

Totally and Slightly Nano $G\delta$ -Continuous Functions

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Abstract- Lellis Thivagar introduced Nano continuous functions in Nano topological spaces and studied some of their properties. Nano $G\delta$ closed sets and Nano δG closed sets are introduced by R. Vijayalakshmi, et., al in Nano topological spaces. Aim of the present paper is we introduce totally Nano $G\delta$ -continuous and slightly Nano $G\delta$ -continuous functions in Nano topological spaces. Also we investigate some of their properties.

Index Terms- Nano $G\delta$ closed sets and Nano $G\delta$ open sets, Totally $G\delta$ -continuous and Slightly Nano $G\delta$ -continuous functions

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1. INTRODUCTION

The concept of Nano topology was introduced by Lellis Thivagar[5] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. Nano $G\delta$ closed sets and Nano δG closed sets are introduced by R. Vijayalakshmi[10], et., al in Nano topological spaces. Lellis Thivagar introduced Nano continuous functions in Nano topological spaces and studies some of their properties. Aim of the present paper is we introduce totally Nano $G\delta$ -continuous and slightly Nano $G\delta$ -continuous functions in Nano topological spaces. Also we investigate some of their properties.

2. PRELIMINARIES

Definition 2.1 [5]:

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$ That is

$$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}. \text{ Where } R(x)$$

denotes the equivalence class determined by $x \in U$.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified

neither as X nor as not- X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [5]: If (U, R) is an approximation space and $X, Y \subseteq U$, then

- i) $L_R(X) \subseteq X \subseteq U_R(X)$
- ii) $L_R(\phi) = U_R(\phi) = \phi$
- iii) $L_R(U) = U_R(U) = U$
- iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- viii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.

$$\text{ix) } U_R(X^c) = [L_R(X)]^c \text{ and } L_R(X^c) = [U_R(X)]^c$$

$$\text{x) } U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$$

$$\text{xi) } L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$$

Definition 2.3 [3]:

Let U be a non-empty, finite universe of objects and R be an equivalence relation on U . Let $X \subseteq U$. Let $\tau_R(X) = N\tau = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$.

Then $\tau_R(X)$ is a topology on U , called as the Nano topology with respect to X .

Elements of the Nano topology are known as the Nano-open sets in U and $(U, N\tau)$ is called the Nano topological space. $[\tau_R(X)]^c$ is called as the dual Nano topology of $N\tau$. Elements of $[N\tau]^c$ are called as Nano closed sets.

Definition 2.4

Let $(U, N\tau)$ be a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then A is said to be

(i). Nano δ -close if $A = Ncl_\delta(A)$, where

$$Ncl_\delta(A) = \{x \in U : Nint(Ncl(Q)) \cap A \neq \phi, Q \in N\tau \text{ and } x \in Q\}.$$

(ii). Nano δG -closed set if $N\delta cl(A) \subseteq Q$ whenever

$$A \subseteq Q, Q \text{ is Nano open in } (U, N\tau).$$

(iii).Nano $G\delta$ -closed set if $Ncl(A)\subseteq Q$ whenever $A\subseteq Q, Q$ is $N\delta$ -open in $(U, N\tau)$.

Definition: 2.5 [4]

A Nano topological space U is said to be connected if U cannot be expressed as the union two disjoint non empty Nano open sets in U .

Definition: 2.6[4]

A Nano topological space U is said to be Nano- $G\delta$ connected if U cannot be expressed as a disjoint union of two non empty Nano- $G\delta$ open sets

Definition: 2.7[4]

A space $(U, N\tau)$ is called a locally indiscrete space if every Nano open set of U is Nano closed in U .

Definition: 2.8 [9]

Every Nano open set is Nano- $G\delta$ open and every Nano closed set is Nano- $G\delta$ closed.

Definition 2.9 [4]

A function $f:(U, N\tau)\rightarrow(V, N\sigma)$ is said to be Nano $G\delta$ -continuous if the inverse image of every open set in $(V, N\sigma)$ is Nano open in $(U, N\tau)$.

3. TOTALLY NANO $G\delta$ -CONTINUOUS FUNCTIONS

Definition: 3.1

A function $f:(U, N\tau)\rightarrow(V, N\sigma)$ is called totally Nano continuous if the inverse image of every Nano open subset of $(V, N\sigma)$ is a Nano clopen subset of $(U, N\tau)$.

Definition: 3.2

A function $f:(U, N\tau)\rightarrow(V, N\sigma)$ is said to be totally Nano- $G\delta$ -continuous, if the inverse image of every Nano open subset of $(V, N\sigma)$ is a Nano- $G\delta$ clopen subset of $(U, N\tau)$

Example :3.3

Let $U = \{a_1, a_2, a_3, a_4\}$ with $U/R = \{\{a_1, a_2\}, \{a_3, a_4\}\}$

Let $X = \{a_1, a_2\} \subseteq U$.

Then $N\tau = \{U, \phi, \{a_1, a_2\}\}$.

Nano $G\delta$ -closed set & Nano $G\delta$ -open set

$= \{U, \phi, \{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_3, a_4\}, \{a_1, a_2, a_3\}, \{a_1, a_3, a_4\}, \{a_1, a_2, a_4\}, \{a_2, a_3, a_4\}\}$

Let $V = \{b_1, b_2, b_3, b_4\}$ with

$V/R = \{\{b_1\}, \{b_2, b_3\}, \{b_4\}\}$

Let $Y = \{b_1, b_3\} \subseteq V$.

Then the Nano topology

$N\sigma = \{V, \phi, \{b_1\}, \{b_2, b_3\}, \{b_1, b_2, b_3\}\}$.

The identity function $f:(U, N\tau)\rightarrow(V, N\sigma)$

Then $f^{-1}(V) = V, f^{-1}(\phi) = \phi, f^{-1}(\{b_1\}) = \{a_1\},$

$f^{-1}(\{b_2, b_3\}) = \{a_1, a_3\}, f^{-1}(\{b_1, b_2, b_3\}) = \{a_1, a_2, a_3\}$

Thus the inverse image of every Nano open set in V is Nano $G\delta$ -clopen in U .

Theorem: 3.4

Every totally Nano continuous functions is totally Nano- $G\delta$ continuous.

Proof:

Let N be an Nano open set of $(V, N\sigma)$. Since f is totally Nano continuous functions, $f^{-1}(N)$ is both Nano open and Nano closed in $(U, N\tau)$ Since every Nano open

set is Nano- $G\delta$ open and every Nano closed set is Nano- $G\delta$ closed, $f^{-1}(N)$ is both Nano- $G\delta$ open and Nano- $G\delta$ closed in

$(U, N\tau)$. Therefore f is totally Nano- $G\delta$ continuous.

Remark: 3.5

The converse of the above theorem need not be true.

Example:3.6

$U = \{a, b, c\}$, with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and

$X = \{a, b\}$ Then the Nano topology,

$N\tau = \{U, \phi, \{a_1, a_2\}, \{a_3\}\}$.

Let $U = \{a_1, a_2, a_3, a_4\}$ with

$U/R = \{\{a_1, a_2\}, \{a_3\}\}$, Let $X = \{a_1, a_2\} \subseteq U$.

Then $N\tau = \{U, \phi, \{a_1, a_2\}\}$.

Nano $G\delta$ -open set =

$\{U, \phi, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_2, a_3\}, \{a_1, a_3\}\}$

Let $V = \{b_1, b_2, b_3\}$ with

$V/R = \{\{b_1\}, \{b_2, b_3\}\}$ Let $Y = \{b_2, b_3\} \subseteq V$.

Then the Nano topology $N\sigma = \{V, \phi, \{b_2, b_3\}\}$

Then the identity function $f:(U, N\tau)\rightarrow(V, N\sigma)$.

$f^{-1}(\{b_1\}) = \{a_1\}$ is totally Nano- $G\delta$ continuous but not totally Nano continuous function

Theorem: 3.7

Every totally $G\delta$ continuous functions is $G\delta$ continuous.

Proof:

Let B be an Nano open set of $(Y, N\sigma)$. since f is totally Nano $G\delta$ continuous functions, $f^{-1}(B)$ is both Nano $G\delta$ - open and Nano $G\delta$ - closed in $(X, N\tau)$. Therefore F is $G\delta$ continuous

Theorem 3.8

If f is a totally Nano- $G\delta$ -continuous function from a Nano- $G\delta$ -connected space U onto any space V , then V is an indiscrete space.

Proof

Suppose that V is not indiscrete. Let A be a proper non-empty Nano open subset of V . Then $f^{-1}(A)$ is a proper non-empty Nano- $G\delta$ -clopen subset of $(U, N\tau)$ which is a contradiction to the fact that U is Nano- $G\delta$ -connected.

Definition 3.9

A space U is said to be Nano- $G\delta$ - T_2 if for any pair of distinct points x, y of U , there exist disjoint Nano- $G\delta$ -open sets M and N such that $x \in M$ and $y \in N$.

Theorem 3.10

A space U is Nano- $G\delta$ - T_2 if and only if for any pair of distinct points x, y of U there exist Nano- $G\delta$ -open sets M and N such that $x \in M$, and $y \in N$ and Nano- $G\delta$ $cl(M) \cap Nano-G\delta cl(N) = \phi$.

Proof

Necessity. Suppose that U is Nano- $G\delta$ - T_2 . Let x and y be distinct points of U . There exist Nano- $G\delta$ -open sets M and N such that $x \in M$, $y \in N$ and $M \cap N = \phi$. Hence Nano- $G\delta$ $cl(M) \cap Nano-G\delta cl(N) = \phi$ and by Definition 3.9, Nano- $G\delta$ $cl(M)$ is Nano- $G\delta$ -open. Therefore, we obtain Nano- $G\delta$ $cl(U) \cap Nano-G\delta cl(N) = \phi$.

Sufficiency. This is obvious.

Theorem 3.11

If $f:(U, N\tau) \rightarrow (V, N\sigma)$ is a totally Nano-G δ continuous injection and V is T_0 then U is Nano-G δ - T_2 .

Proof

Let x and y be any pair of distinct points of U . Then $f(x) \neq f(y)$. Since V is T_0 , there exists a Nano open set M containing say, $f(x)$ but not $f(y)$. Then $x \in f^{-1}(M)$ and $y \notin f^{-1}(M)$. Since f is totally Nano-G δ -continuous, $f^{-1}(M)$ is a Nano-G δ -clopen subset of U . Also, $x \in f^{-1}(M)$ and $y \in U/f^{-1}(M)$. By Theorem 3.10, it follows that U is Nano-G δ - T_2 .

Theorem 3.12

Let $f:(U, N\tau) \rightarrow (V, N\sigma)$ be a totally Nano-G δ -continuous function and V be a T_1 -space. If A is a non-empty Nano-G δ -connected subset of U , then $f(A)$ is a single point.

Definition 3.13

Let $(U, N\tau)$ be a Nano topological space. Then the set of all points y in U such that x and y cannot be separated by a Nano-G δ -separation of U is said to be the quasi Nano-G δ -component of U .

Theorem 3.14

Let $f:(U, N\tau) \rightarrow (V, N\sigma)$ be a totally Nano-G δ -continuous function from a Nano topological space $(U, N\tau)$ into a T_1 -space V . Then f is constant on each quasi Nano-G δ -component of U .

Proof

Let x and y be two points of U that lie in the same quasi-Nano-G δ -component of U . Assume that $f(x) = \alpha \neq \beta = f(y)$. Since V is T_1 , $\{\alpha\}$ is Nano closed in V and so $V/\{\alpha\}$ is an Nano open set. Since f is totally Nano-G δ -continuous, therefore $f^{-1}(\{\alpha\})$ and $f^{-1}(V/\{\alpha\})$ are disjoint Nano-G δ -clopen subsets of U . Further, $x \in f^{-1}(\{\alpha\})$ and $y \in f^{-1}(V/\{\alpha\})$, which is a contradiction in view of the fact that y belongs to the quasi Nano-G δ component of x and hence y must belong to every Nano-G δ -open set containing x . There exists Nano open sets G and A such that $A \subseteq U \subseteq N\text{-int}(N\text{-cl}(G))$.

Definition 3.15

A function $f:(U, N\tau) \rightarrow (V, N\sigma)$ is called Nano G δ -irresolute if $f^{-1}(H)$ is Nano G δ -closed in $(U, N\tau)$ for every Nano G δ -closed set H of $(V, N\sigma)$.

Theorem: 3.16

Let $f:U \rightarrow V$ and $g:V \rightarrow W$ be functions.

Then $g \circ f:U \rightarrow W$.

(i) If f is Nano-G δ -irresolute and g is totally Nano-G δ -continuous then $g \circ f$ is totally Nano-G δ continuous.

(ii) If f is totally Nano-G δ -continuous and g is continuous, then $g \circ f$ is totally Nano-G δ continuous.

Proof:

(i) Let B be an open set in W . since g is totally Nano-G δ -continuous, $g^{-1}(B)$ is Nano-G δ -clopen in V . since f is Nano-G δ -irresolute, $f^{-1}(g^{-1}(B))$ is Nano-G δ -open and Nano-G δ -closed in U .

since $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$.

(ii) Let B be an open set in W . since g is continuous, $g^{-1}(B)$ is open in V . since f is totally Nano-G δ -continuous, $f^{-1}(g^{-1}(B))$ is Nano-G δ -clopen in U . Hence, $g \circ f$ is totally Nano-G δ -continuous

4. SLIGHTLY NANO G δ -CONTINUOUS FUNCTIONS

Definition: 4.1

A function $f:(U, N\tau) \rightarrow (V, N\sigma)$ is called slightly Nano continuous if the inverse image of every Nano clopen set in $(V, N\sigma)$ is open in $(U, N\tau)$.

Definition: 4.2

A function $f:(U, N\tau) \rightarrow (V, N\sigma)$ is said to be slightly Nano-G δ continuous if the inverse image of every Nano clopen set in $(V, N\sigma)$ is Nano-G δ open in $(U, N\tau)$.

Example: 4.3

Let $U = \{a_1, a_2, a_3\}$ with $U/R = \{\{a_1\}, \{a_2, a_3\}\}$

Let $X = \{a_1\} \subseteq U$.

Then $N\tau = \{U, \phi, \{a_1\}\}$.

Nano-G δ -open = $\{U, \phi, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}\}$

Let $V = \{b_1, b_2, b_3\}$ with $V/R = \{\{b_1\}, \{b_2, b_3\}\}$

Let $X = \{b_2, b_3\} \subseteq V$.

Then $N\sigma = \{U, \phi, \{b_2, b_3\}\}$.

Let f is defined by $f(a_1) = b_1, f(a_2) = b_2, f(a_3) = b_3$.

Then the identity function $f:(U, N\tau) \rightarrow (V, N\sigma)$

Then $f^{-1}(V) = U, f^{-1}(\phi) = \phi, f^{-1}(\{b_1\}) = \{a_1\},$

$f^{-1}(\{b_2, b_3\}) = \{a_2, a_3\}$

$f:(U, N\tau) \rightarrow (V, N\sigma)$ is said to be slightly Nano-G δ continuous

Theorem: 4.4

For a function $f:(U, N\tau) \rightarrow (V, N\sigma)$ the following statements are equivalent.

- (i) f is slightly Nano-G δ -continuous.
- (ii) The inverse image of every Nano clopen set N of V is Nano-G δ -open in U .
- (iii) The inverse image of every Nano clopen set N of V is Nano-G δ -closed in U .

- (iv) The inverse image of every Nano clopen set N of V is Nano-G δ -clopen in U .

Proof:

(i) \Rightarrow (ii): Follows from the Definition 4.2.

(ii) \Rightarrow (iii): Let N be a Nano clopen set in V which implies N^c is clopen in V . By (ii), $f^{-1}(N^c) = (f^{-1}(N))^c$ is Nano-G δ -open in U . Therefore, $f^{-1}(N)$ is Nano-G δ -closed in U .

(iii) \Rightarrow (iv): By (ii) and (iii), $f^{-1}(N)$ is Nano-G δ -clopen in U .

(iv) \Rightarrow (i): Let N be a Nano clopen set in V containing $f(x)$, by (iv), $f^{-1}(N)$ is Nano-G δ clopen in U . Take $U = f^{-1}(N)$, then $f(M) \subseteq N$. Hence, f is slightly Nano-G δ -continuous.

Theorem: 4.5

Every slightly Nano continuous function is slightly Nano-G δ -continuous.

Proof:

Let $f:(U, N\tau) \rightarrow (V, N\sigma)$ be a Nano-G δ continuous function. Let N be a Nano clopen set in V . Then, $f^{-1}(N)$ is Nano-G δ -open and Nano-G δ -closed in U . Hence, f is slightly Nano-G δ -continuous.

Remark: 4.6

The converse of the above theorem need not be true as can be seen from the following example

Example :4.7

Let $U = \{a_1, a_2, a_3, a_4\}$ with $U/R = \{\{a_1\}, \{a_2, a_3\}, \{a_4\}\}$.
 Let $X = \{a_1, a_3\} \subseteq U$.
 Then $N\tau = \{U, \phi, \{a_1\}, \{a_2, a_3\}, \{a_1, a_2, a_3\}\}$.
 Let $V = \{b_1, b_2, b_3, b_4\}$ with $V/R = \{\{b_1\}, \{b_2, b_4\}, \{b_3\}\}$.
 Let $Y = \{b_2, b_4\} \subseteq U$. Then, $N\sigma = \{V, \phi, \{b_2, b_4\}\}$.
 Then the identity function $f:(U, N\tau) \rightarrow (V, N\sigma)$ is slightly Nano-G δ -continuous but not slightly Nano continuous function.

Theorem: 4.8

Every contra Nano-G δ continuous function is slightly Nano-G δ continuous.

Proof:

Let $f:(U, N\tau) \rightarrow (V, N\sigma)$ be a contra Nano-G δ continuous function. Let N be a clopen set in V . Then, $f^{-1}(N)$ is Nano-G δ -open in U . Hence, f is slightly Nano-G δ continuous.

Theorem: 4.9

If the function $f:(U, N\tau) \rightarrow (V, N\sigma)$ is slightly Nano-G δ continuous and $(U, N\tau)$ is Nano-G δ $T_{1/2}$ space, then f is slightly Nano continuous.

Proof:

Let N be a Nano clopen set in Z . Since g is slightly Nano-G δ -continuous, $f^{-1}(N)$ is Nano-G δ open in U . Since U is Nano-G δ $T_{1/2}$ space, $f^{-1}(N)$ is Nano open in U . Hence f is slightly Nano continuous.

Theorem: 4.10

Let $f:(U, N\tau) \rightarrow (V, N\sigma)$ be functions. If f is surjective and pre Nano-G δ -open and $(g \circ f):(U, N\tau) \rightarrow (W, N\rho)$ is slightly Nano-G δ continuous, then g is slightly Nano-G δ -continuous.

Proof:

Let N be a Nano clopen set in $(W, N\rho)$. Since $(g \circ f):(U, N\tau) \rightarrow (W, N\rho)$ is slightly Nano-G δ -continuous, $f^{-1}(g^{-1}(N))$ is Nano-G δ -open in U . Since, f is surjective and pre Nano-G δ -open $(f^{-1}(g^{-1}(N))) = g^{-1}(N)$ is Nano-G δ -open. Hence g is slightly Nano-G δ -continuous.

Theorem: 4.11

Let $f:(U, N\tau) \rightarrow (V, N\sigma)$ and $g:(V, N\sigma) \rightarrow (W, N\rho)$ be functions. If f is surjective and pre Nano-G δ -open and Nano-G δ -irresolute, then $(g \circ f):(U, N\tau) \rightarrow (W, N\rho)$ is slightly Nano-G δ continuous if and only if g is slightly Nano-G δ continuous.

Proof:

Let N be a Nano clopen set in $(W, N\rho)$. Since

$(g \circ f):(U, N\tau) \rightarrow (W, N\rho)$ is slightly Nano-G δ continuous, $f^{-1}(g^{-1}(N))$ is Nano-G δ -open in U . Since, f is surjective and pre Nano-G δ open $(f^{-1}(g^{-1}(N))) = g^{-1}(N)$ is Nano-G δ open in V . Hence g is slightly Nano-G δ -continuous.

Conversely, let g is slightly Nano-G δ continuous. Let N be a Nano clopen set in $(W, N\rho)$, then g is Nano-G δ open in V . Since, f is Nano-G δ irresolute, $f^{-1}(g^{-1}(N))$ is Nano-G δ -open in U .

$(g \circ f):(U, N\tau) \rightarrow (W, N\rho)$ is slightly Nano G δ -continuous.

Theorem: 4.12

If f is slightly Nano-G δ continuous from a Nano-G δ -connected space $(U, N\tau)$ onto a space $(V, N\sigma)$ then V is not a discrete space.

Proof:

Suppose that V is a discrete space. Let N be a proper non empty Nano open subset of V . Since, f is slightly Nano-G δ -continuous, $f^{-1}(N)$ is a proper non empty Nano-G δ clopen subset of U which is a contradiction to the fact that U is Nano-G δ -connected.

Theorem: 4.13

If $f:(U, N\tau) \rightarrow (V, N\sigma)$ is a slightly Nano-G δ continuous surjection and U is Nano-G δ connected, then V is connected.

Proof:

Suppose V is not connected, then there exists non empty disjoint N and open sets M and N such that $V = M \cup N$. Therefore, M and N are Nano clopen sets in V . Since, f is slightly Nano-G δ continuous, $f^{-1}(M)$ and $f^{-1}(N)$ are non empty disjoint Nano-G δ open in U and $U = f^{-1}(M) \cup f^{-1}(N)$. This shows that U is not Nano-G δ connected. This is a contradiction and hence, V is connected.

Theorem: 4.14

If $f:(U, N\tau) \rightarrow (V, N\sigma)$ is a slightly Nano-G δ -continuous and $(V, N\sigma)$ is locally indiscrete space then f is Nano-G δ -continuous.

Proof:

Let N be an Nano-open subset of V . Since, $(V, N\sigma)$ is a locally indiscrete space, N is Nano-closed in V . Since, f is slightly Nano-G δ continuous, $f^{-1}(N)$ is Nano-G δ -open in U . Hence, f is Nano-G δ continuous.

5. CONCLUSION

Many different forms of continuous have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, this paper we introduced totally Nano G δ -continuous and slightly Nano G δ -continuous functions in Nano topological spaces. and investigate some of the basic properties. This shall be extended in the future Research with some applications.

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REFERENCES

- [1] S.Chandrasekar, T Rajesh Kannan, M Suresh,, $\delta\omega\alpha$ -Closed Sets in Topological Spaces, Global Journal of Mathematical Sciences: Theory and Practical.9 (2), 103-116.(2017)
- [2] S.Chandrasekar,T Rajesh Kannan,R.Selvaraj , $\delta\omega\alpha$ closed functions and $\delta\omega\alpha$ Closed Functions in Topological Spaces, International Journal of Mathematical Archive, 8(11), 1-6,2017.
- [3] M.LellisThivagar and Carmel Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1(1)(2013),31-37.
- [4] M.LellisThivagar and Carmel Richard, On Nano continuity, Mathematical Theory and Modeling, 3(7)(2013).
- [5] S. Jeyashri,S. Chandrasekar ,G. Ramkumar, Totally Nano Sg Continuous Functions And Slightly Nano Sg Continuous Functions In Nano Topological Spaces,International Journal of Mathematical Archive-9(5),2018,13-18.
- [6] O.Ravi, S. Ganesan and S. Chandrasekar, On Totally sg-Continuity, Strongly sg- Continuity and Contra sg-Continuity, Gen. Math. Notes, Vol. 7, No 1, Nov (2011), pp. 13-24.
- [7] O.Ravi,S.Chandrasekar,S.Ganesan.($\tilde{\delta}$,s) continuous functions between topological spaces. Kyungpook Mathematical Journal, 51 (2011), 323-338.
- [8] O.Ravi,S.Chandrasekar, S.Ganesan On weakly πg -closed sets in topological spaces. Italian journal of pure and applied mathematics (36), 656-666,(2016).
- [9] R.Vijayalakshmi,Mookambika,.A.P.,Nano δG -Closed Sets in Nano Topological Spaces ,International Journal for Research in Engineering Application & Management (IJREAM) (communicated).
- [10] R. Vijayalakshmi ,Mookambika.A.P.,Some Properties Of Nano δG -Closed Sets In Nano Topological Spaces, Journal of Applied Science and Computations,. Volume V, Issue XII, December/2018 ,1260-1264.
- [11] R. Vijayalakshmi ,Mookambika.A.P., Nano δg -Interior And Nano δg -Closure In Nano