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Thermally Induced Vibration of Non-Homogeneous Visco-Elastic Rectangular Plate of Variable Thickness

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Abstract: The analysis presented here is to study the thermally induced vibration of non-homogeneous visco-elastic rectangular plate of variable thickness. The non-homogeneity of the plate material is assumed to arise due to the variation in density, which is assumed to vary parabolically. In this paper, the thickness of the plate varies linearly in both directions and thermal effect is linear in x-direction. Plates are considered to be having clamped boundary conditions on all the four edges. Basic elastic and viscous elements are combined to make visco-elastic materials. We have taken Kelvin model for visco-elasticity, which is a combination of elastic and viscous elements connected in parallel. Using the separation of variable method, the governing differential equation has been solved and Rayleigh-Ritz technique has been used to get the required frequency equation. In the present research, the deflection and time period of vibration of a clamped rectangular plate on the first two modes of vibration are calculated to see the effects of various factors like non-homogeneity constant, aspect ratio, taper constants, and thermal gradient. Results are presented in the tabular form.

Keywords: Vibration, non-homogeneous, visco-elastic, rectangular plate, thickness variation, thermal gradient effect.

1. INTRODUCTION

The vibration problems have been under investigation for quite a long time because when designing structures, the effect of vibration on them is very important factor to consider. In the engineering, almost all machines and engineering structures experienced vibrations so we cannot move without considering the effect of vibration. Obviously, structures used to support heavy centrifugal machines like motors and turbines are subjected to vibration. Vibration causes excessive wear of bearings, noise, fasteners to become loose, material cracking and abrasion of insulation around electrical conductors, resulting in short circuiting. When cutting a metal, vibration can cause chatter, which affects the quality of the surface finish. Structural vibration may cause discomfort and even fear in the occupants working in the building, make it difficult to operate machinery and cause malfunctioning of equipment. In the recent past, there has been a wide increase in the development of visco-elastic materials like fiber-reinforced due to the desirability of lightweight, high strength, corrosion resistance and high temperature performance requirement in modern technology. Visco-elastic is the generalization of viscosity and elasticity. Spring and dashpot are the ideal linear elastic and viscous elements respectively.

Plates of variable thickness are widely used in many engineering fields such as automobile industry, marine and building constructions, aerospace industry, mechanical engineering due to a number of attractive features such as material saving, weight reduction, high strength and also meet the desirability of economy. In particular these plates used in the manufacturing of navy ship interiors, cargo containers, aircraft surfacing and wings, small boats, snow skis, residential construction materials, interior partitions of space vehicles, doors, cabinets and other many everyday items.

As space technology has advanced, the need of the study of vibration of plates of certain aspect ratios with some simple restraints on the boundaries has also increased. For a constitution engineer before finalizing a design, the information for the first few modes of vibrations is very essential.

Thermal effect on the vibration of non-homogeneous plates are of great importance in the field of engineering such as for better designing of space vehicles, gas turbines, nuclear power projects, modern missiles and aircraft wings. Since structures are exposed to high intensity heat fluxes during the heated up period, the properties of the materials of the structures change significantly, therefore the thermal effect on the modulus of elasticity of the material can not be ignored.

Apparently the problem of a vibrating non-homogeneous visco-elastic rectangular plate of variable thickness in two directions with thermal gradient effect has not been considered in the literature. Obviously, the problem is of technical importance in several situations of aeronautical, civil, mechanical and naval engineering.

Sasajima et al. [1] discussed the vibration behavior and simplified design of thick rectangular plates with variable thickness. Leissa [2-4] discussed the vibration of rectangular plates. With thermal gradient effect, the transverse vibrations of clamped visco-elastic rectangular plate with thickness varying in two directions studied by Kaur [5]. Tomar and Gupta [6] discussed the effect of thermal gradient on the frequencies of an orthotropic rectangular plate of linearly varying thickness. Khanna [7] gave the some vibration problems of visco-elastic plate of variable thickness in two directions.

Gupta and Khanna [8] gave the vibration of clamped visco-elastic rectangular plate with parabolic thickness variations. Gupta et al. [9] discussed the problem of thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Effect of thermal gradient on some vibration problems of orthotropic visco-elastic plates of variable thickness was discussed by Kumar [10]. Thermally induced vibrations of a visco-elastic plate ware given by Mazumdar et al. [11]. Nowacki [12] gave the note on thermoelasticity. Transverse vibrations of non-homogeneous orthotropic rectangular plates of variable thickness were given by Lal and Dhanpati [13]. Transverse vibration of rectangular plate with bi-directional thickness variation was given by Singh and Saxena [14]. Sobotka [15] worked on free vibration of visco-elastic orthotropic plates. Rao and Satyanarayana [16] discussed the effect of thermal gradient of frequencies of tapered rectangular plates. Gupta and Kumar [17] analysed the effect of thermal gradient on free vibration of non-homogeneous visco-elastic rectangular plate of parabolically varying thickness. Sonzogni et al. [18] gave free vibrations of rectangular plates of exponentially varying thickness and with free edge.

Nagaya [19] studied vibration and dynamic response of visco-elastic plate on non-periodic elastic supports. Gutierrez et al. [20] solved the problem of vibrations of rectangular plates of bi-linearly varying thickness with general boundary conditions. Thermal effect on vibration problem of non-homogeneous visco-elastic plate of variable thickness has been studied by Kumar [21]. Tomar and Gupta [22] discussed the effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two-directions. The effect of exponential temperature variation on frequencies of an orthotropic rectangular plate of exponentially varying thickness was given by Tomar and Gupta [23]. Gupta and Panwar [24] solved the problem of vibration of non-homogeneous rectangular plate having parabolically varying thickness in both directions with exponentially temperature distribution. Gupta [25] gave the non-linear thermally induced vibrations of non-homogeneous rectangular plate of linearly varying thickness in the presence of external force.

Sobotka [26] discussed the vibration of rectangular orthotropic visco-elastic plates. Transverse vibrations of non uniform rectangular orthotropic plates were given by Tomar et al. [27]. Gupta and Sharma [28] have studied the non-linear thickness variation on exponentially thermal induced vibration of a non-homogeneous rectangular plate. Gupta and Jain [29] discussed the exponential temperature effect on frequencies of a rectangular plate on non-linear varying thickness by Quintic spline technique. Gupta and Kaur [30] discussed the effect of thermal gradient on vibration of clamped visco-elastic rectangular plate with exponentially thickness variation in both the directions.

The aim of this study is to discuss the effect of parabolically varies non-homogeneity on thermally induced vibration of visco-elastic rectangular plate whose thickness varies linearly in both the directions. Plate is considered as clamped supported on all the four edges. The visco-elastic properties of the plates are of the Kelvin type and the assumption of small deflection and linear visco-elastic properties are made. It is further assumed that The deflection and time period are calculated and presented in the tabular form for the different values of aspect ratio, thermal constant, non-homogeneity constant and taper constants for the first two modes of vibration.

2. METHOD OF ANALYSIS AND EQUATION OF MOTION

The differential equations of transverse motion and time function of free vibration of visco-elastic plate of variable thickness have been given by Gupta and Kaur [30]

$$\left[\mathrm{D}\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+2\frac{\partial \mathrm{D}}{\partial x}\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x\partial y^2}\right)+2\frac{\partial \mathrm{D}}{\partial y}\left(\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 w}{\partial x^2\partial y}\right)+\frac{\partial^2 \mathrm{D}}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial^2 \mathrm{D}}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 \mathrm{D}}{\partial x\partial y}\frac{\partial^2 w}{\partial x\partial y}\right]-\rho h k^2 w=0 \quad \ldots\ldots (1)$$

and $\ddot{\mathrm{T}}+k^2\widetilde{\mathrm{D}}\mathrm{T}=0$ …………………………….. (2)

where $\mathrm{D}=\frac{\mathrm{Eh}^3}{12(1-\nu^2)}$ is flexural rigidity,………… (3)

ν is the Poisson's ratio, $\widetilde{\mathrm{D}}$ is unique Rheological operator and E is Young modulus of elasticity.

Consider a symmetrical plate of rectangular form. Let a and b be the length and the width of the plate respectively. Plate is considered as clamped supported on all the four edges. Assuming that the temperature τ of the visco-elastic rectangular plate varies linearly along x-axis only then τ can be expressed as

$$\tau=\tau_0\left(1-\frac{x}{a}\right) \quad \ldots\ldots (4)$$

Where τ and τ_0 denote the increase in temperature above the reference temperature at any point at distance $\frac{x}{a}$ and at the end $x=a$ respectively.

For most of engineering materials, the temperature dependence of the modulus of elasticity can be expressed in the form [12]

$$E=E_0(1-\gamma\tau) \quad \ldots\ldots (5)$$

Where E_0 is the Young's modulus at reference temperature i.e. $\tau=0$ and γ is the slope of the variation of E with τ.

On substituting value of τ from Eq. (4) into Eq. (5), we get

$$E=E_0\left[1-\alpha\left(1-\frac{x}{a}\right)\right] \quad \ldots\ldots (6)$$

Where $\alpha=\gamma\tau_0$ $(0\le\alpha\le 1)$, a parameter known as thermal gradient.

Assuming that thickness of the visco-elastic rectangular plate varies linearly in both x and y directions i.e.

$$h=h_0\left(1+\beta_1\frac{x}{a}\right)\left(1+\beta_2\frac{y}{b}\right) \quad \ldots\ldots (7)$$

where β_1 and β_2 are taper constants along x-axis and y-axis respectively and h_0 is the thickness of the plate at $x=y=0$.

Let's assume that density varies parabolically in x-direction as

$$\rho=\rho_0\left(1+\alpha_1\frac{x^2}{a^2}\right) \quad \ldots\ldots (8)$$

where α_1 is the non-homogeneity constant and $\rho_0=(\rho)_{x=0}$.

For free transverse vibrations of the plate, Deflection function w depend on time can be written in the form of the product of two functions as follows

$$w(x,y,t)=W(x,y)T(t) \quad \ldots\ldots (9)$$

where $W(x,y)$ is the deflection function and $T(t)$ is the time function.

The expression for kinetic energy P and strain energy S as taken by Leissa [2] are

$$P=\frac{1}{2}k^2\int_0^a\int_0^b \rho h W^2\,dx\,dy \quad \ldots\ldots (10)$$

and $$S=\frac{1}{2}\int_0^a\int_0^b D\left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2+\left(\frac{\partial^2 W}{\partial y^2}\right)^2+2\nu\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}+2(1-\nu)\left(\frac{\partial^2 W}{\partial x\partial y}\right)^2\right]dx\,dy \quad \ldots\ldots (11)$$

Now, the flexural rigidity of the plate (assuming Poisson's ratio ν is constant) can be written as

$$D=\frac{1}{12(1-\nu^2)}\left[E_0 {h_0}^3\left\{1-\alpha\left(1-\frac{x}{a}\right)\right\}\left(1+\beta_1\frac{x}{a}\right)^3\left(1+\beta_2\frac{y}{b}\right)^3\right] \quad \ldots\ldots (12)$$

Substituting Eq. (7) and Eq. (8) in Eq. (10), we have

$$P=\frac{1}{2}\rho_0 h_0 k^2\int_0^a\int_0^b\left(1+\alpha_1\frac{x^2}{a^2}\right)\left(1+\beta_1\frac{x}{a}\right)\left(1+\beta_2\frac{y}{b}\right)W^2\,dx\,dy \quad \ldots\ldots (13)$$

According to Rayleigh Ritz technique, the maximum strain energy must be equal to the maximum kinetic energy i.e. $\delta(S-P)=0$ …………………… (14)

The boundary conditions for a rectangular plate clamped along all the four edges are

$$\left.\begin{aligned} W=\frac{\partial W}{\partial x}=0\ at\ x=0,a \\ and\ W=\frac{\partial W}{\partial y}=0\ at\ y=0,b \end{aligned}\right\} \quad \ldots\ldots (15)$$

Taking two term deflection function as

$$W(x,y)=\left[\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]^2\left[B_1+B_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \quad \ldots\ldots (16)$$

where B_1 and B_2 are constants, which are satisfied by Eq. (15).

Now assuming the non-dimensional variables as

$$X=\frac{x}{a},\ Y=\frac{y}{a},\ \bar{W}=\frac{W}{a},\ \bar{h}=\frac{h}{a} \quad \ldots\ldots (17)$$

Using Eq. (12) and Eq. (17) in Eq. (11) and Eq. (13), we obtains

$$P=\frac{1}{2}\rho_0\overline{h_0}k^2a^5\int_0^1\int_0^{b/a}(1+\alpha_1X^2)(1+\beta_1X)\left(1+\beta_2Y\frac{a}{b}\right)\bar{W}^2\,dX\,dY \quad \ldots\ldots (18)$$

and $S = C \int_0^1 \int_0^{b/a} \{1 - \alpha(1 - X)\}(1 + \beta_1 X)^3 \left(1 + \beta_2 Y \frac{a}{b}\right)^3 \left[\left(\frac{\partial^2 \overline{W}}{\partial X^2}\right)^2 + \left(\frac{\partial^2 \overline{W}}{\partial Y^2}\right)^2 + 2\nu \frac{\partial^2 \overline{W}}{\partial X^2}\frac{\partial^2 \overline{W}}{\partial Y^2} + 2(1-\nu)\left(\frac{\partial^2 \overline{W}}{\partial X \partial Y}\right)^2\right] dXdY$ ………………… (19)

where $C = \frac{E_0 \overline{h_0}^3 a^3}{24(1-\nu^2)}$

Here variation of X is 0 to 1 and that of Y is from 0 to $\frac{b}{a}$.

On substituting the values of P and S from Eq. (18) and Eq. (19) Eq. (14), we get

$\delta(S_1 - n^2 k^2 P_1) = 0$ ……………………… (20)

where $S_1 = \int_0^1 \int_0^{b/a} \{1 - \alpha(1 - X)\}(1 + \beta_1 X)^3 \left(1 + \beta_2 Y \frac{a}{b}\right)^3 \left[\left(\frac{\partial^2 \overline{W}}{\partial X^2}\right)^2 + \left(\frac{\partial^2 \overline{W}}{\partial Y^2}\right)^2 + 2\nu \frac{\partial^2 \overline{W}}{\partial X^2}\frac{\partial^2 \overline{W}}{\partial Y^2} + 2(1-\nu)\left(\frac{\partial^2 \overline{W}}{\partial X \partial Y}\right)^2\right] dXdY$ ……… (21)

and $P_1 = \int_0^1 \int_0^{b/a} (1 + \alpha_1 X^2)(1 + \beta_1 X)\left(1 + \beta_2 Y \frac{a}{b}\right) \overline{W}^2 dXdY$ ………………………. (22)

Here $n^2 = \frac{12\rho_0(1-\nu^2)a^2}{E_0 \overline{h_0}^2}$ …………………. (23)

Substituting of W from Eq. (16) in Eq. (20) gives rise to two unknown parameters B_1 and B_2 which can be computed as

$$\left.\begin{array}{l} \frac{\partial}{\partial B_1}(S_1 - n^2k^2P_1) = 0 \\ and \ \frac{\partial}{\partial B_2}(S_1 - n^2k^2P_1) = 0 \end{array}\right\} \text{……………… (24)}$$

After solving Eq. (24), we have

$$\left.\begin{array}{l} b_{11}B_1 + b_{12}B_2 = 0 \\ b_{21}B_1 + b_{22}B_2 = 0 \end{array}\right\} \text{…………………….. (25)}$$

where $b_{11}, b_{12}, b_{21}, b_{22}$ involves parametric constants and the frequency parameter.

For a non-trivial solution, determinant of coefficient of Eq. (25) must be zero.

In this way the frequency equation comes out to be

$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0$ …………………………….. (26)

A quadratic equation in k^2 can be obtain from Eq. (26), which gives the two values of k^2.

On substituting the value of $B_1 = 1$, by choice, in Eq. (25), we get $B_2 = -\frac{b_{11}}{b_{12}}$ and hence W becomes

$W = \left[XY\left(\frac{a}{b}\right)(1 - X)\left(1 - Y\frac{a}{b}\right)\right]^2 \left[1 + \left(-\frac{b_{11}}{b_{12}}\right) XY\left(\frac{a}{b}\right)(1 - X)\left(1 - Y\frac{a}{b}\right)\right]$………… (27)

Time Function of Vibrations of Non-homogeneous Visco-elastic Plate:

The Eq. (2) is defined as general differential equations of time function of free vibration of visco-elastic plate of variable thickness. It depends on the visco-elastic operator $\widetilde{D}$ and which for Kelvin's model, can be taken as

$\widetilde{D} \equiv 1 + \left(\frac{\eta}{G}\right)\left(\frac{d}{dt}\right)$ ………………………. (28)

After substituting Eq. (28) into Eq. (2), one obtains

$\ddot{T} + k^2\left(\frac{\eta}{G}\right)\dot{T} + k^2 T = 0$ ………………… (29)

Eq. (29) is a second order differential equation for the time function T.

Solution of Eq. (29) will be of the form

$T(t) = e^{a_1 t}(A cos b_1 t + B sin b_1 t)$ ………….. (30)

where $a_1 = -\frac{k^2}{2}\left(\frac{\eta}{G}\right)$, $b_1 = k\sqrt{1 - \left(\frac{\rho\eta}{2G}\right)^2}$ …… (31)

and A, B are constants which can be determined easily from the initial conditions of the plate.

Assuming the initial conditions as

$T = 1$ and $\dot{T} = 0$ at $t = 0$ ……………………. (32)

Now using Eq. (32) in Eq. (30), one obtains

$A = 1, B = -\frac{a_1}{b_1}$ ……………………………… (33)

After substituting the values of A, B in Eq. (30), one obtains

$T(t) = e^{a_1 t}\left(cos b_1 t - \frac{a_1}{b_1} sin b_1 t\right)$ ………….. (34)

Thus, deflection of the vibrating mode $w(x, y, t)$, which is equal to $W(x, y)T(t)$, may be expressed using Eq. (27) and Eq. (34) in Eq. (9) as

$w = \left[XY\left(\frac{a}{b}\right)(1 - X)\left(1 - Y\frac{a}{b}\right)\right]^2 \left[1 - \frac{b_{11}}{b_{12}} XY\left(\frac{a}{b}\right)(1 - X)\left(1 - Y\frac{a}{b}\right)\right] . e^{a_1 t}\left(cos b_1 t - \frac{a_1}{b_1} sin b_1 t\right)$ …. (35)

Time period of the vibration of the plate is given by

$K = \frac{2\pi}{k}$ ………………………. (36)

where k is the frequency given by the Eq. (26).

3. RESULT AND DISCUSSION

In the present investigation, the values of the deflection $w(*10^{-5})$ and time period $K(*10^{-6}$in secs) are obtained for a clamped non-homogeneous visco-elastic rectangular plate with linearly thickness variation in both x and y directions and linear temperature effect for different values of taper constants β_1 & β_2 , non-homogeneity constant α_1, aspect ratio a/b, and thermal gradient α for first two modes of vibration. For non-homogeneity of the plate material, density is assumed to vary parabolically in one direction. Poisson's ratio ν is taken as 0.345. All these results are presented in Tables 1-11.

For numerical computation, following material parameters are used [19]:

$E = 7.08 \times 10^{10}$ N/m^2, $G = 2.682 \times 10^{10}$ N/m^2, $\eta = 1.4612 \times 10^6$ Ns/m^2, $\rho = 2.80 \times 10^3$ kg/m^3, $\nu = 0.345$

The thickness of the plate at the centre is $h_0 = 0.01$m.

From Table 1, it can be concluded that as thermal gradient α increases, time period K increases continuously for both the modes of vibration.

Table 2 shows that time period K continuously decreased for both the modes of vibration as aspect ratio a/b increased.

Table 3 shows that time period K decreases as taper constant β_1 increases for both the modes of vibration.

From Table 4, it can be concluded that for both the modes of vibration, time period K decreases with increase in taper constantβ_2.

Table 5 shows that time period K increase when non-homogeneity constant α_1 increase for first two modes of vibration.

Table 6(a), Table 7(a) and Table 8(a) show that deflection w for first mode of vibration first increase and then decrease till zero as X increase for different values of Y.

It can observe from Table 6(b), Table 7(b) and Table 8(b) that deflection w for second mode of vibration, for $Y = 0.2$, first increases then decreases and then increases and finally become zero but for $Y = 0.6$, deflection w first increases and then decreases till zero as X increases.

From Table 9, Table 10, Table 11, we can conclude that deflection w for first mode of vibration continuously increases at initial time $0.K$ and first increases and then slightly decreases at the time $5.K$ but an increase followed by decrease is observed for second mode of vibration, with increase in aspect ratioa/b , at initial time $0.K$ and at time$5.K$.

Table 1

Time period K ($* 10^{-6}$ in secs) of a clamped visco-elastic rectangular plate for different thermal gradient α and constant aspect ratio (a/b=1.5) for all X and Y

	$\alpha_1 = 0$				$\alpha_1 = 0.4$			
	$\beta_1 = 0, \beta_2 = 0$		$\beta_1 = 0.4, \beta_2 = 0.2$		$\beta_1 = 0, \beta_2 = 0$		$\beta_1 = 0.4, \beta_2 = 0.2$	
α	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	6462	1623	4836	1218	6872	1723	5051	1312
0.2	6823	1731	5033	1305	7225	1847	5363	1353
0.4	7236	1863	5381	1358	7720	1995	5675	1458
0.6	7737	2002	5693	1464	8476	2171	6087	1596
0.8	8443	2168	6056	1582	9217	2280	6519	1714
1.0	9231	2304	6543	1703	9653	2432	6931	1818

Table 2

Time period K ($* 10^{-6}$ in secs) of a clamped visco-elastic rectangular plate for different aspect ratio (a/b) for all X and Y

	$\alpha_1 = 0$				$\alpha_1 = 0.4$			
	$\beta_1 = 0, \beta_2 = 0, \alpha = 0$		$\beta_1 = 0.4, \beta_2 = 0.2, \alpha = 0.3$		$\beta_1 = 0, \beta_2 = 0, \alpha = 0$		$\beta_1 = 0.4, \beta_2 = 0.2, \alpha = 0.3$	
a/b	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.5	16486	4098	12928	3193	16932	4217	13327	3321
1.0	11188	2832	8937	2274	11621	2920	9223	2370
1.5	6462	1623	5193	1318	6872	1723	5492	1407
2.0	4063	997	3284	796	4403	1049	3575	879
2.5	2683	644	2185	513	2984	726	2543	611

Table 3

Time period K ($* 10^{-6}$ in secs) of a clamped visco-elastic rectangular plate for different taper constant β_1 and constant aspect ratio (a/b=1.5) for all X and Y

	$\alpha_1 = 0$				$\alpha_1 = 0.4$			
	$\beta_2 = 0, \alpha = 0$		$\beta_2 = 0.2, \alpha = 0.3$		$\beta_2 = 0, \alpha = 0$		$\beta_2 = 0.2, \alpha = 0.3$	
β_1	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	6462	1623	6358	1549	6872	1723	6627	1667
0.2	5843	1467	5774	1407	6348	1617	6120	1505
0.4	5331	1356	5193	1318	5771	1527	5492	1407
0.6	4863	1241	4687	1175	5351	1419	4948	1366
0.8	4478	1138	4239	1053	4822	1269	4546	1208
1.0	4063	1035	3927	987	4449	1119	4216	1094

Table 4

Time period K ($* 10^{-6}$ in secs) of a clamped visco-elastic rectangular plate for different taper constant β_2 and constant aspect ratio (a/b=1.5) for all X and Y

	$\alpha_1 = 0$				$\alpha_1 = 0.4$			
	$\beta_1 = 0, \alpha = 0$		$\beta_1 = 0.4, \alpha = 0.3$		$\beta_1 = 0, \alpha = 0$		$\beta_1 = 0.4, \alpha = 0.3$	
β_2	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	6462	1623	5738	1478	6872	1723	6025	1567
0.2	5832	1449	5193	1318	6334	1604	5492	1407
0.4	5323	1352	4567	1203	5756	1520	4901	1314
0.6	4851	1238	4032	1131	5329	1409	4458	1212
0.8	4463	1127	3765	1042	4809	1261	4090	1147
1.0	3938	1009	3587	961	4404	1092	3843	981

Table 5

Time period K ($* 10^{-6}$ in secs) of a clamped visco-elastic rectangular plate for different non-homogeneity parameter α_1 and constant aspect ratio (a/b=1.5) for all X and Y

	$\alpha = 0$				$\alpha = 0.3$			
	$\beta_1 = 0, \beta_2 = 0$		$\beta_1 = 0.4, \beta_2 = 0.2$		$\beta_1 = 0, \beta_2 = 0$		$\beta_1 = 0.4, \beta_2 = 0.2$	
α_1	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode	First Mode	Second Mode
0.0	6462	1623	4847	1221	7020	1791	5193	1318
0.2	6644	1682	4960	1266	7236	1857	5381	1367
0.4	6872	1723	5053	1312	7476	1914	5492	1407
0.6	7039	1775	5163	1361	7707	1989	5617	1463
0.8	7276	1826	5251	1410	7951	2056	5731	1522
1.0	7441	1878	5373	1471	8209	2121	5841	1586

Table 6(a), 6(b)

Deflection w($* 10^{-5}$) of a clamped visco-elastic rectangular plate for constant aspect ratio (a/b=1.5) and $\alpha = \beta_1 = \beta_2 = \alpha_1 = 0.0$ and for different X and Y

At Initial Time=0.0xK

X	Y=0.20		Y=0.60	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0	0	0	0
0.2	126.32	38.87	427.32	14.52
0.4	303.71	6.12	1420.12	26.73

0.6	303.71	6.12	1420.12	26.73
0.8	126.32	38.87	427.32	14.52
1.0	0	0	0	0

At Time=5.0xK

X	Y=0.20		Y=0.60	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0	0	0	0
0.2	55.12	1.38	187.31	0.53
0.4	132.32	0.21	623.16	1.01
0.6	132.32	0.21	623.16	1.01
0.8	55.12	1.38	187.31	0.53
1.0	0	0	0	0

Table 7(a), 7(b)

Deflection w($* 10^{-5}$) of a clamped visco-elastic rectangular plate for constant aspect ratio (a/b=1.5) and $\alpha = \beta_1 = \beta_2 = 0, \alpha_1 = 0.4$ and for different X and Y

At Initial Time=0.0xK

X	Y=0.20		Y=0.60	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0	0	0	0
0.2	129.02	39.83	429.07	14.71
0.4	306.24	6.23	1422.85	26.83
0.6	306.24	6.23	1422.85	26.83
0.8	129.02	39.83	429.07	14.71
1.0	0	0	0	0

At Time=5.0xK

X	Y=0.20		Y=0.60	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0	0	0	0
0.2	57.84	1.61	188.93	0.60
0.4	134.93	0.25	626.42	1.04
0.6	134.93	0.25	626.42	1.04
0.8	57.84	1.61	188.93	0.60
1.0	0	0	0	0

Table 8(a), 8(b)

Deflection w($* 10^{-5}$) of a clamped visco-elastic rectangular plate for constant aspect ratio (a/b=1.5) and $\alpha = 0.2, \beta_1 = 0.3, \beta_2 = 0.4, \alpha_1 = 0.4$ and for different X and Y

At Initial Time=0.0xK

X	Y=0.20		Y=0.60	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0	0	0	0
0.2	155.12	39.97	1074.07	15.02
0.4	389.53	6.73	3592.61	27.27
0.6	389.53	6.73	3592.61	27.27
0.8	155.12	39.97	1074.07	15.02
1.0	0	0	0	0

At Time=5.0xK

X	Y=0.20		Y=0.60	
	First Mode	Second Mode	First Mode	Second Mode
0.0	0	0	0	0
0.2	59.11	0.78	398.07	0.32
0.4	141.23	0.14	1291.61	0.47
0.6	141.23	0.14	1291.61	0.47
0.8	59.11	0.78	398.07	0.32
1.0	0	0	0	0

Table 9

Deflection w($* 10^{-5}$) of a clamped visco-elastic rectangular plate for different aspect ratio (a/b) and $\alpha = \beta_1 = \beta_2 = \alpha_1 = 0.0$ and for X = Y = 0.2

a/b	At Initial Time=0.0xK		At Time=5.0XK	
	First Mode	Second Mode	First Mode	Second Mode
0.5	20.93	14.02	14.34	3.63
1.0	63.12	32.41	38.67	4.84
1.5	126.32	38.87	55.12	1.38
2.0	233.96	37.37	61.42	0.14
2.5	398.28	36.09	53.21	0.003

Table 10

Deflection w($* 10^{-5}$) of a clamped visco-elastic rectangular plate for different aspect ratio (a/b) and $\alpha = \beta_1 = \beta_2 = 0,\ \alpha_1 = 0.4$ and for X = Y = 0.2

a/b	At Initial Time=0.0xK		At Time=5.0XK	
	First Mode	Second Mode	First Mode	Second Mode
0.5	23.27	15.03	16.24	4.01
1.0	65.48	33.69	40.11	4.99
1.5	129.02	39.83	57.84	1.61
2.0	237.02	38.19	64.07	0.19
2.5	401.13	36.50	55.92	0.006

Table 11

Deflection w($* 10^{-5}$) of a clamped visco-elastic rectangular plate for different aspect ratio (a/b) and $\alpha = 0.3, \beta_1 = 0.3, \beta_2 = 0.4, \alpha_1 = 0.4$ and for X = Y = 0.2

a/b	At Initial Time=0.0xK		At Time=5.0XK	
	First Mode	Second Mode	First Mode	Second Mode
0.5	26.07	15.63	18.37	3.14
1.0	75.33	33.68	43.66	3.53
1.5	156.41	40.21	61.23	0.93
2.0	299.92	38.92	64.13	0.054
2.5	525.23	37.17	52.06	-0.00029

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