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A New Fixed Point theorem satisfying Property *P* in G- Metric Space

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Abstract- In this paper, applying the method of rational contraction we prove general fixed point theorem using the map which satisfy property *P*.

Keywords- G-metric Space; Rational Contraction; Property *P*.

1. INTRODUCTION

The study of fixed point theory has been at the centre of vigorous activity and it has a wide range of applications in applied mathematics and sciences. Various fixed point theorems has been proved in various metric spaces in which one of the important metric space is G-metric space in which triangular inequality was replaced by quadrilateral inequality.

Mustafa and Sims [7] introduced the concept of G-metric space which was generalization of the metric space. The idea of Generalization of metric space were proposed by Gahler [1, 2] (called 2-metric spaces) and Dhage [3,4] (called D-metric spaces). Hsiao [5] showed that, for every contractive definition, with $x_n := T^n x_0$, every orbit is linearly dependent, thus to provide fixed point theorem in such spaces are invalid. So, it was shown that certain theorems involving Dhage's D-metric spaces are flawed, and most of the result claimed by Dhage and other are invalid. These errors were point out by Mustafa and Sims in [6], among others. They also introduced a valid generalized metric space structure, which they call G-metric spaces in which non-negative real number is assigned to every triplet of elements. Some other papers dealing with G-metric spaces are those in [7-11]. To prove the existence of solutions for a class of integral equation, fixed point theorems in G-metric space helps a lot. As before many research paper provides various theorems and broad section of its applications, the main aim of this paper to prove a fixed point theorem for contraction mapping. Some important papers which deal with Property *P* are those in [12-14].

In this paper we prove a new fixed point theorem using contraction based on rational map.

Let Ω be a self-map of a complete metric space (X, d) with a nonempty fixed point set $F(\Omega)$.Then Ω is said to satisfy property *P* if $F(\Omega) = F(\Omega^n)$ for each $n \in \mathbb{N}$. The maps which satisfy property *P* have an interesting property that they have no nontrivial periodic points.

2. PRELIMINARIES

Definition 1.1 ([11]). Let X be a non-empty set and $G : X^3 \to [0,\infty)$ be a function satisfying the following axioms:

(G1) $G(x,y,z) = 0$ if $x = y = z$,

(G2) $0 < G(x,x,y)$ $\forall x,y \in X$ with $x \neq y$

(G3) $G(x,x,y) \leq G(x,y,z)$ $\forall x,y,z \in X$, with $z \neq y$

(G4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \ldots$ (Symmetry in all three variables).

(G5) $G(x,y,z) = G(x,a,a) + G(a,y,z)$ $\forall$ $x,y,z,a \in X$, (rectangular inequality)

Then the function G is called a generalized metric, or specifically a G-metric on X and the pair (X,G) is called a G-metric space.

Definition 1.2 ([11]) Let (X,G) be a G-metric space and let $\{x_n\}$ be a sequence of points in X, a point x in X is said to be the limit of the sequence $\{x_n\}$ and if $G(x, x_n, x_n)=0$ one says that sequence $\{x_n\}$ is *G*-convergent to x. Thus $x_n \to x$ as $n \to \infty$ in a *G*-metric space (X,G), then for each $\in > 0$ there exists a positive integer N such that $G(x,x_n,x_m) < \varepsilon$ for all m, n ≥ N.

Now, we state some results from the papers ([2]-[6]) which are helpful for proving our main results

Proposition 1.3 ([11]). Let (X,G) be a G-metric space. Then the following are equivalent:

$\{x_n\}$ is G-convergent to x,

$G(x_n, x_n, x) \to$ as $n \to \infty$,

$G(x_n, x, x) \to$ as $n \to \infty$,

$G(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Definition 1.4 ([10]). Let (X, G) be G-metric space. A sequence is called G-Cauchy if, for each $\varepsilon > 0$. there exists a positive integer N such that $G(x_n, x_m x_1) < \varepsilon$, for all $n, m, l \in N, i.e., G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.

Definition 1.5 ([10]) A G-Metric space (X, G) is said to be G-complete if every G-Cauchy sequence in (X, G) is G-convergent X.

Definition 1.6 ([11]). If *(X, G) and (X', G') be* two G-metric space and let $f : (X, G) \to (X', G')$ be a function, then f is said to be G-continuous at a point $x_0 \in X$ if given $\varepsilon > 0$, there exists $\delta > 0$ such that for $x, y \in X$ and $G(x_0, x, y) < \delta$ implies $G'(f(x_0), f(x), f(y)) < \varepsilon$. A function f is *G*-continuous at *X* if and only if it is *G*-continuous at all $x_0 \in X$ or function f is said to be *G*-continuous at a point $x_0 \in X$ if and only if it is *G*-sequentially continuous at x_0, that is, whenever $\{x_n\}$ is *G*-convergent to x_0, $\{f(x_n)\}$ is *G*-convergent to $f(x_0)$.

3. Main Results

Theorem 2.1 Let a complete *G*-metric Space

(X, G) and $\Omega: X \to X$ satisfying,

$$G(\Omega x, \Omega y, \Omega z) \le k \max \begin{pmatrix} \frac{G(\Omega y, \Omega x, \Omega y)}{G(\Omega x, y, y) + G(z, z, \Omega y)} . G(x, y, z), \\ \frac{G(\Omega y, \Omega y, z)}{G(y, z, z) + G(\Omega z, \Omega z, z)} . G(z, \Omega y, \Omega y), \\ \frac{G(y, \Omega z, \Omega z) + G(x, \Omega y, \Omega y)}{4}, \\ \frac{G(x, \Omega y, \Omega y) + G(\Omega z, \Omega y, \Omega z)}{4} \end{pmatrix} \tag{2.1}$$

for $\forall\, x, y, z \in X$ and $k \in \mathbb{R}$, such that $0 \le k \le 1$. Then Ω has a unique fixed point (say p) and Ω is *G*-Continuous at *p*.

Proof. Many fundamental ideas of sequencing are introduced within till date papers, in the concrete setting let $x_0 \in X$ be an arbitrary element and we can define the sequence $\{x_n\}$ by $x_n = T^n x_0$. It is assumed that $x_n \ne x_{n+1}$ for each $n \in \mathbb{N} \cup \{0\}$, then there exists an element $N \in \mathbb{N}$ such that $x_N = x_{N+1}$, then x_N is a fixed point of Ω.

From (2.1), with $x = x_{n-1}, y = z = x_n$,

$$G(x_n, x_{n+1}, x_{n+1}) = G(Tx_{n-1}, Tx_n, Tx_n)$$

$$\le k \max \begin{pmatrix} \frac{G(\Omega x_n, \Omega x_{n-1}, \Omega x_n)}{G(\Omega x_{n-1}, x_n, x_n) + G(x_n, x_n, \Omega x_n)} . \\ G(x_{n-1}, x_n, x_n), \\ \frac{G(\Omega x_n, \Omega x_n, x_n)}{G(x_n, x_n, x_n) + G(\Omega x_n, \Omega x_n, x_n)} . \\ G(x_n, \Omega x_n, \Omega x_n), \\ \frac{G(x_n, \Omega x_n, \Omega x_n) + G(x_{n-1}, \Omega x_n, \Omega x_n)}{4}, \\ \frac{G(x_{n-1}, \Omega x_n, \Omega x_n) + G(\Omega x_n, \Omega y, \Omega z)}{4} \end{pmatrix}$$

$$\le k \max \begin{pmatrix} \frac{G(x_{n+1}, x_n, x_{n+1})}{G(x_n, x_n, x_n) + G(x_n, x_n, x_{n+1})} . \\ G(x_{n-1}, x_n, x_n), \\ \frac{G(x_{n+1}, x_{n+1}, x_n)}{G(x_n, x_n, x_n) + G(x_{n+1}, x_{n+1}, x_n)} . \\ G(x_n, x_{n+1}, x_{n+1}), \\ \frac{G(x_n, x_{n+1}, x_{n+1}) + G(x_{n-1}, x_{n+1}, x_{n+1})}{4}, \\ \frac{G(x_{n-1}, x_{n+1}, x_{n+1}) + G(x_{n+1}, x_{n+1}, x_{n+1})}{4} \end{pmatrix}$$

$$\le k \max \begin{pmatrix} G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1}), \\ \frac{G(x_n, x_{n+1}, x_{n+1})}{2}, \\ \frac{G(x_{n-1}, x_{n+1}, x_{n+1})}{4} \end{pmatrix}$$

Let $G(x_n, x_{n+1}, x_{n+1}) \le k\, M_n$, for some $n \in \mathbb{N}$,

if, $M_n = G(x_n, x_{n+1}, x_{n+1})$. Then we have

$$G(x_n, x_{n+1}, x_{n+1}) \le k\, G(x_n, x_{n+1}, x_{n+1})$$

Which is contradiction, since x_n's are distinct.

Suppose that there is an $n \in \mathbb{N}$ for which $M_n = G(x_{n-1}, x_{n+1}, x_{n+1})/2$. Using the property *(G5)*,

$$G(x_{n-1}, x_{n+1}, x_{n+1}) \le G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1}),$$

and hence we have

$$G(x_n, x_{n+1}, x_{n+1}) \le \frac{k}{2}[G(x_{n-1}, x_n, x_n) + G(x_n, x_{n+1}, x_{n+1})]$$

which implies that

$$G(x_n, x_{n+1}, x_{n+1}) \le \frac{k}{2-K} G(x_{n-1}, x_n, x_n) < kG(x_{n-1}, x_n, x_n),$$

Because of k which is less than 1.

we get congruent result if we decided on

$M_n = \frac{G(x_{n-1}, x_{n+1}, x_{n+1})}{4}$

which gives,

$$G(x_n, x_{n+1}, x_{n+1}) \le kG(x_{n-1}, x_n, x_n) \qquad (2.2)$$

Now for every single one of the $n \in \mathbb{N}$, we have

$$G(x_n, x_{n+1}, x_{n+1}) \le k\, G(x_{n-1}, x_n, x_n) \le \cdots \le k^n G(x_0, x_1, x_1). \qquad (2.3)$$

for every $m.n \in N, m > n$, using *(G5)* and (2.3), we have

$$G(x_n, x_m, x_m) \le G(x_n, x_{n+1}, x_{n+1}) + \cdots + G(x_{m-1}, x_m, x_m)$$

$$\le (k^n + \cdots + k^{m-1})\, G(x_0, x_1, x_1) \le \frac{k^n}{1-k} G(x_0, x_1, x_1).$$

$G(x_n, x_m, x_m) \to 0$ as $n \to \infty$, since $k < 1$

Therefore $\{x_n\}$ is G-convergent, since X is a G-complete. The limit point p (say) must belong to X.

Hence $lim_{n\to\infty} x_n = p$

From (2.1) with $x = x_n,\ y = z = p$,

$$G(x_{n+1}, \Omega p, \Omega p) = G(\Omega x_n, \Omega p, \Omega p)$$

$$\le k\ max \begin{pmatrix} \frac{G(\Omega p, \Omega p, \Omega p)}{G(\Omega x_n, p, p) + G(p, p, \Omega p)} . G(x_n, p, p), \\ \frac{G(\Omega p, \Omega p, p)}{G(y, z, z) + G(\Omega z, \Omega z, z)} . G(z, \Omega y, \Omega y), \\ \frac{G(p, \Omega p, \Omega p) + G(x_n, \Omega p, \Omega p)}{4}, \\ \frac{G(x_n, \Omega p, \Omega p) + G(\Omega p, \Omega p, \Omega p)}{4} \end{pmatrix}$$

$$\le k\ max \begin{pmatrix} \frac{G(\Omega p, \Omega p, \Omega p)}{G(x_{n+1}, p, p) + G(p, p, \Omega p)} . G(x_n, p, p), \\ \frac{G(\Omega p, \Omega p, p)}{G(p, p, p) + G(\Omega p, \Omega p, p)} . G(p, \Omega p, \Omega p), \\ \frac{G(p, \Omega p, \Omega p) + G(x_n, \Omega p, \Omega p)}{4}, \\ \frac{G(x_n, \Omega p, \Omega p) + G(\Omega p, \Omega p, \Omega p)}{4} \end{pmatrix}$$

Since, $lim_{n\to\infty} x_n = p$, we can take n approaches to infinity in above equation and on that account we have,

$$G(p, \Omega p, \Omega p)$$

$$\le k\ max \begin{pmatrix} \frac{G(\Omega p, \Omega p, \Omega p)}{G(p, p, p) + G(p, p, \Omega p)} . G(p, p, p), \\ \frac{G(\Omega p, \Omega p, p)}{G(p, p, p) + G(\Omega p, \Omega p, p)} . G(p, \Omega p, \Omega p), \\ \frac{G(p, \Omega p, \Omega p) + G(p, \Omega p, \Omega p)}{4}, \\ \frac{G(p, \Omega p, \Omega p) + G\Omega p, \Omega p, \Omega p)}{4} \end{pmatrix}$$

$$G(p, \Omega p, \Omega p) \le\ k\ max \begin{pmatrix} 0, G(p, \Omega p, \Omega p), \\ \frac{G(p, \Omega p, \Omega p)}{2}, \\ \frac{G(p, \Omega p, \Omega p)}{4} \end{pmatrix}$$

$$G(p, \Omega p, \Omega p) \le k\ G(p, \Omega p, \Omega p),$$

which implies that $G(p, \Omega p, \Omega p) = 0$, since k < 1 and accordingly p = Ωp

Uniqueness

It is proved that p is a fixed point now let q be any other fixed point. Using (2.1), we have $G(\Omega p, \Omega q, \Omega q)$

$$\le k\ max \begin{pmatrix} \frac{G(\Omega q, \Omega p, \Omega q)}{G(\Omega p, q, q) + G(q, q, \Omega q)} . G(p, q, q), \\ \frac{G(\Omega q, \Omega q, q)}{G(q, q, q) + G(\Omega q, \Omega q, q)} . G(q, \Omega q, \Omega q), \\ \frac{G(q, \Omega q, \Omega q) + G(p, \Omega q, \Omega q)}{4}, \\ \frac{G(p, \Omega q, \Omega q) + G(\Omega q, \Omega q, \Omega q)}{4} \end{pmatrix}$$

By the definition of fixed point

$G(p, q, q)$

$$\leq k\ max\begin{pmatrix}\frac{G(q,p,q)}{G(p,q,q)+G(q,q,q)}.G(p,q,q),\\ \frac{G(q,q,q)}{G(q,q,q)+G(q,q,q)}.G(q,q,q),\\ \frac{G(q,q,q)+G(p,q,q)}{4},\\ \frac{G(p,q,q)+G(q,q,q)}{4}\end{pmatrix}$$

$$\leq k\ max\ \left(G(p,q,q),0,\frac{G(p,q,q)}{4},\frac{G(p,q,q)}{4}\right)$$

It is obvious that,

$$G(p,q,q) \leq k\ G(p,q,q)$$

since $k < 1$.

Now, to show that Ω is G-continuous for all the values of n,

Let $\{y_n\} \subset X$ be an arbitrary sequence having one of the limit point as p. using (2.1), we have

$G(\Omega y_n, \Omega p, \Omega y_n)$

$$\leq max\begin{pmatrix}\frac{G(\Omega p,\Omega y_n,\Omega p)}{G(\Omega y_n,p,p)+G(y_n,y_n,\Omega p)}.\\ G(y_n,p,y_n),\\ \frac{G(\Omega p,\Omega p,y_n)}{G(p,y_n,y_n)+G(\Omega y_n,\Omega y_n,y_n)}.\\ G(y_n,\Omega p,\Omega p),\\ \frac{G(p,\Omega y_n,\Omega y_n)+G(y_n,\Omega p,\Omega p)}{4},\\ \frac{G(y_n,\Omega p,\Omega p)+G(\Omega p,\Omega p,\Omega y_n)}{4}\end{pmatrix}$$

since p is the fixed point,

$G(\Omega y_n, p, \Omega y_n)$

$$\leq max\begin{pmatrix}\frac{G(p,\Omega y_n,p)}{G(\Omega y_n,p,p)+G(y_n,y_n,p)}.G(y_n,p,y_n),\\ \frac{G(p,p,y_n)}{G(p,y_n,y_n)+G(\Omega y_n,\Omega y_n,y_n)}.G(y_n,p,p),\\ \frac{G(p,\Omega y_n,\Omega y_n)+G(y_n,p,p)}{4},\\ \frac{G(y_n,p,p)+G(\Omega y_n,p,\Omega y_n)}{4}\end{pmatrix}$$

Using the relation by *(G5)*

$$G(y_n,\Omega y_n,\Omega y_n) \leq G(y_n,p,p) + G(p,\Omega y_n,\Omega y_n)$$

let, $\quad G(\Omega y_n, p, \Omega y_n) \leq k\omega$

Case 1. If, for some n, ω is equal to $\frac{G(p,\Omega y_n,p)}{G(\Omega y_n,p,p)+G(y_n,y_n,p)}.G(y_n,p,y_n)$,

then we have,

$$G(\Omega y_n, p, \Omega y_n) \leq k.\omega$$

since, $\frac{G(p,\Omega y_n,p)}{G(\Omega y_n,p,p)+G(y_n,y_n,p)} \leq 1$

$$G(\Omega y_n, p, \Omega y_n) \leq kG(y_n, p, y_n)$$

Case 2. If, for some n,

$\omega = \frac{G(p,p,y_n)}{G(p,y_n,y_n)+G(\Omega y_n,\Omega y_n,y_n)}.G(y_n,p,p)$, then we have

$$G(\Omega y_n, p, \Omega y_n) \leq k.\omega$$

since, $\frac{G(p,p,y_n)}{G(p,y_n,y_n)+G(\Omega y_n,\Omega y_n,y_n)} \leq 1$

$$G(\Omega y_n, p, \Omega y_n) \leq kG(y_n, p, p)$$

Case 3. If, for some n,

$\omega = \frac{G(y_n,\Omega y_n,\Omega y_n)+G(y_n,p,\Omega y_n)}{4}$

then we have

$$G(\Omega y_n, p, \Omega y_n) \leq k.\omega$$

$$G(\Omega y_n, p, \Omega y_n) \leq \frac{k}{4-k}G(y_n, p, p).$$

Case 4. If, for some n,

$\omega = \frac{G(y_n,p,p)+G(\Omega y_n,p,\Omega y_n)}{4}$, then, we have

$$G(\Omega y_n, p, \Omega y_n) \leq k.\omega,$$

$$G(\Omega y_n, p, \Omega y_n) \leq \frac{K}{4-k}G(y_n, p, p).$$

Therefore, for all n, $lim\, G(p, \Omega y_n, \Omega y_n) = 0$ and Ω is G-continuous at p.

Property P

Let Ω be a self-map of a complete metric space *(X, d)* with a nonempty fixed point set $F(\Omega)$.Then Ω is said to satisfy property P if $F(\Omega) = F(\Omega^n)$ for each $n \in \mathbb{N}$.

In this section we shall show that maps satisfying (2.1) possess property *P*.

Theorem 2.2 Under the condition of theorem 2.1, Ω has property *P*.

Proof. From Theorem 2.1, Ω has a fixed point. Therefore $F(\Omega^n) \neq \emptyset$ for each $n \in \mathbb{N}$. Fix $n > 1$

and assume that $p \in F(\Omega^n)$. We wish to show that $p \in F(\Omega)$.

Suppose that $p \neq Tp$. Using (2.1),

$$G(p, \Omega p, \Omega p) = G(\Omega^n p, \Omega^{n+1} p, \Omega^{n+1} p)$$

$$\leq k\, max \begin{pmatrix} \frac{G(\Omega^{n+1}p,\Omega^n p,\Omega^{n+1}p)}{G(\Omega^n p,\Omega^n p,\Omega^n p)+G(\Omega^n p,\Omega^n p,\Omega^{n+1}p)}. \\ G(\Omega^{n-1}p, \Omega^n p, \Omega^n p), \\ \frac{G(\Omega^{n+1}p,\Omega^{n+1}p,\Omega^n p)}{G(\Omega^n p,\Omega^n p,\Omega^n p)+G(\Omega^{n+1}p,\Omega^{n+1}p,\Omega^n p)}. \\ G(\Omega^n p, \Omega^{n+1}p, \Omega^{n+1}p), \\ \frac{G(\Omega^n p,\Omega^{n+1}p,\Omega^{n+1}p)+G(\Omega^{n-1}p,\Omega^{n+1}p,\Omega^{n+1}p)}{4}, \\ \frac{G(\Omega^{n-1}p,\Omega^{n+1}p,\Omega^{n+1}p)+G(\Omega^{n+1}p,\Omega^{n+1}p,\Omega^{n+1}p)}{4} \end{pmatrix}$$

$$= k\ G(\Omega^{n-1}p, \Omega^n p, \Omega^n p)$$

$$\leq k^2 G(\Omega^{n-2}p, \Omega^{n-1}p, \Omega^{n-1}p)$$

$$\leq \cdots \leq k^n G(p, \Omega p, \Omega p),$$

a contradiction by the reason of value of *k*.

Therefore $p \in F(\Omega)$ and Ω has property *P*.

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