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M-Gonal number $\pm$ n = Nasty number, n = 1,2

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Abstract: This paper concerns with study of obtaining the ranks of M-gonal numbers along with their recurrence relation such that M-gonal number $\pm$ n = Nasty number where n = 1,2.

Key Words: Triangular number, Pentagonal number, Hexagonal number, Octagonal number and Dodecagonal number

1. INTRODUCTION

Numbers exhibit fascinating properties, they form patterns and so on [1]. In [3] the ranks of Triangular, Pentagonal, Heptagonal, Nanogonal and Tridecagonal numbers such that each of these M-gonal number - 2 = a perfect square and the ranks of Pentagonal and Heptagonal such that each of these M-gonal number + 2= a perfect square are obtained. In this context one may refer [3-7]. In this communication an attempt is made to obtain the ranks of M-gonal numbers like Triangular, Pentagonal, Hexagonal, Octagonal and Dodecagonal numbers such that each of these M-gonal number $\pm$ n = Nasty number, n = 1,2. Also the recurrence relations satisfied by the ranks of each of these M-gonal numbers are presented.

2. METHOD OF ANALYSIS

2.1.Pattern 1:

Let the rank of the n^th^ Triangular number be A, then the identity

$$\text{Triangular number} - 2 = 6x^2 \qquad (1)$$

is written as

$$y^2 = 48x^2 + 9 \qquad (2)$$

where $y = 2A + 1$ (3)

whose initial solution is $x_0 = 3,\ y_0 = 21$ (4)

Let $(\tilde{x}_s, \tilde{y}_s)$ be the general solution of the Pellian

$$y^2 = 48x^2 + 1 \qquad (5)$$

where
$$\tilde{x}_s = \frac{1}{2\sqrt{48}}\left(\left(7+\sqrt{48}\right)^{s+1} - \left(7-\sqrt{48}\right)^{s+1}\right)$$
$$\tilde{y}_s = \frac{1}{2}\left(\left(7+\sqrt{48}\right)^{s+1} + \left(7-\sqrt{48}\right)^{s+1}\right),\ s = 0,1,\dots.$$

Applying Brahmagupta's lemma [2] between the solutions (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$ the sequence of values of x and y satisfying equation (2) is given by

$$x_s = \frac{1}{2\sqrt{48}}\left(\left(7+\sqrt{48}\right)^{s+1}\left(21+3\sqrt{48}\right) - \left(7-\sqrt{48}\right)^{s+1}\left(21-3\sqrt{48}\right)\right)$$
$$y_s = \frac{1}{2}\left(\left(7+\sqrt{48}\right)^{s+1}\left(21+3\sqrt{48}\right) + \left(7-\sqrt{48}\right)^{s+1}\left(21-3\sqrt{48}\right)\right),\ s = 0,1,\dots.$$

Inview of (3), the rank of Triangular number is given by

$$A_s=\frac{1}{4}\left(\left(7+\sqrt{48}\right)^{s+1}\left(21+3\sqrt{48}\right)+\left(7-\sqrt{48}\right)^{s+1}\left(21-3\sqrt{48}\right)-2\right),\ s=0,1,2...$$

and the corresponding recurrence relation is found to be

$$A_{2s+3}-194A_{2s+1}+A_{2s-1}-96=0$$

In a similar manner, the ranks of Pentagonal, Hexagonal, Octagonal and Dodecagonal numbers are presented in below table

S.No	M-Gonal number	General form of ranks
1	Pentagonal number (B)	$B_s=\frac{1}{12}\left(\left(5+\sqrt{24}\right)^{s+1}\left(7+\sqrt{24}\right)+\left(5-\sqrt{24}\right)^{s+1}\left(7-\sqrt{24}\right)+2\right),\ s=0,2,4...$
2	Hexagonal number (C)	$C_s=\frac{1}{8}\left(\left(7+\sqrt{48}\right)^{s+1}\left(21+3\sqrt{48}\right)+\left(7-\sqrt{48}\right)^{s+1}\left(21-3\sqrt{48}\right)+2\right), s=0,2,4...$
3	Octagonal number (D)	$D_s=\frac{1}{12}\left(\left(17+2\sqrt{72}\right)^{s+1}\left(68+8\sqrt{72}\right)+\left(17-2\sqrt{72}\right)^{s+1}\left(68-8\sqrt{72}\right)+4\right)$ $s=0,2,4...$
4	Dodecagonal number (E)	$E_s=\frac{1}{20}\left(\left(11+\sqrt{120}\right)^{s+1}\left(66+6\sqrt{120}\right)+\left(11-\sqrt{120}\right)^{s+1}\left(66-6\sqrt{120}\right)+8\right)$ $s=0,1,2...$

The recurrence relations satisfied by the ranks of each of these M-Gonal numbers are presented in the below table

S.NO	RECURRENCE RELATIONS
1	$B_{2s+3}-98B_{2s+1}+B_{2s-1}+16=0$
2	$C_{2s+3}-194C_{2s+1}+C_{2s-1}+48=0$
3	$D_{2s+3}-1154D_{2s+1}+D_{2s-1}+384=0$
4	$E_{2s+3}-482E_{2s+1}+E_{2s-1}+192=0$

2.2.Pattern 2:

Assume the rank of the n^{th} Pentagonal number to be B, then the equation

Pentagonal number + 1 = 6 x^2 (6)

is written as

$$y^2=24x^2-23 \quad (7)$$

where $y=6B-1$ (8)

the initial solution of (7) is $x_0=4$, $y_0=19$ (9)

Let $\left(\tilde{x}_s,\tilde{y}_s\right)$ be the general solution of the Pellian

$$y^2=24x^2+1 \quad (10)$$

where $\tilde{x}_s=\frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1}-\left(5-\sqrt{24}\right)^{s+1}\right)$

$$\tilde{y}_s=\frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1}+\left(5-\sqrt{24}\right)^{s+1}\right),\ s=0,1,....$$

On applying Brahmagupta's lemma [2] between the solutions $\left(x_0, y_0\right)$ and $\left(\tilde{x}_s, \tilde{y}_s\right)$ the sequence of values of x and y satisfying equation (7) is given by

$$x_s = \frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1}\left(19+4\sqrt{24}\right)-\left(5-\sqrt{24}\right)^{s+1}\left(19-4\sqrt{24}\right)\right)$$

$$y_s = \frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1}\left(19+4\sqrt{24}\right)+\left(5-\sqrt{24}\right)^{s+1}\left(19-4\sqrt{24}\right)\right)$$

The rank of Pentagonal number from (8) is given by

$$Bs = \frac{1}{12}\left(\left(5+\sqrt{24}\right)^{s-1}\left(19+4\sqrt{24}\right)+\left(5-\sqrt{24}\right)^{s-1}\left(19-4\sqrt{24}\right)+2\right),\ s=0,2,4,...$$

and the consequently recurrence relation is found to be

$$B_{2s+3} - 98B_{2s+1} + B_{2s-1} + 16 = 0$$

On following the procedure similar to the above for Octagonal number, we get the rank and recurrence relation which are given below

$$D_s = \frac{1}{12}\left(\left(17+2\sqrt{72}\right)^{s+1}\left(8+\sqrt{72}\right)+\left(17-2\sqrt{72}\right)^{s+1}\left(8-\sqrt{72}\right)+4\right),\ s=0,2...$$

$$D_{2s+3} - 1154C_{2s+1} + D_{2s-1} + 384 = 0$$

2.3.Pattern 3:

Consider the rank of the n[th] Pentagonal number to be B, then the identity ,

Pentagonal number -2 = 6 x^2 (11)

is written as

$$y^2 = 24x^2 + 49 \quad (12)$$

where $y = 6B - 1$ (13)

whose initial solution is $x_0 = 7$, $y_0 = 35$ (14)

Let $\left(\tilde{x}_s, \tilde{y}_s\right)$ be the general solution of the Pellian

$$y^2 = 24x^2 + 1 \quad (15)$$

where $\tilde{x}_s = \frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1}-\left(5-\sqrt{24}\right)^{s+1}\right)$

$$\tilde{y}_s = \frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1}+\left(5-\sqrt{24}\right)^{s+1}\right),\ s=0,1,....$$

On using Brahmagupta's lemma [2] between the solutions $\left(x_0, y_0\right)$ and $\left(\tilde{x}_s, \tilde{y}_s\right)$ the sequence of values of x and y satisfying equation (12) is given by

$$x_s = \frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1}\left(35+7\sqrt{24}\right)-\left(5-\sqrt{24}\right)^{s+1}\left(35-7\sqrt{24}\right)\right)$$

$$y_s = \frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1}\left(35+7\sqrt{24}\right)+\left(5-\sqrt{24}\right)^{s+1}\left(35-7\sqrt{24}\right)\right)$$

Inview of (13), the ranks of Pentagonal number is given by

$$Bs = \frac{1}{12}\left(\left(5+\sqrt{24}\right)^{s+1}\left(35+7\sqrt{24}\right)+\left(5-\sqrt{24}\right)^{s-1}\left(35-7\sqrt{24}\right)+2\right),\ s=1,3,5,....$$

and the corresponding recurrence relation is found to be

$$B_{2s+3} - 98B_{2s+1} + B_{2s-1} + 16 = 0$$

On applying the same procedure to Octagonal number, the rank and recurrence relation are found which are given below

$$D_s = \frac{1}{12}\left(\left(17+2\sqrt{72}\right)^{s+1}\left(10+\sqrt{72}\right) + \left(17-2\sqrt{72}\right)^{s+1}\left(10-\sqrt{72}\right) + 4\right),\ s = 1,3,5,....$$

$$D_{2s+3} - 1154C_{2s+1} + D_{2s-1} + 384 = 0$$

2.4.Pattern 4:

Suppose the rank of the n[th] Pentagonal number to be B, then the identity,

Pentagonal number +2 = 6 x^2 (16)

is written as

$$y^2 = 24x^2 - 47 \quad (17)$$

where $y = 6B - 1$ (18)

whose initial solution is $x_0 = 2, y_0 = 7$ (19)

Let $(\tilde{x}_s, \tilde{y}_s)$ be the general solution of the Pellian

$$y^2 = 24x^2 + 1 \quad (20)$$

where $$\tilde{x}_s = \frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1} - \left(5-\sqrt{24}\right)^{s+1}\right)$$

$$\tilde{y}_s = \frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1} + \left(5-\sqrt{24}\right)^{s+1}\right),\ s = 0,1,....$$

By means of Brahmagupta's lemma [2] among the solutions of (x_0, y_0) and $(\tilde{x}_s, \tilde{y}_s)$ the sequence of values of x and y satisfying equation (17) is given by

$$x_s = \frac{1}{2\sqrt{24}}\left(\left(5+\sqrt{24}\right)^{s+1}\left(7+2\sqrt{24}\right) - \left(5-\sqrt{24}\right)^{s+1}\left(7-2\sqrt{24}\right)\right)$$

$$y_s = \frac{1}{2}\left(\left(5+\sqrt{24}\right)^{s+1}\left(7+2\sqrt{24}\right) + \left(5-\sqrt{24}\right)^{s+1}\left(7-2\sqrt{24}\right)\right)$$

Therefore from (18), the rank of Pentagonal number is given by

$$Bs = \frac{1}{12}\left(\left(5+\sqrt{24}\right)^{s+1}\left(7+2\sqrt{24}\right) + \left(5-\sqrt{24}\right)^{s-1}\left(7-2\sqrt{24}\right) + 2\right),\ s = 0,2,4,....$$

and the corresponding recurrence relation is found to be

$$B_{2s+3} - 98B_{2s+1} + B_{2s-1} + 16 = 0$$

3. CONCLUSION

In this paper the ranks of M-gonal numbers such that M-gonal number ± n = nasty number, n=1,2. In this manner one can scrutinize the ranks of M-gonal number satisfying various properties along with recurrence relation.

REFERENCE

[1] L.E.Dikson(1952): "History of Theory of Numbers", Chelsa Publishing Company, New York, 2.
[2] T.S.Bhanumathy(1995): Ancient Indian Mathematics, New Age Publishers Ltd, New Delhi, 1995.
[3] V.E. Hoggatt Jr, Fibonacci and Lucas numbers, Houghton-Mifflin, Boston 1969.

[4] B.L. Bhatia, and Mohanty, Supriya , Nasty numbers and their characterization, Mathematical Education, II (1), 1985,34-37.

[5] M.A.Gopalan., K.Geetha and Manju Somanath (Oct- Nov 2014): M-gonal $\pm$ 2 = A perfect square, International journal of Innovative Technology and Research, Vol.2, Issue 6, Pp.1618-1620.

[6] M.A.Gopalan., K.Geetha and Manju Somanath, "Equality of centered hexagonal number with special M-gonal number", Global journal of Mathematics and Mathematical Science, Vol.3, No.1, PP.41-45, 2013.

[7] M.A.Gopalan., K.Geetha and Manju Somanath, M-gonal $\pm$ n = A perfect square, n= 5,17, Jamal Academic research journal: An interdisciplinary special issue, Pp.259-264, February 2016.