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Unsteady MHD flow of a viscoelastic fluid with Oscillating temperature, Chemical reaction and Radiation effect in a Porous Medium

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Abstract: The paper researches about the impact of radiation parameter and chemical reaction of unsteady MHD flow of a viscoelastic fluid in a porous medium with oscillating temperature. The flow is assumed to be incompressible electrically conducting and transmitting viscoelastic fluid in the presence of uniform magnetic field. A uniform magnetic field is applied normal to the plate. The velocity, temperature and concentration distributions are derived, solved analytically and their profiles for various physical parameters are appeared through graphs. The coefficient of Skin friction, Nusselt number and Sherwood number are determined.

Keywords: Viscoelastic; porous medium; magnetohydrodynamics; chemical reaction.

1. INTRODUCTION

A magnetohydrodynamic generator is a device that changes thermal energy and kinetic energy into electricity. MHD fluid flow in different geometries relevant to many interesting and important scientific because of essential applications in medical sciences and industrial technology. The MHD study has considerable interest in specialized field like MHD power generators, cooling of nuclear reactors, liquid metal flow control, micro MHD pumps, high temperature plasmas, biological transportation, drying processes and solidification of binary alloy etc. The interesting new problem generates from their importance in liquid metals, electrolytes and ionized gases. On account of their varied importance, these flows have been studied by several authors: Aboeldahab and Elbarby (2001) examined hall current effect on MHD free convection flow past a semi -infinite vertical plate with mass transfer. Chitra.M and suhasini.M (2018) examined the Effect of unsteady oscillatory MHD flow through a porous medium in porous vertical channel with chemical reaction and concentration. Effect of Heat and Mass transfer on free convection flow near a infinite vertical plate embedded in a porous medium, which moves with time dependent velocity in a viscous, electrically conducting incompressible fluid under the influence of uniform magnetic field, applied normal to the plate was studied by K. Das and S. Jana (2010). Das (2010) investigated the exact solution of MHD free convection flow and mass transfer near a moving vertical porous plate in the presence of thermal radiation. Radiation and mass transfer effects on MHD flow through porous medium past an exponentially accelerated inclined plate with variable temperature was studied by Jyotsna Rani Pattnail et.al.,(2017). Omokhuale. E and Onwuka.G.S (2012) investigated about the effect of mass transfer and hall current on unsteady MHD flow of a viscoelastic fluid in a porous medium. Effect of Hall current and chemical reaction and Hydromagnetic flow of a stretching vertical surface with internal heat generation/ absorption was examined by M.Salem and Mohamed Abd El-Aziz (2008). The effect of Hall current on MHD mixed convective flow of a viscous incompressible fluid past a vertical porous plate immersed in a porous medium with heat source/ sink studied by Sharma et al (2007). Shateyi et al (2010) worked on the effect of thermal radiation, hall current, Soret and Dufour on MHD flow by mixed convection over a vertical surface in porous medium. . Singh et al (2013) explained about the Hall current effect on viscoelastic MHD oscillatory convective flow through a porous medium in a vertical channel with heat radiation. Takhar (2006) studied about unsteady flow free convection flow over an infinite vertical porous plate due to the combined effects of thermal and mass diffusion, magnetic field and hall current.

2. MATHEMATICAL FORMULATION

We have considered an unsteady MHD two dimensional flow of a viscoelastic fluid which is incompressible and electrically conducting with oscillating temperature and mass transfer over a infinite porous plate. Let x-axis is assumed to be vertically upward direction of flow along the plate and y-axis is taken normal to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is very small, so that the produced magnetic field is insignificant. Likewise it is supposed that there is no applied voltage, so that the electric field is vanished. The plate is assumed to be electrically non-conducting with uniform magnetic field, B_0 is applied normal to the plate and the entire region of the fluid are at same temperature T_∞ and concentration C_∞. The governing equation for this investigation is based on the balance of linear momentum energy. All the fluid properties are considered to be constant except the influence of the density variation caused by the temperature changes in the body force term. Taking into consideration the assumption made above the flow field is governed by the following set of equations.

The governing equations for the momentum, energy and concentration are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + v\frac{\partial \bar{u}}{\partial \bar{y}} = \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - k_1\frac{\partial^3 \bar{u}}{\partial \bar{y}^2 \partial \bar{t}} - \sigma B_0{}^2\frac{(u+m\omega)}{\rho(1+m^2)} + g\beta(\bar{T} - \bar{T}_\infty) + g\beta^*(\bar{C} - \bar{C}_\infty) - \frac{\nu u}{k^*} \tag{1}$$

$$\frac{\partial \bar{\omega}}{\partial \bar{t}} + v\frac{\partial \bar{\omega}}{\partial \bar{y}} = \upsilon\frac{\partial^2 \bar{\omega}}{\partial \bar{y}^2} - k_1\frac{\partial^3 \bar{\omega}}{\partial \bar{y}^2 \partial \bar{t}} - \sigma B_0{}^2\frac{(\omega - mu)}{\rho(1+m^2)} - \frac{u\bar{\omega}}{k^*} \tag{2}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + v\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{K_T}{\rho Cp}\frac{\partial^2 \bar{T}}{\partial y^2} - \frac{1}{\rho Cp}\frac{\partial q_r}{\partial \bar{y}} - \frac{Q_0}{\rho Cp}(\bar{T} - \bar{T}_\infty) \tag{3}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + v\frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + D_1\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - K'_c(\bar{C} - \bar{C}_\infty) \tag{4}$$

The boundary conditions for the problem are:

$u = 0$, $\omega = 0$, $T = T_\infty + (T_\omega - T_\infty)\, e^{int}$, $C = C_\infty + (C_\omega - C_\infty)\, e^{int}$ at $y \to 0$

$$u \to 0,\ \omega \to 0,\ T \to 0,\ C \to 0, \text{ as } y \to \infty \tag{5}$$

where u and v are the components of velocity in the x and y direction respectively, g is the acceleration due to gravity, β and β* are the coefficient of volume expansion, K is the kinematic viscoelasticity, ρ is the density, μ is the viscosity, υ is the kinematic viscosity, K_T is the thermal conductivity, Cp is the specific heat in the fluid at constant pressure, σ is the electrical conductivity of the fluid, μ_e is the magnetic permeability, D is the molecular diffusivity, D_1 is the thermal diffusivity, T_ω is the temperature of the plane and T_∞ is the temperature of the fluid far away from plane. C_ω is the concentration of the plane and C_∞ is the concentration of the fluid far away from plane, K'_c is the chemical reaction. And $v = -v_0$, the negative sign indicates that the suction is towards the plane.

By using the Rosseland approximation, the radioactive flux vector q_r can be written as:

$$q_r' = \frac{4\sigma^*}{3k_1'}\frac{\partial T_W'^4}{\partial y'}$$

where σ^* and k_1' are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. Assume that the temperature difference within the flow is sufficiently small such that
T'^4 may be expressed as linear function of the temperature. This is accomplished by expanding in a Taylor series about the free stream temperature T_∞' and neglecting higher order terms, thus

$$T_w'^4 \cong 4T_\infty'^3 T_w' - 3T_\infty'^4 \tag{6}$$

In view of eq. (5) and (6), eq. (3) reduces to:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + v\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{K_T}{\rho Cp}\frac{\partial^2 \bar{T}}{\partial y^2} + \frac{1}{\rho Cp^{\infty}}\frac{16\sigma^* T'}{3k_1'}\frac{\partial^2 \bar{T}}{\partial y^2} \tag{7}$$

Introducing the following non-dimensionless parameters:

$$\eta = \frac{v_0 \bar{y}}{\upsilon},\ t = \frac{v_0{}^2 \bar{t}}{4\upsilon},\ u = \frac{\bar{u}}{v_0},\ \omega = \frac{\bar{\omega}}{v_0},\ \theta = \frac{\bar{T} - T_\infty}{T_\omega - T_\infty},\ C = \frac{\bar{C} - C_\infty}{C_\omega - C_\infty},\ Gr = \frac{g\beta\upsilon(T_\omega - T_\infty)}{v_0{}^2},\ Gc = \frac{g\beta\upsilon(C_\omega - C_\infty)}{v_0{}^2},\ M = \frac{\sigma B_0{}^2\upsilon}{\rho v_0{}^2},\ Pr = \frac{\mu Cp}{K_T},\ Sc = \frac{\upsilon}{D},\ Sc_1 = \frac{\upsilon_1}{D_1},\ K = \frac{k_1 v_0{}^2}{4\upsilon},\ k = \frac{k^* v_0{}^2}{\upsilon^2},\ K_c = \frac{\upsilon K'_c}{v_0{}^2},\ F = \frac{Q_0}{\rho Cp v_0{}^2},\ N = \frac{k_1' k}{4\sigma^* T_\infty'^3} \tag{8}$$

Substituting the dimensionless variables in eq.(10) into eq. (3), eq.(4), eq.(6) and eq. (9) and dropping the bars

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \eta^2} - \frac{K}{4}\frac{\partial^2 u}{\partial \eta^2 \partial t} - \frac{M(u+m\omega)}{(1+m^2)} - \frac{u}{k} + Gr\theta + GcC \tag{9}$$

$$\frac{1}{4}\frac{\partial \omega}{\partial t} - \frac{\partial \omega}{\partial \eta} = \frac{\partial^2 \omega}{\partial \eta^2} - \frac{K}{4}\frac{\partial^2 \omega}{\partial \eta^2 \partial t} - \frac{M(\omega - mu)}{(1+m^2)} - \frac{u}{k} \tag{10}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial \eta} = \frac{1}{Pr}\left(1 + \frac{4}{3N}\right)\frac{\partial^2 \theta}{\partial \eta^2} - F\theta \tag{11}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \frac{\partial C}{\partial \eta} = \frac{1}{Sc}\frac{\partial^2 C}{\partial \eta^2} + \frac{1}{Sc_1}\frac{\partial^2 C}{\partial \eta^2} - K_C c \tag{12}$$

The corresponding boundary conditions are

$u(0, t) = 0$, $\omega(0,t) = 0$, $\theta(0,t) = e^{int}$, $C(0,t) = e^{int}$ at $y \to 0$

$$u(\infty, t) = \omega(\infty,t) = \theta(\infty,t) = C(\infty,t) = 0 \text{ as } y \to \infty \tag{13}$$

Eq. (9) and eq. (10) can be obtained into a single equation by introducing the complex velocity.

$$U=u(\eta,t)+i\omega(\eta,t) \tag{14}$$

where $i = \sqrt{-1}$

Thus,

$$\frac{1}{4}\frac{\partial U}{\partial t} + \frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial \eta^2} - \frac{K}{4}\frac{\partial^2 U}{\partial \eta^2 \partial t} - \frac{M(1-m\omega)U}{(1+m^2)} - \frac{U}{k} + Gr\theta + GcC \tag{15}$$

with boundary conditions:

$U(0,t) = 0$, $\theta(0,t) = e^{int}$, $C(0,t) = e^{int}$ at $\eta \to 0$

$$U(\infty,t) = \theta(\infty,t) = C(\infty,t) \to 0 \text{ as } \eta \to \infty \tag{16}$$

where Gr is the thermal Grashof number, Gc is the mass Grashof number, Sc is the Schmidt number, Pr is the Prandtl number, K is the viscoelastic parameter, M is the Hartmann number and k is the permeability, K_r is the chemical reaction number and N is the radiation parameter.

3. METHOD OF SOLUTIONS

In order to obtain the analytical solution of the system of differential eq. (11), eq. (12) and eq. (15) subject to boundary conditions eq. (16) we shall use the perturbation technique

$$U(\eta,t)=U_0+U_1(\eta)e^{int} \tag{17}$$

$$\theta(\eta,t)= \theta_0+\theta_1(\eta)e^{int} \tag{18}$$

$$C(\eta,t)=C_0+C_1(\eta)e^{int} \tag{19}$$

Substituting eq. (17) to eq. (19) into eq. (11), eq. (12) and eq. (15) and comparing harmonic and non-harmonic terms, we obtain

$$P_5U_0''+U_0' - P_6U_0 = - Gr\theta - GcC \tag{20}$$

$$U_1'' + \frac{U_1'}{P_1} - P_3U_1 = -\frac{Gr\theta}{P_1 e^{int}} - \frac{GcC}{P_1 e^{int}} \tag{21}$$

$$P_4\theta_0'' + \theta_0' - F\theta_0 = 0 \tag{22}$$

$$P_4\theta_1''+\theta_1'-\left(\frac{in}{4} + F\right)\theta_0=0 \tag{23}$$

$$P_7C_0''+C_0'-K_CC_0=0 \tag{24}$$

$$P_7C_1''+C_1'-P_8C_1=0 \tag{25}$$

and boundary conditions give

$U_1(0) =0$, $\theta_1(0) = C_1(0) = 1$ at $\eta \to 0$

$U_1(\infty) \to 0$, $\theta_1(\infty) \to 0$, $C_1(\infty) \to 0$ as $\eta \to \infty$ (26)

where the primes represents differentiation with respect to η.

Solving eq. (20) to eq. (25) subject to the boundary conditions eq. (26) substituting the obtained solutions into eq. (17) to eq. (19) respectively.

Then the velocity field can be expressed as

$$U(\eta,t) = [\, A_5e^{-m_5\eta} + A_1e^{-m_3\eta} + A_2e^{-m_4\eta}] + [A_6e^{-m_6\eta} + A_3e^{-m_4\eta} + A_4e^{-m_2\eta}]\, e^{int} \tag{27}$$

And, the temperature field is given by

$$\theta(\eta,t)=e^{-m_3\eta}+e^{-m_4\eta}.e^{int} \tag{28}$$

Similarly, the concentration distribution gives

$$C(\eta,t)=e^{-m_1\eta}+e^{-m_2\eta}.e^{int} \tag{29}$$

The Skin friction, Nusselt number and Sherwood number is obtained by differentiating

eq. (27) to eq. (29) and evaluated at η = 0 respectively.

$$-\left.\frac{\partial U(\eta,t)}{\partial \eta}\right|_{\eta=0} =[\, m_5A_5 + m_3A_1 + m_4A_2 \,] + [\, m_6A_6 + m_4A_3 + m_2A_4 \,] \tag{30}$$

$$-\left.\frac{\partial \theta(\eta,t)}{\partial \eta}\right|_{\eta=0} =m_3+ m_4e^{int} \tag{31}$$

$$-\left.\frac{\partial C(\eta,t)}{\partial \eta}\right|_{\eta=0} =m_1 + m_2e^{int} \tag{32}$$

4. RESULT AND DISCUSSION

The impact of Hall current, heat source and chemical reaction on unsteady MHD flow of a viscoelastic fluid through porous medium filled in a vertical channel is analysed. Using various parameter like Prandtl number, Grashoff number, Schmidt number , radiation effect, heat flux etc the flow of the fluid computations are performed on velocity, temperature and concentration profiles are demonstrated graphically.
Figure 1 shows the velocity profile decrease with increase in prandtl number. In a significant number

of the heat transfer problems, the prandtl number controls the relative thickness of the momentum and thermal boundary layers**.** Figure 2 and Figure 3 displays the velocity profiles for various values of Schmidt number and it observed that velocity profile decrease with increase in Sc and Sc1. This is physically true because, the Sc is a dimensionless number which is the ratio of momentum diffusivity and mass diffusivity. Figure 4 portrays the estimation of n increases, the velocity decreases. Figure 5 signifies the radiation parameter effect on velocity, as radiation increases, velocity profile decreases. Figure 6 shows the different values of m increases while decreasing the velocity profile. Figure 7 demonstrates the different values of M to be noted as Hartmann number, while velocity decreases with increase of Hartmann number. Figure 8 presents the velocity profiles for different values of viscoelastic parameter K, velocity is found to be decreasing with the increase in viscoelastic parameter. In Figure 9 effect of thermal Grashof number on velocity is presented. As Gr increases, the velocity decreases. This is due to the buoyancy which is acting on the fluid particle. Figure 10 is noticed that in presence of mass Grashof number which increases the fluid velocity as its value decreases.

Figure 11 speaks to the dimensionless temperature profiles for different values of prandtl number with constant chemical reaction and permeability parameter. It is clear that the temperature of the fluid decreases as prandtl number increases. It is seen from Figure 12 that the temperature decreases with increase in heat source/sink. From Figure 13 the radiation parameter increasing with the decrease of temperature. The impact of Schmidt number Sc and Sc1 on the concentration profiles is shown in Figure 14 and Figure 15. It is observed that the species concentration decreases as the Schmidt number increases. Effect of chemical reaction on concentration are presented in Figure 16. From this figure it is noticed that the concentration boundary layer shrink when the values of chemical reaction parameter increases.

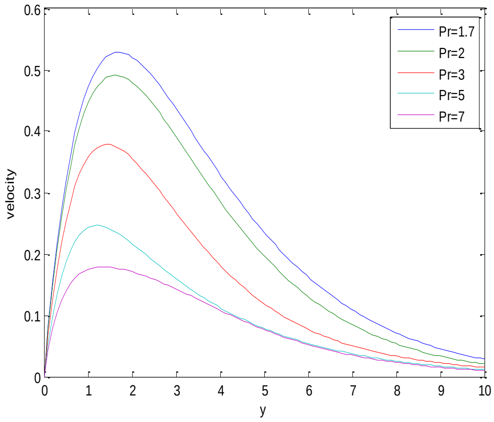

Figure 1. Velocity profile for different values of Pr

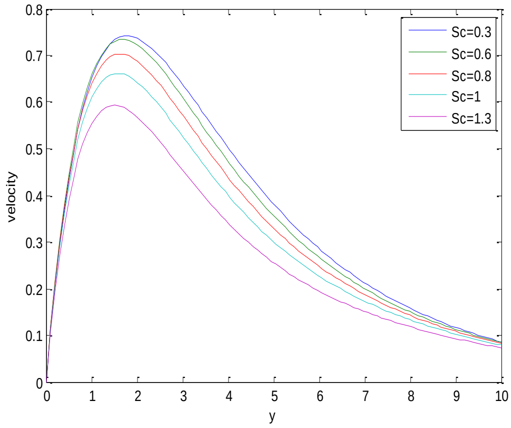

Figure 2. Velocity profile for different values of Sc

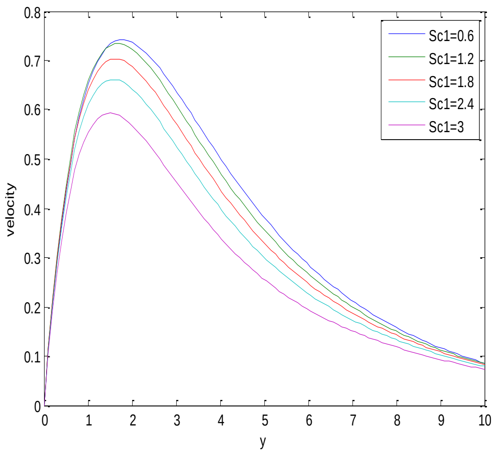

Figure 3. Velocity profile for different values of Sc1

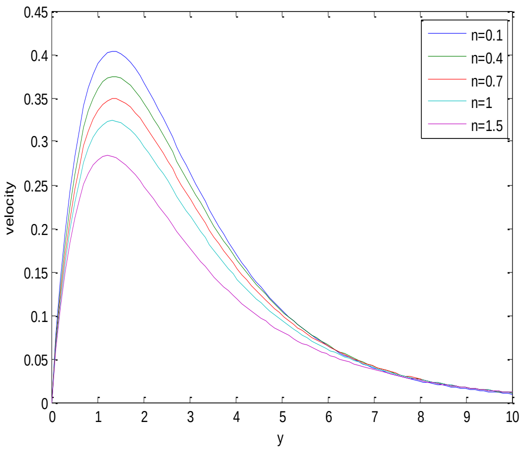

Figure 4. Velocity profile for different values of n

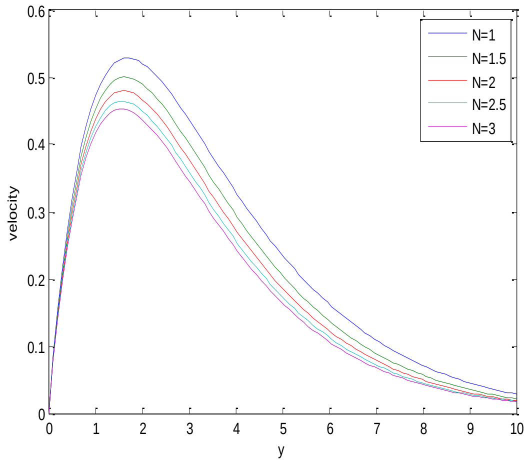

Figure 5. Velocity profile for different values of N

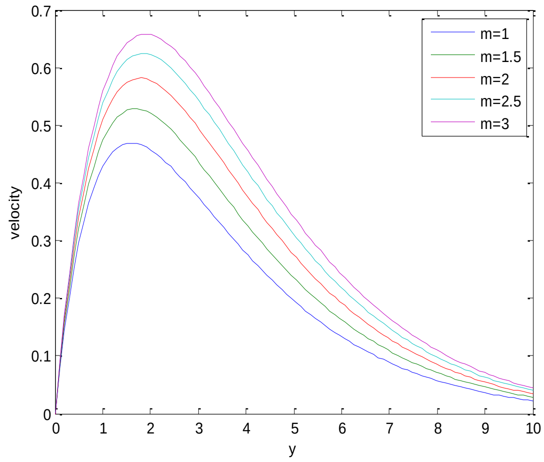

Figure 6. Velocity profile for different values of m

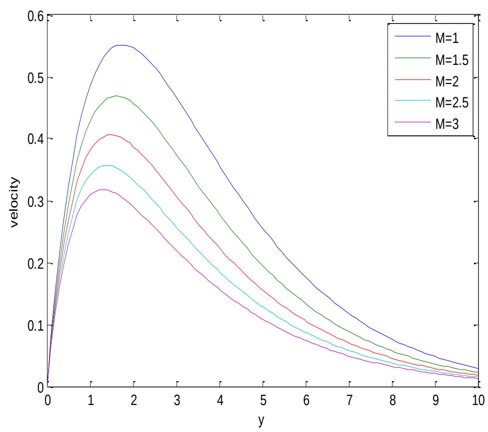

Figure 7. Velocity profile for different values of M

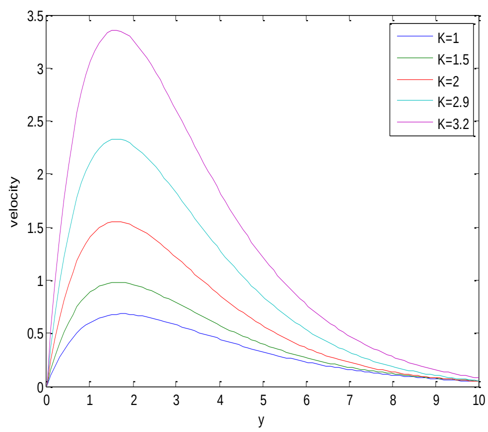

Figure 8. Velocity profile for different values of K

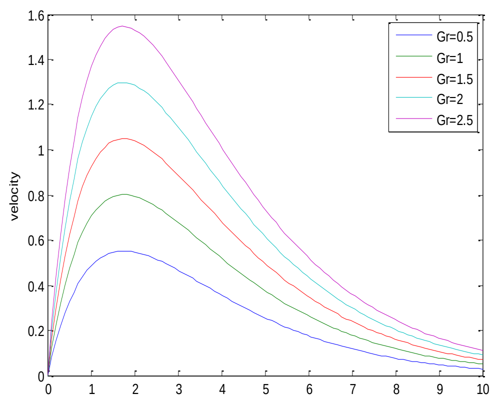

Figure 9. Velocity profile for different values of Gr

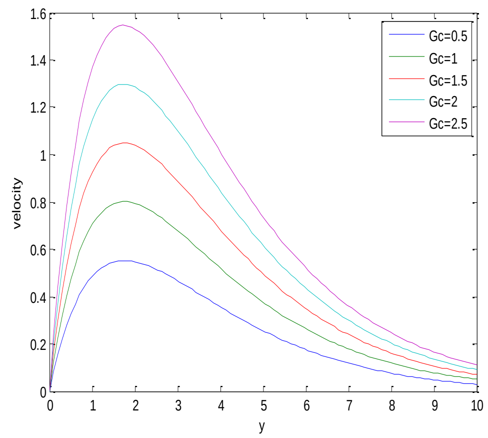

Figure 10. Velocity profile for different values of Gc

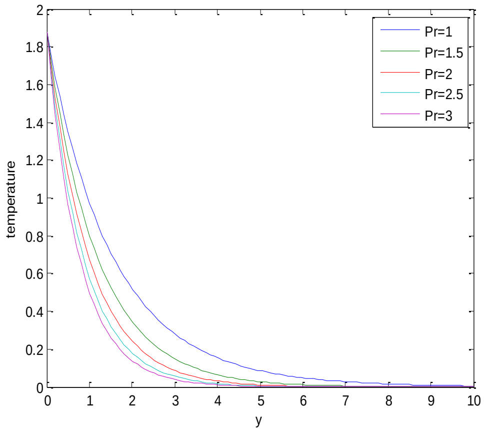

Figure 11. Temperature profile for different values of Pr

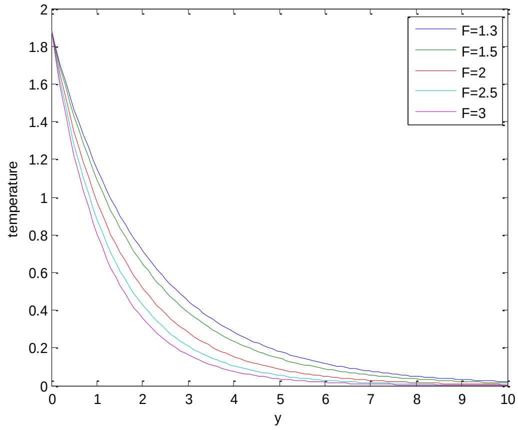

Figure 12. Temperature profile for different values of F

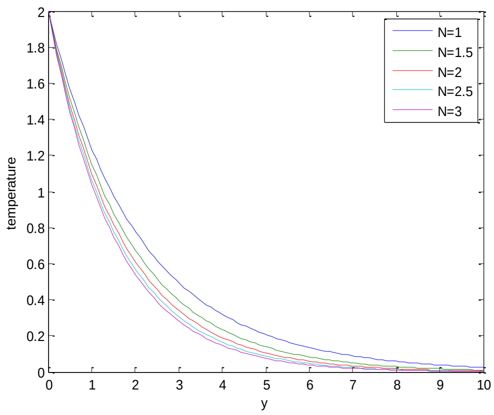

Figure 13. Temperature profile for different values of N

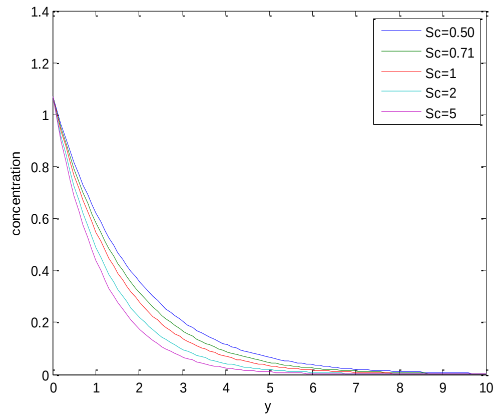

Figure 14. Concentration profile for different values of Sc

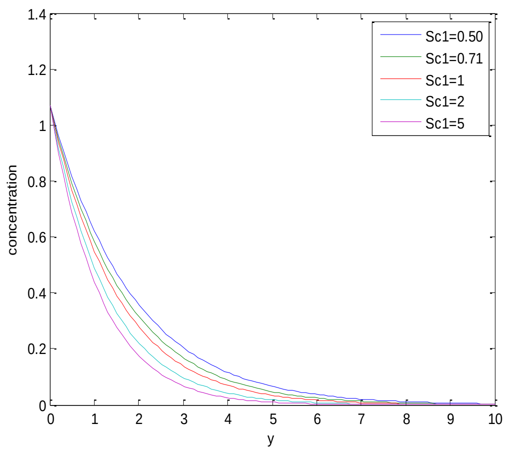

Figure 15. Concentration profile for different values of Sc1

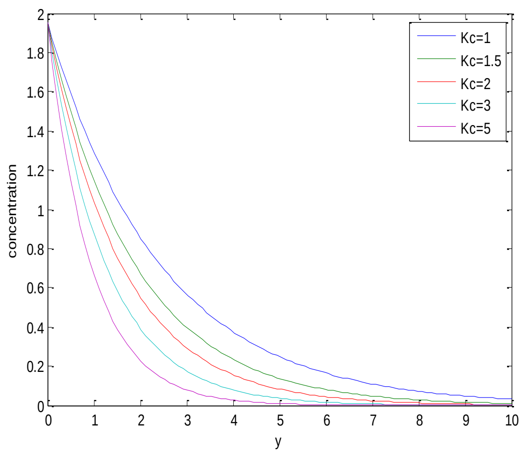

Figure 16. Concentration profile for different values of Kc

5. CONCLUSION

We have analysed and solved diagnostically the effects of MHD flow of viscoelastic fluid through porous medium in the presence of vertical channel with Chemical reaction and Heat Source. The governing equations are approximated to a system of non-linear ordinary differential equation with the similar transformations. The result is presented graphically and the flow field is concluded. It is observed that the velocity increase with increase of Hartmann number, Radiative Heat flux, Hall parameter, Chemical reaction and permeability. The temperature profile decreases with decrease of Heat source and Radiative heat flux and the concentration profile decreases with decrease in Chemical reaction and Schmidt number.

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