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A controller design using Fractional Order Sliding Mode for Quadcopter

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Abstract- This paper incorporates design and synthesis of fractional order sliding mode-based controller for Quadcopter. The Quadcopter having six degrees of freedom is stabilized with respect to parametric uncertainty and provides the great robustness to the quadcopter under disturbances. In comparison to PI based controllers the transient behavior of FOSMC is improved and the impact of same will lead to reduce tracking response error. The designing of FOSMC controller proves it's optimal and robust behavior that has reduced reachability time, improved transient response and high robustness to disturbances. The controller stability is test by lyapunov stability criteria. To reflect disturbance during simulation the parametric inputs are changed accordingly to check the converge of error sliding surface. The entire controller calculation is exhibited according to the error sliding surface which is based on variable structure dynamics. The controller is further tested through Simulink tool in MATLAB. The results show that FOSMC provide better execution with lower parametric sensitivity and strong withstanding towards parameter varieties and unsettling influences.

Index Terms- *Sliding Mode Control, lyapunov stability criteria, Advance Control theory, Simulink MATLAB, Quadcopter (UAV).*

1. INTRODUCTION

Quadcopter, also called Unmanned Aerial Vehicles (UAV), Quadrotor or Quadrotor Helicopter, is widely used in various fields, in both civilian markets and military. For external disturbances UAV is vulnerable, nonlinear and unstable. It introduces control dynamics where the vulnerabilities on variable dynamic driving and physical parameters as the automaton make it appealing and testing control issue. Distinctive techniques and systems have prescribed to control Quadcopter. The following methodologies used: [1] Fuzzy Logic (FL) control. [2] Linear quadratic regulator control (LQR). [1-4] Proportional Derivative Integral (PID) control.

SMC (Sliding Mode Control) is a clear generous framework which licenses arranging controllers for immediate and for non-linear strategies [6-8]. Sliding Mode Control has ended up being picked one answer for a few, valuable diagrams for control, for instance, equipment contraptions, mechanical self-governance, engineered systems [8, 9]. Sliding Mode Control involves in given two segments: immediately, a violated law control to actualize the misstep vector decision toward oversee, known as sliding surface, in the midst of the accomplishing stage. This section control the trading on a particular sliding surface side condition what's more, when the sliding surface is bound to the misstep vector, by the conditions depicting the sliding surface a corresponding bit of the controller exhibitions to take after the movement constrained [6].

There are a couple of works of SMC associated with following and bearing errands for Quadcopters, let us make a rapidly indicate of them, for events: [10], a flexible back stepping algorithm is used to sliding mode control design. [11], in this paper; the key thought relies upon the dynamic showing of Quadcopter considering high-organize non-holonomic objectives. For the amalgamation of following botches and Lyapunov limits, a sliding mode controller is gotten by using the Back stepping approach. [12] Controller based on feedback linearization with a sliding mode high-organize onlooker. The observation goes about as a passerby and external disrupting impacts affect the estimator. [13] The controller in perspective of HOSMC (high demand sliding mode control) is proposed for bearing after. [14] Back venturing and the control designs are shown and considered as sliding mode. [15] A sliding mode control is created which is direct adaptable. The developed control system is associated by not considering parameter vulnerabilities and pondering aggravations. The arrangement of the modification laws relies upon Lyapunov plot standard. [16] The makers show a control system algorithm gotten by isolating the diagram in the two phases: immediately, a DSMC (dynamic sliding mode control system) is developed and furthermore a high get observation is considered. [17] This article is proposed a control plot in perspective of backstepping and its mix with fluffy system for jabbering diminish.

The target of this paper involves in arranging a FOSMC and applying it by reenactment in a Quadcopter. For vertical take-off and landing (VTOL) plane an ERROR sliding surface is considered, similarly changes in points are done, and incorporate a few aggravations. As needs be, as the sliding surface the controller can be executed utilizing an ERROR controller and including some factor based math the whole controller count is shown on the grounds that: The FOSMC is used to expel structure vulnerabilities and lessen the span of control prattling. . Diverged from customary entire number demand sliding mode control, the high-repeat jabbering of the control information can be made to drastically demoralize. FOSMC is pertinent for both single-input single-output system and a normal multi-input multi-output structure Global quality of the close circle control system can be deductively shown using Lyapunov security theory while Sliding mode control (SMC) any straight sliding surface can guarantee the asymptotic soundness and needed execution of the close circle control structures. Regardless, dull trading input control get causes the settling time of the close circle control system to extend, suggesting that the structure state can't accomplish the adjust point in a constrained time. Furthermore, SMC offers high-repeat chattering of control input, which prompts undesirable loads on control actuators.

The paper is classified as. In Section II Dynamics of Quadcopter, Section IV Fractional order control design, Section V Fractional calculations, Section VI Results, Section VII conclusions

2. DYNAMICS OF QUADCOPTER

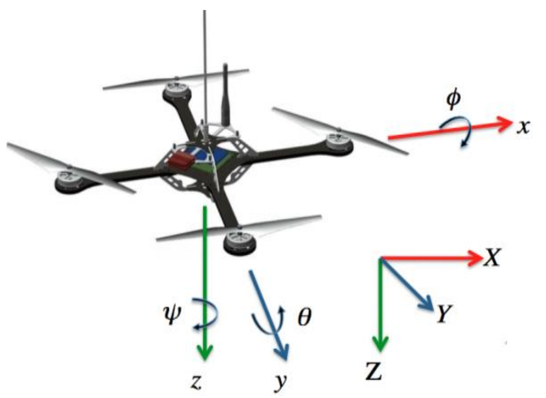

Fig. 1, Quadcopter dynamics and axes system

The dynamics and working of the Quadcopter is shown above. The motion equations are;
The Quadcopter has twelve equations of motions, which are following:

$$X=[x\dot{x}y\dot{y}z\dot{z}\o\dot{\o}\theta\dot{\theta}\Psi\dot{\Psi}] \quad (1)$$

Where, $\dot{x}$ $\dot{y}$ and $\dot{z}$ are the speed in the x, y, and z axes. $\dot{\o}$ $\dot{\theta}$ $\dot{\Psi}$ Parameters are the Speed for roll, pitch and yaw. x, y and z are the location in the x, y, and z axes. Ø, Ψ and θ are the roll, yaw and pitch angles

$$\ddot{z} = -g + \frac{\cos(\o).\cos(\theta)}{m}U_1 \quad (2)$$

$$\ddot{y} = \frac{\cos(\o).\sin(\theta).\sin(\Psi) - \sin(\o).\cos(\Psi)}{m}U_1 \quad (3)$$

$$\ddot{x} = \frac{\cos(\o).\sin(\theta).\cos(\Psi) + \sin(\o).\sin(\Psi)}{m}U_1 \quad (4)$$

$$\ddot{\o} = \dot{\o}\,\dot{\Psi}\,(\frac{Iy-Iz}{Ix}) - \frac{Jr}{Ix}\dot{\theta}\text{€} + \frac{l}{Ix}U_2 \quad (5)$$

$$\ddot{\theta} = \dot{\o}\,\dot{\Psi}\,(\frac{Iz-Ix}{Iy}) + \frac{Jr}{Iy}\dot{\theta}\text{€} + \frac{l}{Iy}U_3 \quad (6)$$

$$\ddot{\Psi} = \dot{\theta}\,\dot{\o}\,(\frac{Ix-Iy}{Iz}) + \frac{l}{Iz}U_4 \quad (7)$$

Where, Ix, Iy and Iz are the inertia of the Quadcopter in 'x', 'y' and 'z' individually, Jr is the inertia of the quadcopter and m is the mass of the quadcopter, The info flag U1is the aggregate drag of the copters.U2, U3 and U4 Are the moments for pitch, roll and yaw separately.

3. FRACTIONAL ORDER CONTROL DESIGN

A control procedure called fractional-order terminal SMC (FO-TSMC) was presented for a class of indeterminate dynamical frameworks. In light of the Lyapunov solidness hypothesis, a fractional-order exchanging complex was proposed to ensure the sliding condition. This procedure guaranteed limited time solidness for the shut circle framework. FOSMC conspire was proposed for control of a solitary connection adaptable controller. The exchanging surface can be developed in view of the fractional subsidiaries in the differential condition. The FOSMC won't just give better execution with little control gabbing, yet in addition vigorous even with outer load unsettling influence and parameter varieties. The FOSMC is utilized to dismiss framework vulnerabilities and lessen the size of control gabbing. FOSMC is appropriate for both single-input single-yield framework and an average multi-input multi-yield frameworks Global solidness of the shut circle control framework can be scientifically demonstrated utilizing Lyapunov strength hypothesis. Contrasted with regular whole number order sliding mode control, the high-recurrence gabbing of the control information can be made to definitely discourage.

Four control conditions are utilized to keep the quadcopter on the reference an incentive notwithstanding outer aggravations. The flag U1 is utilized to ensure that the elevation takes after the reference esteem, in spite of the fact that the signs U2, U3 and U4 are utilized for control movement, pitch and yaw of the structure. Looking conditions (2-7), we have four input control signals. The action of these data signals makes the quadcopter moves advances, backward, to the other side, to the other side, upwards or down.

As specified some time recently, the sliding surface or choice manage must be chosen. This choice is made in view of execution criteria, because of the sliding surface condition decides the flow of framework.

Thus, it will be:

$$s= \dot{e} + \lambda_{RL}\, D^{\alpha-1}(\text{sign}\,(e)^{r}) \tag{8}$$

Where, λ is a tuning design parameter λ > 0.

And sign represents signum function which means for any parameter x is

$$\text{Signum}(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

Taking derivative of equation (12)

$$\dot{s}= \ddot{e} + \lambda_{RL}\, D^{\alpha}\, (\text{sign}\,(e)^{r}) \tag{9}$$

But, $\ddot{e} = \ddot{z}_d - \ddot{z}$, therefore

$$\dot{s}= (\ddot{z}_d - \ddot{z}) + \lambda_{RL}\, D^{\alpha}\, (\text{sign}\,(e)^{r}) \tag{10}$$

And $\dot{s}$ = -k sign(s)

Where, k > 0, which gives $\dot{s} \le 0$ at any value of s, which means system is stable.

Now putting the value of $\dot{s}$ and $\ddot{z}$ in equation (14)

$$-k\,\text{sign}(s) = \ddot{z}_d + g - \frac{\cos(\emptyset).\cos(\theta)}{m}U_1 + \lambda_{RL}\, D^{\alpha}\, (\text{sign}\,(e)^{r}) \tag{11}$$

Rearranging the terms to calculate value of U_1

Therefore,

$$U_1= \frac{m}{\cos(\emptyset).\cos(\theta)}(\ddot{z}_d + g + k\,\text{sign}(s) + \lambda_{RL}\, D^{\alpha}\, (\text{sign}\,(e)^{r})) \tag{12}$$

Similarly from rest of the equations, the values of U_2, U_3 and U_4 can be determined.

$$-k\,\text{sign}(s) = \ddot{\emptyset}_d - \dot{\emptyset}\dot{\Psi}\, (\frac{Iy-Iz}{Ix}) + \frac{Jr}{Ix}\dot{\theta}€ - \frac{l}{Ix}U_2 + \lambda_{RL}\, D^{\alpha}\, (\text{sign}\,(e)^{r}) \tag{13}$$

$$U_2= \frac{Ix}{l}[\ \ddot{\emptyset}_d - \dot{\emptyset}\ \dot{\Psi}\ (\frac{Iy-Iz}{Ix}) + \frac{Jr}{Ix}\dot{\theta}€ + k\,\text{sign}(s) + \lambda_{RL}\, D^{\alpha}\, (\text{sign}\,(e)^{r})] \tag{14}$$

$$U_3 = \frac{Iy}{l}[\ddot{\theta}_d - \dot{\emptyset}\ \dot{\Psi}\ (\frac{Iz-Ix}{Iy}) - \frac{Jr}{Iy}\dot{\theta}€ + k\,\text{sign}(s) + \lambda_{RL}\, D^{\alpha}\, (\text{sign}\,(e)^{r})] \tag{15}$$

$$U_4= \frac{Iz}{l}[\ddot{\Psi}_d - \dot{\theta}\dot{\emptyset}\ (\frac{Ix-Iy}{Iz}) + k\,\text{sign}(s) + \lambda_{RL} D^{\alpha}(\text{sign}(e)^{r})] \tag{16}$$

The equations for input signals are given below;

$$U_1 = p\ (€_1^2 + €_2^2 + €_3^2 + €_4^2) \tag{17}$$

$$U_2 = p\ (- €_2^2 + €_4^2) \tag{18}$$

$$U_3 = p\ (-€_1^2 + €_3^2) \tag{19}$$

$$U_4 = p\ (-€_1^2 + €_2^2 - €_3^2 + €_4^2) \tag{20}$$

The angular velocity for each copter is €1, €2, €3, €4. *l* is the separation between the copter and the focal point of the Quadcopter and p is the drag factor.

Now from the equations (17), (18), (19) and (20) the € factor can be written as :

$$\begin{bmatrix} €1^2 \\ €2^2 \\ €3^2 \\ €4^2 \end{bmatrix} = \begin{bmatrix} p & p & p & p \\ 0 & -p & 0 & p \\ -p & 0 & p & 0 \\ -d & d & -d & d \end{bmatrix} * \begin{bmatrix} U1 \\ U2 \\ U3 \\ U4 \end{bmatrix}$$

And final value of € is:

€ = - €1 + €2 - €3 + €4

Lyapunov Stability of system;

W is taken as Lyapunov Function which should be positive for stability, now by using error sliding surface "s"

$W = \frac{1}{2}s^2$

This means W > 0, due to square term of sliding surface.

Now, according to the stability criteria $\dot{w}$ should be negative

$$\dot{w} = s\dot{s} = s\,(-k\ \text{sign}(s))$$

Where k > 0 for t > 0, this means $\dot{w} < 0$

This satisfies the Lyapunov Stability.

4. FRACTIONAL CALCULATIONS

Fractional investigation has been known since the change of the entire number request math, however for a long time, it is assumed as a sole numerical issue. In late decades, fractional examination has transformed into an intriguing subject among systems examinations and control fields.

4.1. *Mathematical Analysis of fractional calculus*

Fractional math is a speculation of whole number order coordination and separation to order ones non-whole number. Assuming a symbol aDt^{λ} which represents fractional-order fundamental operator, it can be written as;

$$D^{\lambda} \triangleq aDt^{\lambda} = \begin{cases} \frac{d^{\lambda}}{dt^{\lambda}} & Q(\lambda) > 0, \\ 1 & Q(\lambda) = 0, \\ \int_a^t (d\tau)^{-\lambda} & Q(\lambda) < 0 \end{cases}$$

Where λ is known as order of the system, and the value of $\lambda \in Q$, Q is rational numbers and the value of λ can be in the form of x + i y (complex number), t and a are the upper and lower limits of the system respectively.
There are two definitions mostly used for fractional integration and differentiation is the Riemann-Liouville (RL) and the Grunwald-Letnikov (GL). The RL definition can be written as

$$aDt^{\lambda} f(t) = \frac{1}{\Gamma(n-\lambda)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\lambda-\pi+1}} d\tau$$

Where, the value of λ lies between, $n - 1 < \lambda < n$.
The GL can be written as-

$$aDt^{\lambda} f(t) = \lim_{h \to 0} h^{-\lambda} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^i \binom{\lambda}{i} f(t-ih)$$

Now taking the Laplace transformation for fractional-order λ of RL definition and with zero initial condition is given by

$$L\{aDt^{\lambda} f(t)\} = s^{\lambda} F(s)$$

Since, the Laplace transform of the function f (t) is $F(s)$.
Unmistakably, the number of degrees of freedom (DOF) of fractional-order administrator is greater than the whole number order. The superior execution can be acquired by taking best possible decision of order.

4.2. *Fractional Differentiator Approximation*

This area introduces the Oustaloups recursive estimate calculation. The calculation is utilized to understand the non-number integrator and differentiator squares of FOSMC (D^{α} and $D^{\alpha-1}$). Thus, it will approximate a transfer function form of
$H(s) = s^r$, $r \in [-1, 1]$, $r \in Q$
Now in form of rational transfer function

$$\hat{H}(s) = C_0 \prod_{k=-N}^{N} \left(\frac{s+\omega' k}{s+\omega k}\right)$$

The frequency range of $\omega_b < \omega < \omega_h$, on the basis of the given algorithm, the parameters which are unknowns of rotational transformation $\hat{H}(s)$ (i.e., C_0, ω_k, and ω'_k) can be found out by taking recursive dispersion of complex poles and zeros described by complex recursive components

$$\omega_k = \omega_b \left(\frac{\omega h}{\omega b}\right)^{\frac{0.5(1+r)+N+k}{1+2N}}$$

$$\omega'_k = \omega_b \left(\frac{\omega h}{\omega b}\right)^{\frac{0.5(1+r)+N+k}{1+2N}}$$

$$C_0 = \left(\frac{\omega h}{\omega b}\right)^{-\frac{r}{2}} \prod_{k=-N}^{N} \left(\frac{\omega k}{\omega' k}\right)$$

Where, the order of $\hat{H}(s)$ is 1+2N. For e.g. the fractional differentiator $H(s) = s^{1/2}$ is the 5-th order approximation, between the range of frequency from $\omega_b = 10^{-4}$ rad/s to $\omega_h = 10^4$ rad/s is

$$\hat{H}(s) = \frac{s^5 + 647 s^4 + 10260 s^3 + 4086 s^2 + 40 s + 0.01}{0.01 s^5 + 40 s^4 + 4086 s^3 + 10260 s^2 + 647 s + 1}$$

5. RESULTS

This section demonstrates the execution of FOSMC (ERROR) controller. The following pictures displays the behavior of framework to various set focuses, for example, elevation, points ψ, ɵ and ϕ. Additionally the framework execution is tried to unsettling influences.
The FOSMC controller execution for height attain by the quadcopter, Fig. 2. shows the given outcome. Tuning parameters for controller are

Altitude and Angle Parameters	Tunings		
	k_D	λ	δ
Ø	110	6	0.3
Ψ	110	6	0.3
ɵ	110	6	0.3
Z	30	72	0.3

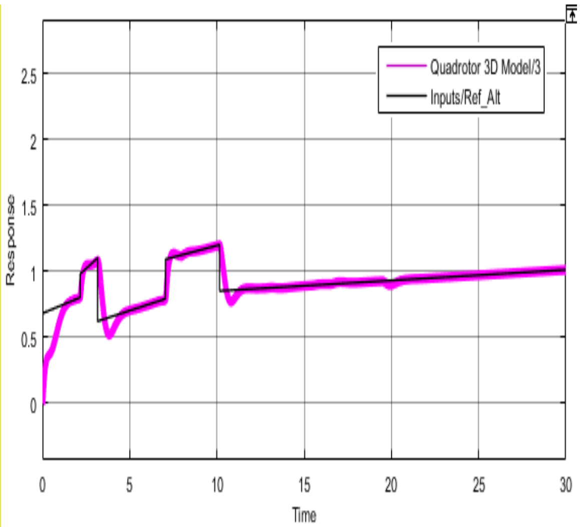

Fig. 2, Altitude and response

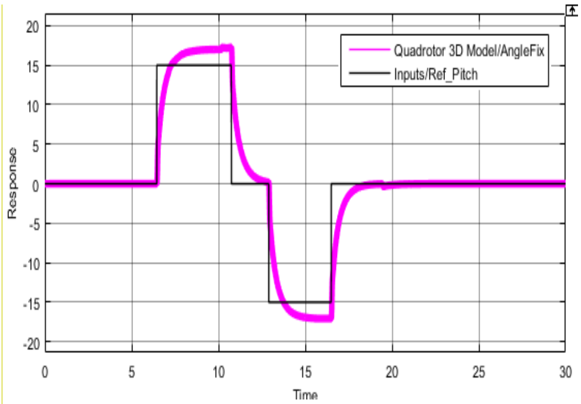

Fig. 3, Pitch Response

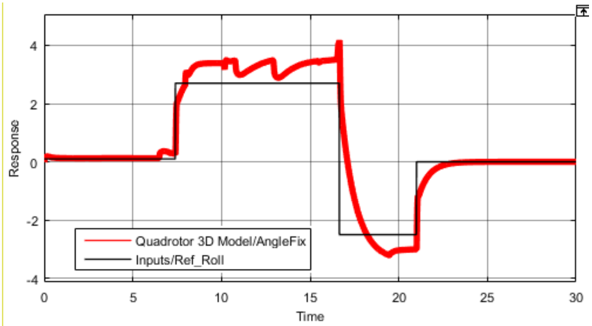

Fig. 4, Roll Response

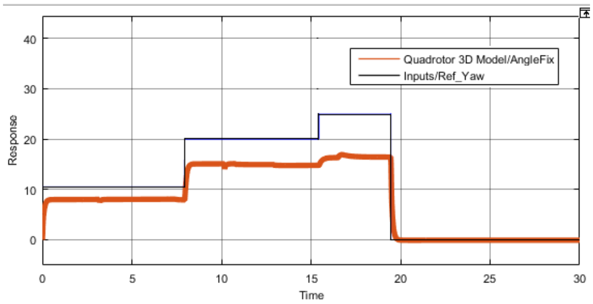

Fig. 5, Yaw Response

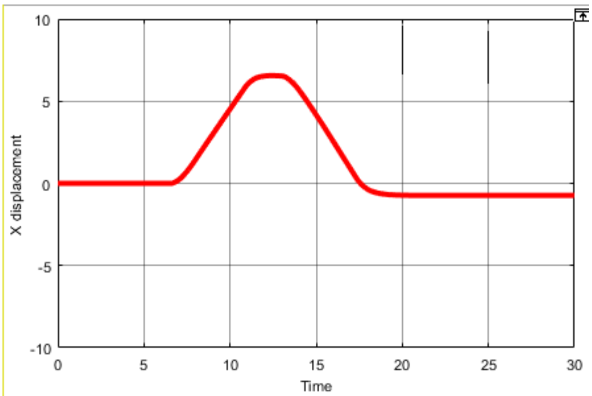

Fig. 6, Response in x direction

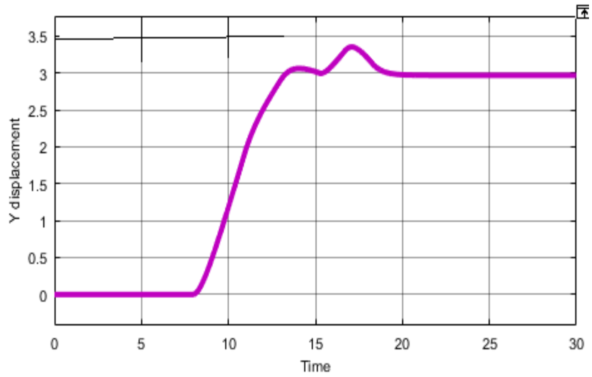

Fig. 7, Response in y direction

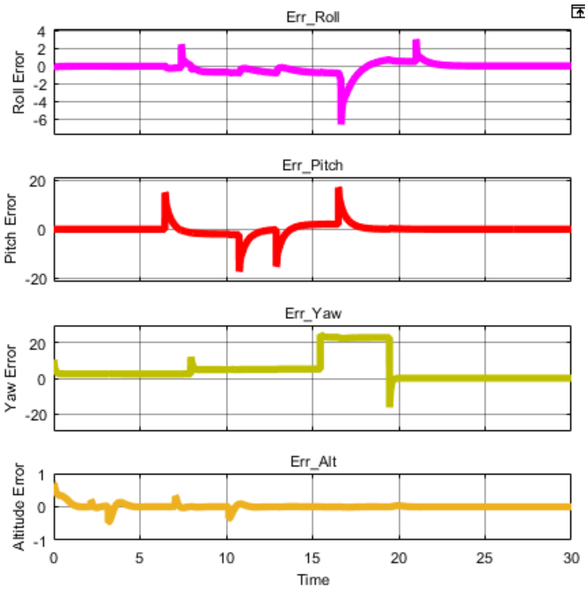

Fig. 8, Error Analysis

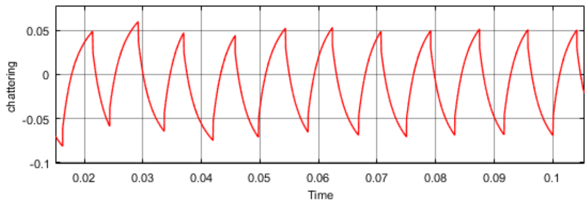

Fig. 9, Chattering

6. CONCLUSIONS

This paper provides a completely controlled Quadcopter by using Fractional Order Sliding Mode Control (FOSMC) by considering an ERROR sliding surface. By using ERROR sliding surface the development of FOSMC is done and tested with the help of Simulink MATLAB. The results of tested FOSMC show regulating performance and good tracking which is better than previous controller. The FOSMC controller is easy to implement used as an ERROR sliding surface. The FOSMC isn't just gives better execution with little control prattling, yet in addition strong notwithstanding outer load unsettling influence and parameter varieties. FOSMC controller is optimal and robust controller which has reachability time very less, transient response very fast and removes minor disturbances. As shown in error analysis graphs that all the error tending to zero within finite time, it made error sliding surface to converge smoothly the surface is stable and error is converging proved by Lyapunov Stability.

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