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Optimization of Fuzzy Inventory Model with Minimum Transportation to Reduce Carbon Emissions

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Abstract: In this paper, an inventory model is offered under fuzzy atmosphere. Our objective is to decrease the transportation cost so as to diminish the carbon emissions. We consider holding cost, setup cost, ordering cost and carbon emission cost are trapezoidal fuzzy numbers. Ranking function technique is used for defuzzification. Numerical example is provided to illustrate the suggested model.

Keywords: Carbon Emissions, Function Principle, Ranking Function Method, Trapezoidal Fuzzy Numbers.

1. INTRODUCTION

The fitness threats of air pollution are tremendously serious. For lone air ascendancy intensify respiratory infirmity like bronchitis & asthma, reinforce the venture of life threatening ambience like cancer, and burdens our health care system with extensive medical costs.

Emissions of many air toxins have been shown to have variety of undesirable possessions on civic strength and the normal environment. There are emissions of carbon by burning oil, coal, and gas .carbon dioxide also leftover materials and from some industrial processes. Such as cement production etc. Transportation plays a vigorous protagonist in carbon emissions nowadays. By minimizing the transportation, control carbon emissions are controlled from which it would contribute a little for the maintenance of the green environment in our atmosphere.

Cardenas Barron (2007) established a multi stage supply chain model and he used an algebraic approach to solve the model.) by The above mentioned two authors did not consider the variable transportation and carbon emission cost. Park el al (2011) adduced an optimal shipment strategy for inexact items in a stock out situation.

A fuzzy conviction is a premise of which the restraint of requisition can vary appreciably in consonance with to conditions or contexts, preferably being agreed once for all. Further in a series of papers, Yao et al considered the fuzzified problems for the inventory with or without backorder models.

In this paper, we recommend an inventory model to control the carbon emissions using fuzzy conceptions. The solution is discussed in both crisp and fuzzy sense.

2. DEFINITIONS AND METHODOLOGIES

Fuzzy Set: A fuzzy set $\tilde{A}$ in a universe of discourse X is defined as the following set of pair's $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ .Here $\mu_{\tilde{A}} : X \to [0,1]$ is a mapping called the membership value of $x \in X$ in a fuzzy set $\tilde{A}$.

Trapezoidal Fuzzy Numbers: A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is symbolized with membership function $\mu_{\tilde{A}}$ as

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a}{b-a}, a \le x \le b \\ 1, b \le x \le a \\ R(x) = \dfrac{d-x}{d-a}, a \le x \le d \\ 0, otherwise \end{cases}$$

Ranking Function Method: A ranking function $R : f(R) \to R$ which maps each fuzzy numbers to the real line: f(R) signifies the set of all Trapezoidal fuzzy numbers. If R be any linear ranking function, then

$$R(\tilde{A}) = \left(\frac{a_1 + a_2 + a_3 + a_4}{4} \right)$$

Arithmetic Operations using Function Principle:

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then

- Addition
  $$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are any real numbers.

- Multiplication
  $\tilde{A}\otimes\tilde{B} = \left(a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4\right)$
- Subtraction
  $\tilde{A}\Theta\tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$
  where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$ are any real numbers.
- Division $\tilde{A}\varnothing\tilde{B} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$
- Let $\alpha \in R$ then,

$$\alpha \geq 0, \alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4)$$

$$\alpha < 0, \alpha \otimes \tilde{A} = \left(\alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1\right)$$

- The square root is defined as

$$\sqrt{\tilde{A}} = \sqrt{(a_1, a_2, a_3, a_4)} = \left(\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_3}, \sqrt{a_4}\right)$$

Notations:

D - Demand rate
P – Production rate
d – Distance in km
F – Fuel Cost per unit distance
H – Holding cost per unit per unit time
S – Setup cost
A – Ordering cost
T – Transportation cost per unit distance
C – Carbon emission cost
T_C - Total Cost
Q – Order quantity per unit time
$\tilde{H}$ - Fuzzy Holding cost per unit per unit time
$\tilde{S}$ - Fuzzy Setup cost
$\tilde{A}$ - Fuzzy Ordering cost
$\tilde{T}$ - Fuzzy Transportation cost per unit distance
$\tilde{C}$ - Fuzzy Carbon emission cost
$\tilde{T}_C$ - Fuzzy Total Cost
$\tilde{Q}$ - Fuzzy Order quantity per unit time

Assumptions:

- Single type of products model is considered.
- Holding cost is increasing in the downstream direction.
- Carbon emission is increased due to the number of transportation increased.

Mathematical Model in Crisp Sense:

The total cost is given by

$$T_C = \frac{Q}{2}\left[HD + \left(1 - \frac{P}{D}\right)A\right] + \frac{1}{Q}\left[AD + S + \left(dF + T\right)C\right] + CD$$ -------------------------- (1)

By partially differentiating equation (1) w.r.t. Q, equating it to zero, we get the optimal order quantity

$$Q^* = \sqrt{\frac{2\left[\left(AD + S\right) + \left(dF + T\right)C\right]}{\left[HD + \left(1 - \frac{P}{D}\right)A\right]}}$$ -- (2)

3. MATHEMATICAL MODEL IN FUZZY SENSE

Now, we derive the same mathematical model in fuzzy environment. Here, the holding cost, setup cost, ordering cost, carbon emission cost and transportation cost are taken in fuzzy nature.

Hence, the fuzzy total cost is given by

$$\tilde{T}_C = \frac{Q}{2}\left[\tilde{H}D + \left(1 - \frac{P}{D}\right)\tilde{A}\right] + \frac{1}{Q}\left[\tilde{A}D + \tilde{S} + \left(dF + \tilde{T}\right)\tilde{C}\right] + \tilde{C}D$$ ------------------------- (3)

Suppose

$$\tilde{H} = \left(h_1, h_2, h_3, h_4\right)$$
$$\tilde{S} = \left(s_1, s_2, s_3, s_4\right)$$
$$\tilde{A} = \left(a_1, a_2, a_3, a_4\right)$$
$$\tilde{T} = \left(t_1, t_2, t_3, t_4\right)$$
$$\tilde{C} = \left(c_1, c_2, c_3, c_4\right)$$

From equation (3), we have

$$\tilde{T}_C = \frac{Q}{2}\left[\left(\tilde{H}\otimes D\right)\oplus\left(1 - \frac{P}{D}\right)\otimes\tilde{A}\right]\oplus\frac{1}{Q}\left[\left(\tilde{A}\otimes D\right)\oplus\tilde{S}\oplus\left[\left(dF\otimes\tilde{C}\right)\oplus\left(\tilde{T}\otimes\right.\right.\right.$$ --(4)

By substituting the trapezoidal fuzzy numbers in the above equation and simplifying, we get

$$\tilde{T}_C = \begin{pmatrix} \frac{Q}{2}\left[\left(h_1 D\right)+\left(1-\frac{P}{D}\right)a_1\right]+\frac{1}{Q}\left[a_1 D+s_1+\left[dFc_1+t_1c_1\right]\right]+c_1 D, \\ \frac{Q}{2}\left[\left(h_2 D\right)+\left(1-\frac{P}{D}\right)a_2\right]+\frac{1}{Q}\left[a_2 D+s_2+\left[dFc_2+t_2c_2\right]\right]+c_2 D, \\ \frac{Q}{2}\left[\left(h_3 D\right)+\left(1-\frac{P}{D}\right)a_3\right]+\frac{1}{Q}\left[a_3 D+s_3+\left[dFc_3+t_3c_3\right]\right]+c_3 D, \\ \frac{Q}{2}\left[\left(h_4 D\right)+\left(1-\frac{P}{D}\right)a_4\right]+\frac{1}{Q}\left[a_4 D+s_4+\left[dFc_4+t_4c_4\right]\right]+c_4 D \end{pmatrix}$$

------------------------- (5)

By applying ranking function method, we are going to defuzzify equation (5). Hence, after defuzzification, equation (5) becomes

$$\tilde{T}_C^* = \frac{Q}{8}\left[\left(h_1+h_2+h_3+h_4\right)D+\left(1-\frac{P}{D}\right)\left(a_1+a_2+a_3+a_4\right)\right]+$$
$$\frac{1}{4Q}\left[\begin{array}{l}\left(a_1+a_2+a_3+a_4\right)D+\left(s_1+s_2+s_3+s_4\right)+ \\ \left[dF\left(c_1+c_2+c_3+c_4\right)+\left(t_1c_1+t_2c_2+t_3c_3+t_4c_4\right)\right]\end{array}\right]$$
$$+\frac{D}{4}\left(c_1+c_2+c_3+c_4\right)$$

----------------------------------- (6)

= F (q) say.

F (q) is minimum when $\frac{\partial F(q)}{\partial Q}=0$ and when $\frac{\partial^2 F(q)}{\partial Q^2}>0$.

Now, differentiating equation (6) w.r.t. Q, we get

$$\frac{\partial F(q)}{\partial Q} = \frac{1}{8}\left[\left(h_1+h_2+h_3+h_4\right)D+\left(1-\frac{P}{D}\right)\left(a_1+a_2+a_3+a_4\right)\right]-$$
$$\frac{1}{4Q^2}\left[\begin{array}{l}\left(a_1+a_2+a_3+a_4\right)D+\left(s_1+s_2+s_3+s_4\right)+ \\ \left[dF\left(c_1+c_2+c_3+c_4\right)+\left(t_1c_1+t_2c_2+t_3c_3+t_4c_4\right)\right]\end{array}\right]$$

------------------------------ (7)

Equating equation (7) to zero, we get

$$\tilde{Q}^* = \sqrt{\frac{2\left[\left(a_1+a_2+a_3+a_4\right)D+\left(s_1+s_2+s_3+s_4\right)+dF\left(c_1+c_2+c_3+c_4\right)+\left(t_1c_1+t_2c_2+t_3c_3+t_4c_4\right)\right]}{\left[\left(h_1+h_2+h_3+h_4\right)D+\left(1-\frac{P}{D}\right)\left(a_1+a_2+a_3+a_4\right)\right]}}$$

------- (8)

This shows that $\frac{\partial^2 F(q)}{\partial Q^2}>0$.

Therefore, equation (8) gives the fuzzy optimal order quantity and equation (6) gives the fuzzy total cost for the given period [0, t].

4. NUMERICAL EXAMPLE

i. In Crisp Sense:

Let D = 1500 units per year
P = 1250 units per year
d = 350 km
F = Rs. 4 per km
H = Rs. 5 per unit
S = Rs. 500
A = Rs. 300
T = Rs. 100 per unit
C = Rs. 0.2

Then, the optimal order quantity

$$Q^* = \sqrt{\frac{2\left[\left(AD+S\right)+\left(dF+T\right)C\right]}{\left[HD+\left(1-\frac{P}{D}\right)A\right]}}$$

$$Q^* = 10.927$$

And, the total cost

$$T_C^* = \frac{Q}{2}\left[HD+\left(1-\frac{P}{D}\right)A\right]+\frac{1}{Q}\left[AD+S+\left(dF+T\right)C\right]+CD$$

$$T_C^* = Rs.82805.09/-$$

ii. In Fuzzy Sense:

Let D = 1500 units per year
P = 1250 units per year
d = 350 km
F = Rs. 4 per km
$\tilde{H}$ = (2, 4, 6, 8)
$\tilde{S}$ = (200, 400, 600, 800)
$\tilde{A}$ = (100, 200, 400, 500)
$\tilde{T}$ = (70, 80, 120, 130)
$\tilde{C}$ = (0.1, 0.2, 0.3, 0.4)

Then, the fuzzy optimal order quantity

$$\tilde{Q}^* = \sqrt{\frac{2\left[\left(a_1+a_2+a_3+a_4\right)D+\left(s_1+s_2+s_3+s_4\right)+dF\left(c_1+c_2+c_3+c_4\right)+\left(t_1c_1+t_2c_2+t_3c_3+t_4c_4\right)\right]}{\left[\left(h_1+h_2+h_3+h_4\right)D+\left(1-\frac{P}{D}\right)\left(a_1+a_2+a_3+a_4\right)\right]}}$$

$$\tilde{Q}^* = 10.928$$

And, the fuzzy total cost

$$\tilde{T}_C^* = \frac{Q}{8}\left[\left(h_1+h_2+h_3+h_4\right)D+\left(1-\frac{P}{D}\right)\left(a_1+a_2+a_3+a_4\right)\right]+$$
$$\frac{1}{4Q}\left[\begin{array}{l}\left(a_1+a_2+a_3+a_4\right)D+\left(s_1+s_2+s_3+s_4\right)+ \\ \left[dF\left(c_1+c_2+c_3+c_4\right)+\left(t_1c_1+t_2c_2+t_3c_3+t_4c_4\right)\right]\end{array}\right]$$
$$+\frac{D}{4}\left(c_1+c_2+c_3+c_4\right)$$

$$\tilde{T}_C^* = Rs.82887.2/-$$

5. CONCLUSION

A mathematical model is derived to calculate economic order quantity and the total cost for the given period in both crisp and fuzzy senses. Then to minimize the costof transportation and to reduce the carbon emissions so as to maintain a green environment. The suggested model is verified using numerical examples. This work can be established in future for more investigation interpretations.

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