\,(G,i)\, x^i$ Where $d_g(G)$ is the Detour number of G.

Theorem 4.2
The Detour geodetic polynomial of the bistar graph $B_{n,n}$ is $n^2(x^2 + x^3)$**.**
i.e $D\,\mathcal{G}\,(B_{n,n}, x) = n^2(x^2 + x^3)$ *, $n \geq 2$.*
Proof:
In the bistar graph $B_{n,n}$ has 2n+2 vertices, $n \geq 2$. It has 2n pendent vertices and 1 cut vertex.
Let $X = \{v_1, v_2, v_3, \ldots\ldots\ldots v_{2n+2}\}$. There is n^2 geodetic sets with cardinality 2 in X , and n^2 geodetic sets with cardinality 3 in X**.** $d_g(G) = 2$ **,** $dg^+(G) = 3$ **.**
Hence the detour geodetic polynomial of the bistar graph is $n^2(x^2 + x^3)$,$n \geq 2$.
i.e $D\,\mathcal{G}\,(B_{n,n}, x) = n^2(x^2 + x^3)$, $n \geq 2$.

Theorem 4.3
The Detour geodetic polynomial of radial graph of the bistar graph $B_{n,n}$ is $\frac{n^2(n+1)^2}{4}\ x^4$**.**
i.e $D\,\mathcal{G}\,(R(B_{n,n}), x) = \frac{n^2(n+1)^2}{4}\ x^4$ *, $n \geq 2$.*
Proof
Let $B_{n,n}$ be a bistar graph with 2n+2 vertices. Since
$R(B_{n,n}) = K_{n+1} \cup K_{n+1}$
$D\,\mathcal{G}\,(R(B_{n,n}), x) = D\,\mathcal{G}\,(K_{n+1}, x)\ .\ D\,\mathcal{G}\,(K_{n+1}, x)$
$= (n+1)C_2\, x^2 . (n+1)C_2\, x^2$
$= \frac{n(n+1)x^2}{2} \cdot \frac{n(n+1)x^2}{2}$
$D\,\mathcal{G}\,(R(B_{n,n}), x) = \frac{1}{4}\, n^2(n+1)^2 x^4$.

4.4 *Example*
Let $B_{3,3}$ is the bistar graph with 8 vertices.

$B_{3,3}$:



$DS_1 = \{\ v_2, v_6\}$, $DS_2 = \{\ v_2, v_7\}$, $DS_3 = \{\ v_2, v_8\}$,
$DS_4 = \{\ v_3, v_6\}$, $DS_5 = \{\ v_3, v_7\}$, $DS_6 = \{\ v_3, v_8\}$,
$DS_7 = \{\ v_4, v_6\}$, $DS_8 = \{\ v_4, v_8\}$, $DS_9 = \{\ v_4, v_7\}$,
$DS_{10} = \{\ v_2, v_5, v_6\ \}$, $DS_{11} = \{\ v_2, v_5, v_7\ \}$,
$DS_{12} = \{\ v_2, v_5, v_8\ \}$, $DS_{13} = \{\ v_3, v_5, v_6\}$,
$DS_{14} = \{\ v_3, v_5, v_7\ \}$, $DS_{15} = \{\ v_3, v_5, v_8\ \}$,
$DS_{16} = \{\ v_4, v_5, v_6\ \}$, $DS_{17} = \{\ v_4, v_5, v_7\ \}$,
$DS_{18} = \{\ v_4, v_5, v_8\ \}$
$Dg_e\left(B_{3,3}, 2\right) = \{\{\ v_2, v_6\}, \{\ v_2, v_7\}, \{\ v_2, v_8\}$,
$\{v_3, v_6\}$,$\{\ v_3, v_7\}, \{\ v_3, v_8\}, \{\ v_4, v_6\}, \{\ v_4, v_8\}$,
$\{\ v_4, v_7\}\ \}$
$Dg_e\,(B_{3,3}\ 3) = \{\ \{\ v_2, v_5, v_6\ \}, \{\ v_2, v_5, v_7\ \}$,
$\{\ v_2, v_5, v_8\ \}, \{\ v_3, v_5, v_6\}, \{\ v_3, v_5, v_7\ \}, \{\ v_3, v_5, v_8\ \}$,
$\{\ v_4, v_5, v_6\ \}, \{\ v_4, v_5, v_7\ \}, \{\ v_4, v_5, v_8\ \}\ \}$
$d_g(B_{3,3}) = 2$,$dg^+(B_{3,3}) = 3$
Detour geodetic polynomial of bistar graph $B_{3,3}$ is
$D\,\mathcal{G}\,(B_{3,3}\ x) = \sum_{i=2}^{2} Dg_e\left(B_{3,3}, i\right) x^i$
$D\,\mathcal{G}\,(B_{3,3}, x) = 9x^2 + 9x^3$
The detour geodetic polynomial of bistar graph $B_{3,3}$ is

$D\,\mathcal{G}\,(B_{3,3}, x) = 9x^2 + 9x^3$.

5. CONCLUSION

Thus in this paper radial graph of bistar graphs and geodetic polynomial, detour geodetic polynomial of bistar graphs have been studied. Further we can find the geodetic polynomial and detour geodetic polynomial of other graphs.

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