

Performance Evaluation Bulk Arrival and Bulk Service with Multi Server using Queue Model

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Abstract- This paper deal bulk arrival and bulk service queueing model. Arrival and service rate is considered in batch size α and β respectively for multi-server model. Performance measure have been carried out viz average number of customers in the queue, average number of customers in the system, average waiting time of customers in queue, average waiting time of customers in the system, response time and efficiency of the server corresponding to customers. Numerical illustrations have been provided to validate our results.

Index Terms - Bulk arrival, Bulk service, Multi-servers queue model and Transient states.

1. INTRODUCTION

In manufacturing process the work-pieces arrive at a centre in batches and they leave in batches. A batch consists of identical work-pieces that are processed and then transported in batches for further processing. Such a situation can be modeled as queues with bulk arrivals. There is a discipline within the mathematical theory of probability, called a bulk queue (also called batch queue). Various researchers have been focused such issues in dimension. Pang and Whitt [1] motivated by large-scale service systems, and considered an infinite-server queue with batch arrivals, where the service times are dependent within each batch. Zwart et al. [2] examined the existing modal, the dependent between queueing time and wait – to – batch time has been identified. Khalaf et al. [3] introduced four different main servers with interruptions and a stand-by. Bagyam et al. [4] considered bulk arrival general service retrial queueing system where server provides two phases of service-essential and optimal. After each service completion, the server searches for customers in the orbit. Customers may balk or renege at particular times and accidental and active breakdown of the server. Chen et al. [5] Markovian bulk-arrival and bulk service queue incorporating state-dependent control and obtain the behavior of queue length regarding to hitting and busy period are also explored. Ghimire et al. [6] formulated mathematical model with the bulk queueing model with fixed batch size, and also obtained mean waiting time and mean time spent in the system and queue. Briere and Chaudhry [7] and Kambo and Chaudhry [11] used numerical approaches to get the performance indices. Chaudhry and

Templeton [8] gave more extensive study on batch arrival/service queues. Downton [10] derived the relation between limiting queue size distributions at arrival and departure epochs. Dragovic et al. [12] Developed this modeled by $M^X/M/n/m$ queue with finite waiting areas and identical and independent cargo-handling capacities. Gupta and Goswani [13] analyzed a discrete-time infinite buffer bulk-service. Some analytic computational results for discrete-time bulk service queue have been reported. Gupta et al. [14] have discussed the same queueing model for EAS and LAS- DA, and developed a recursive procedure to obtain system length distribution at pervasive arbitrary and outside observer's observation epochs. Kumar et al. [15] proposed various performance indices for multi-server model.

We organized this paper as follows. Section 2 & 3 presented the mathematical notations and model of the queueing system. In section, 4 described mathematical models corresponding to queue system. Performance measure as average number of customers in the queue, average number of customers in the system, average waiting time of customers in the queue and average waiting time of customers in the system is obtained in section 5. In section 6, numerical illustration and graphical representation have been drawn and finally Section, 7 conclude the paper.

2. NOTATION

n = Number of customers in the system

λ = Arrival rate

α = Customers in Group or Batch (as different size)
for arrival rate

μ = Service rate

β = Customers in Group or Batch (as different size) for service rate
 c = Serving rate when $c > 1$ in a system
 ρ = System intensity or utilization factor ($\rho = \alpha\lambda / \beta c\mu$)
 Lq = Average number of customers in the queue
 Ls = Average number of customers in the system
 Wq = Average waiting time of customers in the queue
 Ws = Average waiting time of customers in the system
 Es = Efficiency of the system

3. MODEL DESCRIPTION

Different structures of queuing model have been discussed. Customers requiring services are generated over time by an input source. This service mechanism is described in two ways:

- Single queue with multiple server model
- Multiple queue with multiple server model

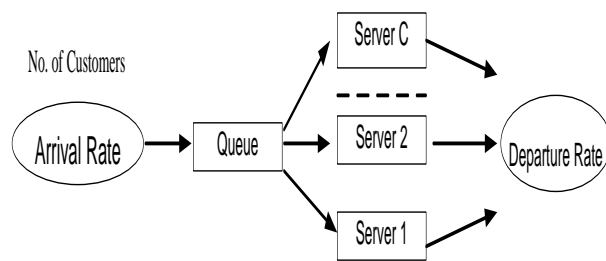


Figure 1: Queuing Model for Single-Queue with Multiple Parallel Servers

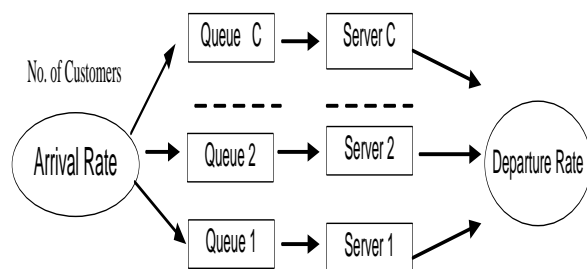


Figure 2: Queuing Model for Multi Queues with Multiple Parallel Servers

We are discussing two types queue system multi queue and multi queue with multi servers are illustrated in figures 1 & 2.

4. MATHEMATICAL MODEL

In $M^a|M^b|C$ queue model, the arrival rate remains same as $M|M|1$ queues but the service rate will depend on the number of servers. The service rate will be $n\mu$ for $n \leq C$. As soon as the number of customers exceeds C , the service rate becomes $C\mu$ as shown in figure 3.

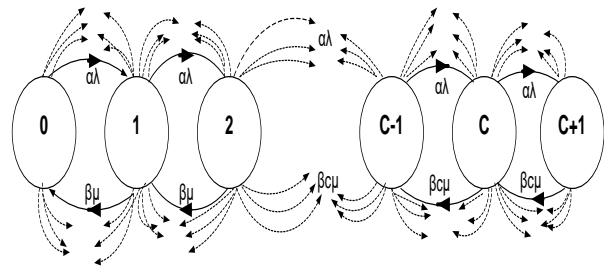


Figure 3 Steady –state diagram

If there is a single server, mean service rate $\mu_n = \mu$ for all n . but there are c servers working independently of each other. Therefore, over all service rate, when there are n customers in the system, may be obtain two situations

- (i) If $n < C$, all the customers may be served simultaneously and there will be no queue. Hence $(C - n)$ number of servers may remain idle and then mean service rate $\mu_n = n\mu$.
- (ii) If $n \geq C$, all the servers are busy, then the maximum number of customers waiting in the queue will be $(n - C)$ and the mean service rate $\mu_n = c\mu$.

(iii) Therefore we assume,

- (a) The average arrival rate $\lambda_n = \lambda$ for all n .
- (b) The average service rate

$$\mu_n = \begin{cases} n \mu & \text{if } 0 \leq n \leq C \\ C \mu & \text{if } n \geq C \end{cases}$$

- (c) The average arrival rate is less than $c \mu$ i.e., $\lambda < c\mu$.

We have for Poisson queue system,

$$P_n = \frac{\lambda_0 \lambda_1 \lambda_2 \dots \lambda_{n-1}}{\mu_1 \mu_2 \mu_3 \dots \mu_n} \times P_0, \quad \text{for } n \geq 1 \quad (1)$$

where $P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \frac{\lambda_0 \lambda_1 \lambda_2 \dots \lambda_{n-1}}{\mu_1 \mu_2 \mu_3 \dots \mu_n}}$ or

$$P_0 = \left[1 + \sum_{n=1}^{\infty} \frac{\lambda_0 \lambda_1 \lambda_2 \dots \lambda_{n-1}}{\mu_1 \mu_2 \mu_3 \dots \mu_n} \right]^{-1} \quad (2)$$

Substituting the assumption (a) and (b) in (1) we get

$$P_n = \begin{cases} \frac{\lambda^n}{1 \cdot \mu \cdot 2 \cdot \mu \cdot 3 \cdot \mu \dots n \mu} P_0 & \text{if } 0 \leq n < C \\ \frac{\lambda^n}{n! \mu^n} P_0 & \text{if } 0 \leq n \leq C \\ \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0 & \text{if } 0 \leq n \leq C \end{cases} \quad (3)$$

and

$$P_n = \frac{\lambda^n}{\{1 \cdot \mu \cdot 2 \cdot \mu \cdot 3 \cdot \mu \dots (c-1) \mu\} \{c \cdot \mu \cdot c \cdot \mu \dots (n-(c-1)) \mu \text{ times}\}} P_0$$

$$\begin{aligned}
 P_n &= \frac{\lambda^n}{\{1.\mu \ 2.\mu \ 3.\mu \dots (c-1)\mu\} \{c.\mu \ c.\mu \ \dots (n-(c+1))\mu \ \text{times}\}} P_0 \\
 &= \frac{\lambda^n}{(c-1)! \mu^{c-1} (c\mu)^{n-c+1}} \quad \text{if } n \geq c \\
 &= \frac{1}{c! c^{n-c}} \left(\frac{\lambda}{\mu}\right)^n P_0 \quad \text{if } n \geq c \quad (4)
 \end{aligned}$$

To find the value of P_0 , we use the fact $\sum_{n=0}^{\infty} P_n = 1$

$$\text{i.e., } \sum_{n=0}^{c-1} P_n + \sum_{n=c}^{\infty} P_n = 1$$

$$\text{i.e., } \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!} \sum_{n=c}^{\infty} \left(\frac{\lambda}{\mu}\right)^n \right] P_0 = 1$$

$$\text{i.e., } \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{c^c}{c!} \sum_{n=c}^{\infty} \left(\frac{\lambda}{c\mu}\right)^n \right] P_0 = 1$$

$$\text{i.e., } \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{c^c}{c!} \sum_{n=c}^{\infty} \left(\frac{\lambda}{c\mu}\right)^n \frac{1}{1-\frac{\lambda}{c\mu}} \right] P_0 = 1$$

$$\therefore \left(\frac{\lambda}{c\mu}\right)^c \left\{ 1 + \left(\frac{\lambda}{c\mu}\right) + \left(\frac{\lambda}{c\mu}\right)^2 + \dots \dots \right\} =$$

$$\left(\frac{\lambda}{c\mu}\right)^c \left\{ \frac{1}{1-\frac{\lambda}{c\mu}} \right\} \quad \text{if } \frac{\lambda}{c\mu} < 1$$

$$\text{i.e., } \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c! \left(1-\frac{\lambda}{c\mu}\right)} \left(\frac{\lambda}{\mu}\right)^c \right] P_0 = 1$$

$$\therefore P_0 = \frac{1}{\left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c! \left(1-\frac{\lambda}{c\mu}\right)} \left(\frac{\lambda}{\mu}\right)^c \right]}$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu - \lambda} \right]^{-1}$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{c^c}{c!} \left(\frac{\rho^c}{1-\rho}\right) \right]^{-1} \quad \text{where } \rho = \frac{\alpha\lambda}{c\beta\mu} \quad (5)$$

5. PERFORMANCE MEASURES OF $M^a|M^b|C$ QUEUEING MODEL

In this section, we calculate the mathematical formulae with $M^a|M^b|C$ queue model for average number of customers in queue, average number of customers in the system, average waiting time of customers in the queue and average waiting time of customers in the system. Also, obtain average number of idle servers corresponding to customers and efficiency of system with queue model with utilization factor. All these mathematical expressions of this queue model are termed as the performance evaluate of the system and described as follows:

5.1 Average number of customers in the queue : (Lq)

$$L_q = \frac{1}{c.c!} \frac{(\rho)^{c+1}}{(1-\rho)^2} P_0$$

5.2 Average number of customers in the system: (Ls)

$$L_s = \frac{1}{c.c!} \frac{(\rho)^{c+1}}{(1-\rho)^2} P_0 + \rho$$

5.3 Average waiting time of a customers in the queue: (Wq)

$$W_q = \frac{1}{\mu} \frac{1}{c.c!} \frac{(\rho)^c}{(1-\rho)^2} P_0$$

5.4 Average waiting time of a customers in the system: (Ws)

$$W_s = \frac{1}{\mu} + \frac{1}{\mu} \frac{1}{c.c!} \frac{(\rho)^c}{(1-\rho)^2} P_0$$

5.5 Average Response time : (Rt)

$$R_t = \frac{C(c, \lambda/\mu)}{c\mu - \lambda} + \frac{1}{\mu}$$

Where $C(c, \lambda/\mu)$ is the probability that an arrival customer is forced to join the (all servers are occupied), referred to as Erlang's C formula

$$C\left(c, \frac{\lambda}{\mu}\right) = \frac{1}{1 + (1-\rho) \frac{c!}{(c\rho)^c} \sum_{n=0}^{c-1} \frac{(c\rho)^n}{c!}}$$

5.6 Efficiency of system with queue model : (Es)

$$E_s = \frac{\text{Average number of customers served}}{\text{Total number of customers}}$$

Special Cases:

In this section, we described two special cases

$M^a/M/1$ and $M/M^b/1$. If

(i) $M^a/M/1$ (see Ref. 17)

(ii) $M/M^b/1$ (see Ref. 11)

6. NUMERICAL APPROACH AND GRAPHIC INTERPRETATIONS

We consider some numerical parameters for average number of customers in queue, average number of customers in the system, average waiting time of customers in the queue and average waiting time of customers in the system. Also, obtain average number of idle servers corresponding to customers and efficiency of system with queue model with utilization factor. In this section, we discuss some cases with arrival rate, server rate, batch size of arrival rate, batch size of service rate with number of servers. We consider the $\lambda = 200$, $\mu = 154$, $C = 2, 3, 4, 5, 6$, $\alpha = 3, 4, 6, 8$, and $\beta = 2, 3, 4, 5, 7, 8, 9$ and obtained the L_q , L_s , W_q , W_s , response time, efficiency of server and probability with utilization factor.

Table 1 Performance measure response time, efficiency of server with utilization

Arrival & Service rate	No. of Servers	ρ	Lq	Ls	Wq	Ws	Rt	Ef
$\alpha = 3$ $\beta = 2$	C= 2	0.974026	4.50567	5.4797	0.0300378	0.0365313	0.0095919	0.0273985
	C= 3	0.649351	0.00961011	0.658961	9.61011e-005	0.00658961	0.0071067	0.0032948
	C=4	0.487013	0.000149637	0.487163	1.99516e-006	0.0064955	0.00670674	0.00243581
	C=5	0.38961	2.21601e-006	0.389613	3.69334e-008	0.00649354	0.00656627	0.00194806
	C=6	0.324675	2.74828e-008	0.324675	5.49655e-010	0.00649351	0.0065137	0.00162338
$\alpha = 3$ $\beta = 4$	C= 2	0.487013	0.0378565	0.524869	0.000504753	0.00699826	0.00874704	3.49913e-005
	C= 3	0.324675	0.000505755	0.325181	1.01151e-005	0.00650362	0.00678301	3.25181e-005
	C=4	0.243506	5.87626e-006	0.243512	1.567e-007	0.00649366	0.00653714	3.24683e-005
	C=5	0.194805	5.30361e-008	0.194805	1.76787e-009	0.00649351	0.00649948	3.24675e-005
	C=6	0.162338	3.70077e-010	0.162338	1.48031e-011	0.00649351	0.00649424	3.24675e-005
$\alpha = 3$ $\beta = 5$	C= 2	0.38961	0.0174314	0.407042	0.000290523	0.00678403	0.00822213	3.39201e-005
	C= 3	0.25974	0.000210768	0.259951	5.26919e-006	0.00649878	0.00666992	3.24939e-005
	C=4	0.194805	2.06692e-006	0.194807	6.88974e-008	0.00649358	0.00651432	3.24679e-005
	C=5	0.155844	1.53715e-008	0.155844	6.4048e-010	0.00649351	0.00649579	3.24675e-005
	C=6	0.12987	8.73977e-011	0.12987	4.36989e-012	0.00649351	0.00649373	3.24675e-005
$\alpha = 3$ $\beta = 8$	C= 2	0.243506	0.00383723	0.247344	0.000102326	0.00659583	0.00733735	3.29792e-005
	C= 3	0.162338	3.37642e-005	0.162371	1.35057e-006	0.00649486	0.00654691	3.24743e-005
	C=4	0.121753	2.22009e-007	0.121753	1.18405e-008	0.00649352	0.00649744	3.24676e-005
	C=5	0.0974026	1.07343e-009	0.0974026	7.15623e-011	0.0064935	0.00649378	3.24675e-005
	C=6	0.0811688	3.91077e-012	0.0811688	3.12862e-013	0.00649351	0.00649352	3.24675e-005

Table 2 Performance measure response time, efficiency of server with utilization

Arrival & Service rate	No. of Servers	ρ	Lq	Ls	Wq	Ws	Rt	Ef
$\alpha = 4$ $\beta = 3$	C= 2	0.865801	0.648006	1.51381	0.00486005	0.0113536	0.0096047	5.67678e-005
	C= 3	0.577201	0.00549511	0.582696	6.18199e-005	0.00655533	0.00709954	3.27766e-005
	C=4	0.4329	8.55407e-005	0.432986	1.28311e-006	0.00649479	0.00668115	3.24739e-005
	C=5	0.34632	1.18703e-006	0.346322	2.22568e-008	0.00649353	0.00654661	3.24676e-005
	C=6	0.2886	1.3492e-008	0.2886	3.03569e-010	0.00649351	0.00650584	3.24675e-005
$\alpha = 4$ $\beta = 5$	C= 2	0.519481	0.048	0.567481	0.000600001	0.00709351	0.00889584	3.54675e-005
	C= 3	0.34632	0.000652893	0.346973	1.22417e-005	0.00650575	0.00682263	3.25287e-005
	C=4	0.25974	7.9376e-006	0.259748	1.9844e-007	0.0064937	0.006547	3.24685e-005
	C=5	0.207792	7.56203e-008	0.207792	2.36314e-009	0.00649351	0.00650135	3.24675e-005
	C=6	0.17316	5.59222e-010	0.17316	2.09708e-011	0.00649351	0.00649454	3.24675e-005
$\alpha = 4$ $\beta = 7$	C= 2	0.371058	0.0148114	0.385869	0.0002592	0.00675271	0.00811194	3.37635e-005
	C= 3	0.247372	0.00017422	0.247546	4.57327e-006	0.00649808	0.00665052	3.24904e-005
	C=4	0.185529	1.64279e-006	0.18553	5.74978e-008	0.00649356	0.00651112	3.24678e-005
	C=5	0.148423	1.16976e-008	0.148423	5.11771e-010	0.00649351	0.00649534	3.24675e-005
	C=6	0.123686	6.35534e-011	0.123686	3.33655e-012	0.00649351	0.00649368	3.24675e-005
$\alpha = 4$ $\beta = 9$	C= 2	0.2886	0.00655539	0.295156	0.000147496	0.006641	0.00760736	3.3205e-005
	C= 3	0.1924	6.54474e-005	0.192466	2.20885e-006	0.00649572	0.00657705	3.24786e-005
	C=4	0.1443	4.99687e-007	0.144301	2.24859e-008	0.00649353	0.00650078	3.24676e-005
	C=5	0.11544	2.83046e-009	0.11544	1.59213e-010	0.00649351	0.0064941	3.24675e-005
	C=6	0.0962001	1.21318e-011	0.0962001	8.18896e-013	0.00649351	0.00649355	3.24675e-005

Table 3 Performance measure response time, efficiency of server with utilization

Arrival & Service rate	No. of Servers	ρ	Lq	Ls	Wq	Ws	Rt	Ef
$\alpha = 4$ $\beta = 3$	C= 2	0.865801	0.648006	1.51381	0.00486005	0.0113536	0.0096047	5.67678e-005
	C= 3	0.577201	0.00549511	0.582696	6.18199e-005	0.00655533	0.00709954	3.27766e-005
	C=4	0.4329	8.55407e-005	0.432986	1.28311e-006	0.00649479	0.00668115	3.24739e-005
	C=5	0.34632	1.18703e-006	0.346322	2.22568e-008	0.00649353	0.00654661	3.24676e-005
	C=6	0.2886	1.3492e-008	0.2886	3.03569e-010	0.00649351	0.00650584	3.24675e-005
$\alpha = 4$ $\beta = 5$	C= 2	0.519481	0.048	0.567481	0.000600001	0.00709351	0.00889584	3.54675e-005
	C= 3	0.34632	0.000652893	0.346973	1.22417e-005	0.00650575	0.00682263	3.25287e-005
	C=4	0.25974	7.9376e-006	0.259748	1.9844e-007	0.0064937	0.006547	3.24685e-005
	C=5	0.207792	7.56203e-008	0.207792	2.36314e-009	0.00649351	0.00650135	3.24675e-005
	C=6	0.17316	5.59222e-010	0.17316	2.09708e-011	0.00649351	0.00649454	3.24675e-005
$\alpha = 4$ $\beta = 7$	C= 2	0.371058	0.0148114	0.385869	0.0002592	0.00675271	0.00811194	3.37635e-005
	C= 3	0.247372	0.00017422	0.247546	4.57327e-006	0.00649808	0.00665052	3.24904e-005
	C=4	0.185529	1.64279e-006	0.18553	5.74978e-008	0.00649356	0.00651112	3.24678e-005
	C=5	0.148423	1.16976e-008	0.148423	5.11771e-010	0.00649351	0.00649534	3.24675e-005
	C=6	0.123686	6.35534e-011	0.123686	3.33655e-012	0.00649351	0.00649368	3.24675e-005
$\alpha = 4$ $\beta = 9$	C= 2	0.2886	0.00655539	0.295156	0.000147496	0.006641	0.00760736	3.3205e-005
	C= 3	0.1924	6.54474e-005	0.192466	2.20885e-006	0.00649572	0.00657705	3.24786e-005
	C=4	0.1443	4.99687e-007	0.144301	2.24859e-008	0.00649353	0.00650078	3.24676e-005
	C=5	0.11544	2.83046e-009	0.11544	1.59213e-010	0.00649351	0.0064941	3.24675e-005
	C=6	0.0962001	1.21318e-011	0.0962001	8.18896e-013	0.00649351	0.00649355	3.24675e-005

Table 4 Performance measure response time, efficiency of server with utilization

Arrival & Service rate	No. of Servers	ρ	Lq	Ls	Wq	Ws	Rt	Ef
$\alpha = 3$ $\beta = 7$	C= 2	0.278293	0.00584058	0.284134	0.00013628	0.00662979	0.00754459	3.31489e-005
	C= 3	0.185529	5.6805e-005	0.185586	1.98818e-006	0.00649549	0.00656951	3.24775e-005
	C=4	0.139147	4.20215e-007	0.139147	1.961e-008	0.00649353	0.00649989	3.24676e-005
	C=5	0.111317	2.30147e-009	0.111317	1.34252e-010	0.00649351	0.00649401	3.24675e-005
	C=6	0.0927644	9.52845e-012	0.0927644	6.66991e-013	0.00649351	0.00649354	3.24675e-005
$\alpha = 5$ $\beta = 7$	C= 2	0.463822	0.0317831	0.495605	0.000444964	0.00693847	0.008632	3.46924e-005
	C= 3	0.309215	0.000417307	0.309632	8.76344e-006	0.00650227	0.00675492	3.25113e-005
	C=4	0.231911	4.6795e-006	0.231916	1.31026e-007	0.00649364	0.00653077	3.24682e-005
	C=5	0.185529	4.05177e-008	0.185529	1.41812e-009	0.00649351	0.00649836	3.24675e-005
	C=6	0.154607	2.70479e-010	0.154607	1.13601e-011	0.00649351	0.00649408	3.24675e-005
$\alpha = 6$ $\beta = 7$	C= 2	0.556586	0.0624533	0.61904	0.000728621	0.00722213	0.00904732	3.61106e-005
	C= 3	0.371058	0.00085931	0.371917	1.50379e-005	0.00650854	0.00686738	3.25427e-005
	C=4	0.278293	1.09404e-005	0.278304	2.55277e-007	0.00649376	0.00655958	3.24688e-005
	C=5	0.222635	1.10299e-007	0.222635	3.21706e-009	0.00649351	0.00650394	3.24675e-005
	C=6	0.185529	8.67326e-010	0.185529	3.03564e-011	0.00649351	0.00649498	3.24675e-005
$\alpha = 9$ $\beta = 7$	C= 2	0.834879	0.480178	1.31506	0.00373472	0.0102282	0.00959441	5.11411e-005
	C= 3	0.556586	0.00466434	0.561251	5.44173e-005	0.00654792	0.00709117	3.27396e-005
	C=4	0.41744	7.21055e-005	0.417512	1.12164e-006	0.00649463	0.00667098	3.24731e-005
	C=5	0.333952	9.78449e-007	0.333953	1.90254e-008	0.00649353	0.00654097	3.24676e-005
	C=6	0.278293	1.08127e-008	0.278293	2.52296e-010	0.00649351	0.006504	3.24675e-005

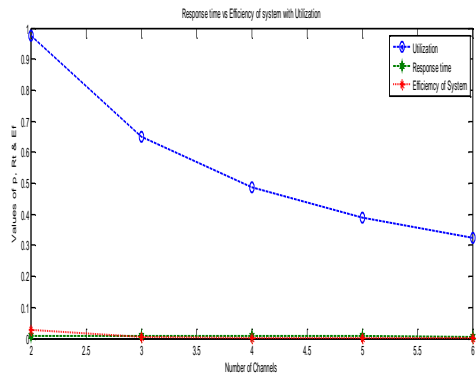


Figure 4

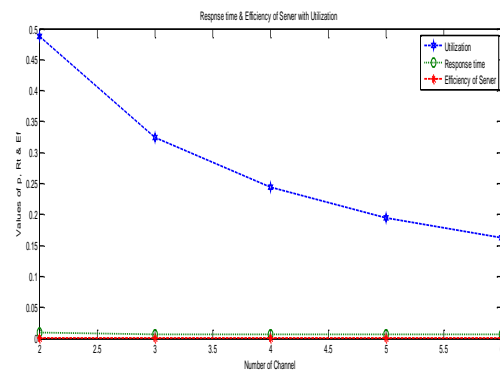


Figure 5

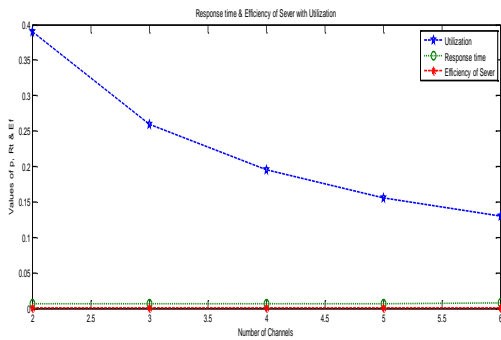


Figure 6

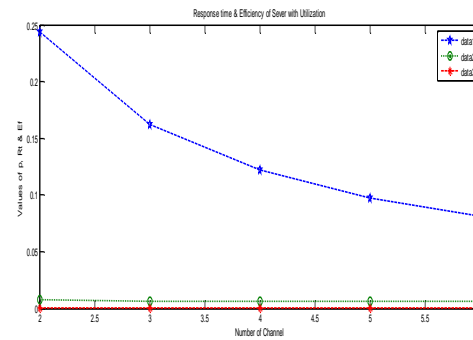


Figure 7

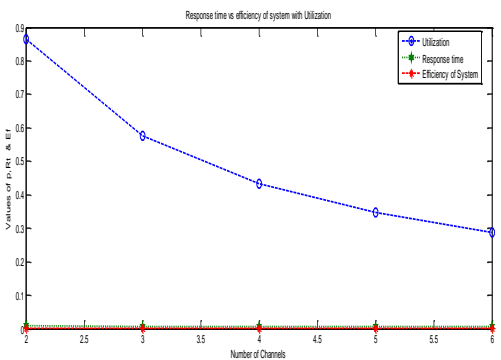


Figure 8

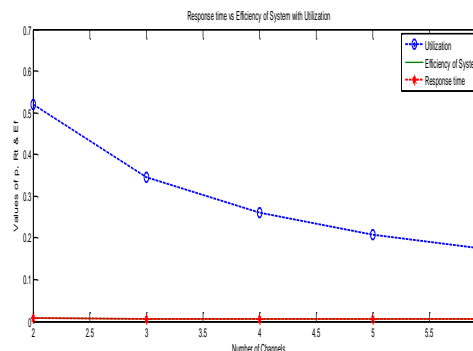


Figure 9

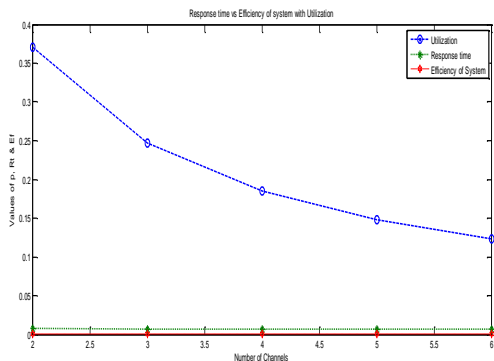


Figure 10

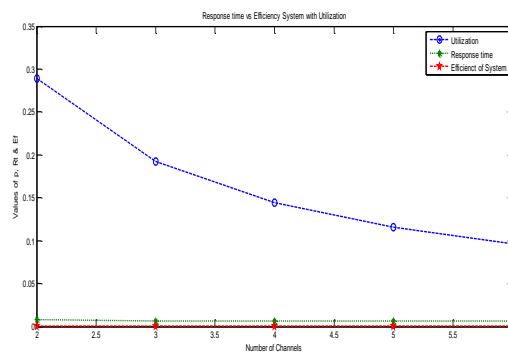


Figure 11

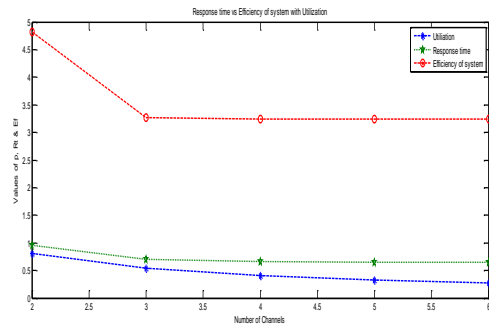


Figure 12

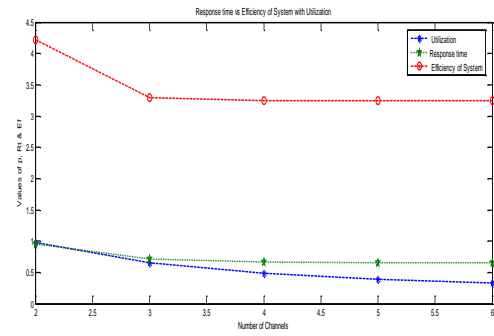


Figure 13

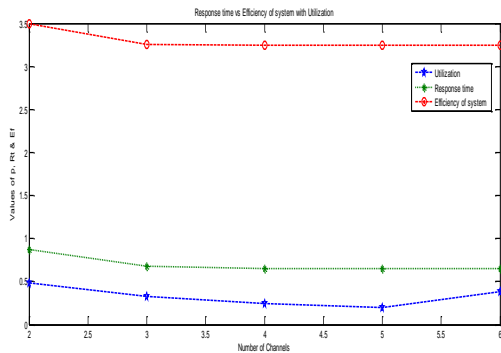


Figure 14

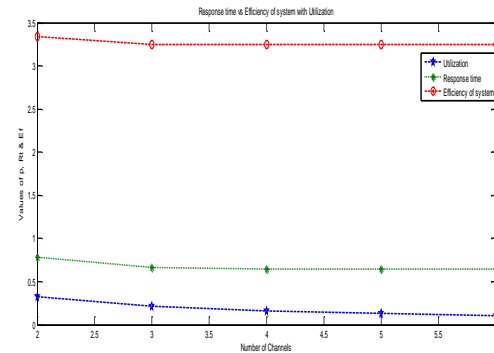


Figure 15

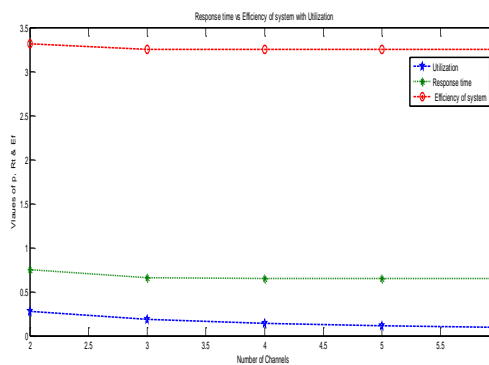


Figure 16

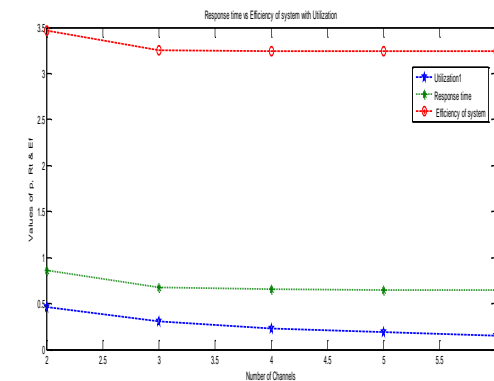


Figure 17

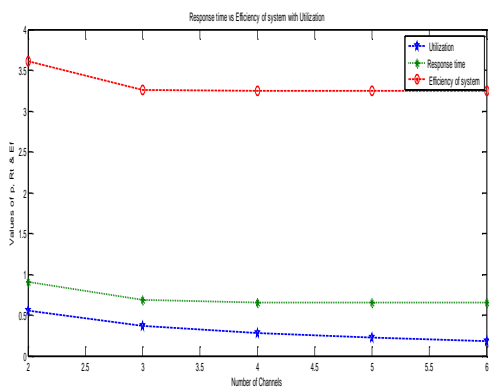


Figure 18

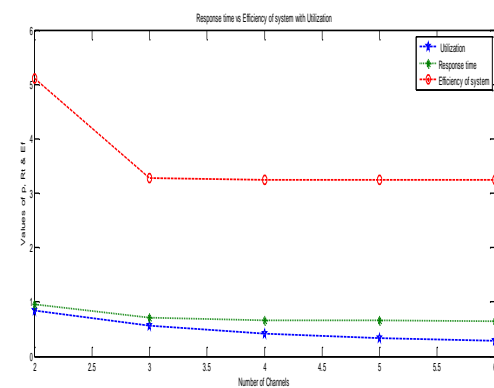


Figure 19

Here, we represented the graphical interpretation between response time vs efficiency of system (server) with utilization in figures 4 to 19 by varying various parameters of bulk arrival and bulk service with multiple number of servers. It has been observed that the performance of system (server) response time and utilization of system goes down when the number of channels increases.

7. CONCLUSION

In this paper, we obtained explicit mathematical formulae for real life problems such as customer's dispatching strategies for bulk arrival and bulk service with multi-server which consist of some combination of customers with system holding and cancellation strategies. The numerical results have been carried out by using MATLAB-9. Result shows applicability in several real-world situations such as passport office, airport, manufacturing system, transportation system, assembly line system and other place in overall supply chain management systems. This model can be studied under the provision of time dependent arrival and service rate which make our model more realistic environment.

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