Solution of Population Growth and Decay Problems by Using Mohand Transform

[1]*Sudhanshu Aggarwal, [2]Nidhi Sharma, [3] Raman Chauhan
[1]Assistant Professor, Department of Mathematics, National P.G. College Barhalganj, Gorakhpur-273402, U.P., India
[2, 3]Assistant Professor, Department of Mathematics, Noida Institute of Engineering & Technology, Greater Noida-201306, U.P., India
sudhanshu30187@gmail.com, 6nidhisharma@gmail.com, raman28chauhan@gmail.com

Abstract: The population growth and decay problems generally appear in the field of zoology, chemistry, social science, biology, physics etc. In the present paper, we applied Mohand transform for solving population growth and decay problems and in application section, some numerical applications are given to explain the usefulness of Mohand transform for solving population growth and decay problems.

Keywords: Mohand transform, Inverse Mohand transform, Population growth problem, Decay problem, Half-life.

1. INTRODUCTION

Mathematically the population growth (growth of a country, or bacteria or a plant, or a cell, or an organ, or a species) is governed by the first order linear ordinary differential equation [1-10]

$$\frac{dN}{dt} = kN \ldots\ldots\ldots\ldots\ldots(1)$$

with initial condition as

$$N(t_0) = N_0 \ldots\ldots\ldots..(2)$$

where k is a positive real number, N and N_0 are the amount of populations at time t and initial time t_0.

Equation (1) is called the Malthusian law of population growth.

On the other hand the decay problem of the material (radioactive substance) is mathematically defined by the first order linear ordinary differential equation [7, 9-10]

$$\frac{dN}{dt} = -kN \ldots\ldots\ldots\ldots\ldots(3)$$

with initial condition as

$$N(t_0) = N_0 \ldots\ldots\ldots..(4)$$

where N is the amount of the material at time t, k is a positive real number and N_0 is the amount of the material at initial time t_0. Since the mass of the substance decreases with time and so the time derivative of N denoted by $\frac{dN}{dt}$ must be negative. This is the reason behind taking the negative sign in the R.H.S. of equation (3).

The Mohand transform of the function $F(t)$ is defined as [11]:

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt$$

$$= R(v), t \geq 0, k_1 \leq v \leq k_2$$

where M is Mohand transform operator.

The sufficient conditions for the existence of Mohand transform of the function $F(t)$ for $t \geq 0$ are

a) $F(t)$ must be piecewise continuous.
b) $F(t)$ must be of exponential order.

Kumar et al. [12] used Mohand transform for solving linear Volterra integro-differential equations. Aggarwal et al. [13] solved population growth and decay problems using Laplace transform. Aggarwal et al. [14] applied Kamal transform to solve the population growth and decay problems. Application of Elzaki transform for solving population growth and decay problems wa given by Aggarwal et al. [15]. Aggarwal et al. [16] used Mahgoub transform for solving population growth and decay problems. Aggarwal et al. [17] applied Aboodh transform for solving the population growth and decay problems. Aggarwal et al. [18] gave the solution of linear Volterra integral equations of second kind using Mohand transform. Mohand transform of Bessel's functions was defined by Aggarwal et al. [19].

The aim of the present work is to finding the solution of population growth and decay problems by applying Mohand transform without any large calculation work.

2. LINEARITY PROPERTY OF MOHAND TRANSFORM

If $M\{F(t)\} = H(v)$ and $M\{G(t)\} = I(v)$ then $M\{aF(t) + bG(t)\} = aM\{F(t)\} + bM\{G(t)\}$

$\Rightarrow M\{aF(t) + bG(t)\} = aH(v) + bI(v)$,

where a, b are arbitrary constants.

3. MOHAND TRANSFORM OF SOME ELEMENTARY FUNCTIONS [11-12]

S.N.	$F(t)$	$M\{F(t)\} = R(v)$
1.	1	v
2.	t	1
3.	t^2	$\frac{2!}{v}$
4.	$t^n, n\epsilon N$	$\frac{n!}{v^{n-1}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$
6.	e^{at}	$\frac{v^2}{v-a}$
7.	$sinat$	$\frac{av^2}{(v^2+a^2)}$
8.	$cosat$	$\frac{v^3}{(v^2+a^2)}$
9.	$sinhat$	$\frac{av^2}{(v^2-a^2)}$
10.	$coshat$	$\frac{v^3}{(v^2-a^2)}$

4. INVERSE MOHAND TRANSFORM

If $M\{F(t)\} = R(v)$ then inverse Mohand transform of $R(v)$ is given by $F(t)$ and mathematically it is expressed as $F(t) = M^{-1}\{R(v)\}$

where M^{-1} is the inverse Mohand transform operator.

5. LINEARITY PROPERTY OF INVERSE MOHAND TRANSFORM

If $M^{-1}\{H(v)\} = F(t)$ and $M^{-1}\{I(v)\} = G(t)$ then $M^{-1}\{aH(v) + bI(v)\}$

$= aM^{-1}\{H(v)\} + bM^{-1}\{I(v)\}$

$\Rightarrow M^{-1}\{aH(v) + bI(v)\} = aF(t) + bG(t)$,

where a, b are arbitrary constants.

6. INVERSE MOHAND TRANSFORM OF SOME ELEMENTARY FUNCTIONS

S.N.	$R(v)$	$F(t) = M^{-1}\{R(v)\}$
1.	v	1
2.	1	t
3.	$\frac{1}{v}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^{n-1}}$	$\frac{t^n}{n!}, n\epsilon N$
5.	$\frac{1}{v^{n-1}}$	$\frac{t^n}{\Gamma(n+1)}, n > -1$
6.	$\frac{v^2}{v-a}$	e^{at}
7.	$\frac{v^2}{(v^2+a^2)}$	$\frac{sinat}{a}$

8.	$\frac{v^3}{(v^2+a^2)}$	$cosat$
9.	$\frac{v^2}{(v^2-a^2)}$	$\frac{sinhat}{a}$
10.	$\frac{v^3}{(v^2-a^2)}$	$coshat$

7. MOHAND TRNANSFORM OF THE DERIVATIVES OF THE FUNCTION $F(t)$ [11-12]

If $M\{F(t)\} = R(v)$ then

a) $M\{F'(t)\} = vR(v) - v^2F(0)$
b) $M\{F''(t)\} = v^2R(v) - v^3F(0) - v^2F'(0)$
c) $M\{F^{(n)}(t)\} = v^nR(v) - v^{n+1}F(0) - v^nF'(0) - \cdots \ldots - v^2F^{(n-1)}(0)$

8. MOHAND TRANSFORM METHOD FOR POPULATION GROWTH PROBLEM

In this section, we explain the procedure of solving population growth problem which is mathematically given by (1) and (2) by applying Mohand transform.

Applying the Mohand transform on both sides of (1), we have

$M\left\{\frac{dN}{dt}\right\} = kM\{N(t)\} \ldots \ldots \ldots \ldots \ldots \ldots . (5)$

Now applying the property, Mohand transform of derivative of function, on (5), we have

$vM\{N(t)\} - v^2N(0) = kM\{N(t)\} \ldots \ldots (6)$

Using (2) in (6) and on simplification, we have

$(v-k)\, M\{N(t)\} = v^2N_0$

$\Rightarrow M\{N(t)\} = \frac{v^2N_0}{(v-k)} \ldots \ldots \ldots \ldots \ldots (7)$

Operating inverse Mohand transform on both sides of (7), we get the required amount of the population at time t.

$N(t) = N_0M^{-1}\left\{\frac{v^2}{(v-k)}\right\} = N_0e^{kt} \ldots . . (8)$

9. MOHAND TRANSFORM METHOD FOR DECAY PROBLEM

In this section, we explain the procedure of solving decay problem which is given by (3) and (4) by applying Mohand transform..

Applying the Mohand transform on both sides of (3), we have

$M\left\{\frac{dN}{dt}\right\} = -kM\{N(t)\} \ldots \ldots \ldots \ldots \ldots \ldots . (9)$

Now applying the property, Mohand transform of derivative of function, on (9), we have

$vM\{N(t)\} - v^2N(0) = -kM\{N(t)\} \ldots (10)$

Using (4) in (10) and on simplification, we have

$(v+k)\, M\{N(t)\} = v^2N_0$

$\Rightarrow M\{N(t)\} = \frac{v^2N_0}{(v+k)} \ldots \ldots \ldots \ldots \ldots (11)$

Operating inverse Mohand transform on both sides of (11), we get the required amount of substance at time t.

$N(t) = N_0M^{-1}\left\{\frac{v^2}{(v+k)}\right\}$

$\Rightarrow N(t) = N_0e^{-kt} \ldots \ldots \ldots \ldots \ldots \ldots (12)$

10. APPLICATIONS

In this section, some numerical applications are given to explain the usefulness of Mohand transform for solving population growth and decay problems.

Application: 10.1 The population of Australia grows at a rate proportional to the number of people presently living in Australia. If after five years, the population has doubled, and after ten years the population is 20,000, then estimate the number of people initially living in Australia.

Mathematically the above problem can be written as:

$\frac{dN(t)}{dt} = kN(t) \ldots \ldots \ldots \ldots \ldots . (13)$

where N denote the number of people living in the country at any time t and k is the constant

of proportionality. Consider N_0 is the number of people initially living in Australia at $t = 0$.
Applying the Mohand transform on both sides of (13), we have

$$M\left\{\frac{dN(t)}{dt}\right\} = kM\{N(t)\} \dots\dots\dots\dots\dots\dots(14)$$

Now applying the property, Mohand transform of derivative of function, on (14), we have

$$vM\{N(t)\} - v^2 N(0) = kM\{N(t)\} \dots\dots(15)$$

Since at $t = 0, N = N_0$, so using this in (15), we have

$$(v - k)\, M\{N(t)\} = v^2 N_0$$

$$\Rightarrow\ M\{N(t)\} = \frac{v^2 N_0}{(v-k)} \dots\dots\dots\dots..(16)$$

Operating inverse Mohand transform on both sides of (16), we have

$$N(t) = N_0 M^{-1}\left\{\frac{v^2}{(v-k)}\right\} = N_0 e^{kt} \dots.(17)$$

Now at t = 5, $N = 2N_0$, so using this in (17), we have

$$2N_0 = N_0 e^{5k}$$

$$\Rightarrow e^{5k} = 2$$

$$\Rightarrow k = \frac{1}{5} log_e 2 = 0.1386 \dots\dots\dots\dots\dots(18)$$

Now using the condition at $t = 10$, $N = 20{,}000$, in (17), we have

$$20{,}000 = N_0 e^{10k} \dots\dots\dots\dots.(19)$$

Now substituting the value of k from (18) to (19), we get the required number of people initially living in Australia.

$$20{,}000 = N_0 e^{10\times 0.1386}$$

$$\Rightarrow 20{,}000 = 3.9988 N_0$$

$$\Rightarrow N_0 \simeq 5001.50 \dots\dots\dots\dots(20)$$

Application: 10.2 A radioactive material is known to decay at a rate proportional to the amount present. If initially there is 500 milligrams of the radioactive material present and after five hours it is observed that the radioactive material has lost 25 percent of its original mass, find the half life of the radioactive material.

Mathematically the above problem can be written as:

$$\frac{dN(t)}{dt} = -kN(t) \dots\dots\dots\dots\dots.(21)$$

where N denote the amount of radioactive material at time t and k is the constant of proportionality. Consider N_0 is the initial amount of the radioactive material at time $t = 0$.
Applying the Mohand transform on both sides of (21), we have

$$M\left\{\frac{dN(t)}{dt}\right\} = -kM\{N(t)\} \dots\dots\dots\dots\dots\dots(22)$$

Now applying the property, Mohand transform of derivative of function, on (22), we have

$$vM\{N(t)\} - v^2 N(0) = -kM\{N(t)\} \dots(23)$$

Since at $t = 0, N = N_0 = 500$, so using this in (23), we have

$$vM\{N(t)\} - 500v^2 = -kM\{N(t)\}$$

$$\Rightarrow (v + k)\, M\{N(t)\} = 500v^2$$

$$\Rightarrow\ M\{N(t)\} = \frac{500v^2}{(v+k)} \dots\dots\dots\dots..(24)$$

Operating inverse Mohand transform on both sides of (24), we have

$$N(t) = 500M^{-1}\left\{\frac{v^2}{(v+k)}\right\}$$

$$\Rightarrow N(t) = 500e^{-kt} \dots\dots..(25)$$

Now at $t = 5$, the radioactive material has lost 25 percent of its original mass 500 mg so $N = 500 - 125 = 375$, using this in (25), we have

$$375 = 500e^{-5k}$$

$$\Rightarrow e^{-5k} = 0.75$$

$$\Rightarrow k = -\frac{1}{5} log_e 0.75 = 0.0575 \dots\dots\dots(26)$$

We required t when $N = \frac{N_0}{2} = \frac{500}{2} = 250$ so from (25), we have

$$250 = 500e^{-kt} \dots\dots\dots\dots\dots.(27)$$

Now substituting the value of k from (26) to (27), we get the required half-life of the radioactive material.

$$250 = 500e^{-0.0575t}$$

$\Rightarrow e^{-0.0575t} = 0.50$

$\Rightarrow t = -\frac{1}{0.0575} log_e 0.50$

$$\Rightarrow t = 12.05\ hours \ldots\ldots\ldots\ldots..(28)$$

11. CONCLUSION

In the present paper, we have successfully discussed the Mohand transform for solving the population growth and decay problems. The given numerical applications in application section show that the usefulness of Mohand transform for solving population growth and decay problems. These applications also show that there is no need of very large computational work for solving population growth and decay problems using Mohand transform. In the future,we can easily solve the problems of mechanics and electrical circuit using the present scheme.

REFERENCES

[1] Weigelhofer, W.S.; Lindsay, K.A. (1999): Ordinary Differential Equations & Applications: Mathematical Methods for Applied Mathematicians, Physicists, Engineers and Bioscientists, Woodhead.

[2] Roberts, C. (2010): Ordinary Differential Equations: Applications, Models and Computing, Chapman and Hall/ CRC.

[3] Ahsan, Z. (2006): Differential Equations and Their Applications, PHI.

[4] Braun, M. (1975): Differential Equations and Their Applications, Springer.

[5] Abell, M.L.; Braselton, J.P. (2014): Introductory Differential Equations, Academic Press.

[6] Ang, W.T.; Park, Y.S. (2008): Ordinary Differential Equations: Methods and Applications, Universal Publishers.

[7] Gorain, G.C. (2014): Introductory Course on Differential Equations, Narosa.

[8] Zill, D.G.; Cullen, M.R. (1996): Differential Equations with Boundary Value Problems, Thomson Brooks/ Cole.

[9] Bronson, R.; Costa, G.B. (2006): Schaum's Outline of Differential Equations, McGraw-Hill.

[10] Kapur, J.N. (2005): Mathematical Modelling, New-Age.

[11]Mohand, M.; Mahgoub, A. (2017): The new integral transform "Mohand Transform", Advances in Theoretical and Applied Mathematics, 12(2), pp. 113 – 120.

[12] Kumar, P.S.; Gnanavel, M.G.; Viswanathan, A. (2018): Application of Mohand transform for solving linear Volterra integro-differential equations, International Journal of Research in Advent Technology, 6(10), pp. 2554-2556.

[13] Aggarwal, S.; Gupta, A.R.; Singh, D.P.; Asthana, N.; Kumar, N. (2018): Application of Laplace transform for solving population growth and decay problems, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(9), pp. 141-145.

[14] Aggarwal, S.; Gupta, A.R.; Asthana, N.; Singh, D.P. (2018): Application of Kamal transform for solving population growth and decay problems, Global Journal of Engineering Science and Researches, 5(9), pp. 254-260.

[15] Aggarwal, S.; Singh, D.P.; Asthana, N.; Gupta, A.R. (2018): Application of Elzaki transform for solving population growth and decay problems, Journal of Emerging Technologies and Innovative Research, 5(9), pp. 281-284.

[16] Aggarwal, S.; Pandey, M.; Asthana, N.; Singh, D.P.; Kumar, A. (2018): Application of Mahgoub transform for solving population growth and decay problems, Journal of Computer and

Mathematical Sciences, 9(10), pp. 1490-1496.
[17] Aggarwal, S.; Asthana, N.; Singh, D.P. (2018): Solution of population growth and decay problems by using Aboodh transform method, International Journal of Research in Advent Technology, 6(10), pp. 2706-2710.
[18] Aggarwal, S.; Sharma, N;Chauhan, R. (2018): Solution of linear Volterra integral equations of second kind using Mohand transform, International Journal of Research in Advent Technology, 6(11), pp. 3098-3102.
[19] Aggarwal, S.; Raman, R.; Sharma, N. (2018): Mohand transform of Bessel's functions, International Journal of Research in Advent Technology, 6(11), pp. 3034-3038.

Sudhanshu Aggarwal received his M.Sc. degree from M.S. College, Saharanpur in 2007. He has also qualified CSIR NET examination (June-2010, June-2012, June-2013, June-2014 & June-2015) in Mathematical Sciences. He is working as an Assistant Professor in National P.G. College Barhalganj Gorakhpur. He is equipped with an extraordinary caliber and appreciable academic potency. He has around ten years of teaching experience at various engineering colleges affiliated to AKTU. His fields of interest include Integral Transform Methods, Differential and Partial Differential Equations, Integral Equations and Number Theory.

Nidhi Sharma received her M.Sc. degree from Banasthali University Rajasthan in 2015. She is working as an Assistant Professor in Noida Institute of Engineering & Technology, Greater Noida since 2015. She has also qualified IIT Jam exam (2013).She has published many research papers in national and international journals. Her fields of interest include Optimization, Fuzzy Logic and Topological Spaces.

Raman Chauhan has completed M.Sc. (Mathematics) from Indian Institute of Technology, Roorkee in 2011.He is working at NIET, Greater Noida since 2013 as a dynamic, successful and responsible faculty member in the department of Mathematics. He has also qualified IIT Jam exam (2009) and Gate exam (2011, 2013, 2014 & 2015).His fields of interest include Optimization, Integral Equations and Abstract Algebra.