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Solution of Linear Volterra Integral Equations of Second Kind Using Mohand Transform

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Abstract: Volterra integral equations appear when we convert initial value problem to an integral equation. The solution of Volterra integral equation is much easier than the original initial value problem. Many problems of science and engineering like neutron diffusion problem, heat transfer problem, radiation transfer problem, electric circuit problem etc can be represent mathematically in terms of Volterra integral equation. In the present paper, we discussed the solution of linear Volterra integral equations of second kind using Mohand transform. In this paper, we have taken convolution type linear Volterra integral equations of second kind. In application section, some applications are given to explain the procedure of solution of linear Volterra integral equations of second kind using Mohand transform.

Keywords: Mohand transform, Volterra integral equation, Inverse Mohand Transform, Convolution theorem.

1. INTRODUCTION

Linear Volterra integral equations of second kind has the form [1-6]

$y(x) = f(x) + \lambda \int_0^x K(x,t) y(t) dt$……. (1)

where $y(x)$:unknown function

λ: a non- zero parameter

$K(x,t)$: kernel of the integral equation

$f(x)$:known real-valued function

A the new integral transform "Mohand transform'' of the function $F(t)$ for $t \geq 0$ was given by Mohand and Mahgoub [7] as

$M\{F(t)\} = v^2 \int_0^\infty F(t) e^{-vt} dt = R(v), k_1 \leq v \leq k_2$…………….. (2)

where M is called Mohand transform operator.

If the function $F(t)$ for $t \geq 0$ is piecewise continuous and of exponential order then Mohand transform of the function $F(t)$ for $t \geq 0$ exists. These conditions are sufficient conditions for the existence of Mohand transform of the function $F(t)$ for $t \geq 0$

Kumar et al. [8] used Mohand transform for solving linear Volterra integro-differential equations. Aggarwal et al. [9] applied Aboodh transform for solving linear Volterra integral equations. A new application of Kamal transform for solving linear Volterra integral equations was given by Aggarwal et al. [10]. Aggarwal et al. [11] discussed the new application of Mahgoub transform for solving linear Volterra integral equations.

The object of the present research is to determine the solution of linear Volterra integral equations of second kind using Mohand transform without large computational work and spending a little time.

2. SOME USEFUL PROPERTIES OF MOHAND TRANSFORM:

2.1 Linearity property:

If $M\{F_1(t)\} = R_1(v)$ and $M\{F_2(t)\} = R_2(v)$ then

$M\{aF_1(t) + bF_2(t)\} = aM\{F_1(t)\} + bM\{F_2(t)\}$

$= aR_1(v) + bR_2(v)$, where a, b are arbitrary constants.

Proof: By the definition of Mohand transform, we have

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = v^2 \int_0^\infty [aF_1(t) + bF_2(t)]e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = av^2 \int_0^\infty F_1(t)e^{-vt}dt + bv^2 \int_0^\infty F_2(t)e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = aM\{F_1(t)\} + bM\{F_2(t)\}$$

$$\Rightarrow M\{aF_1(t) + bF_2(t)\} = aR_1(v) + bR_2(v),$$

where a, b are arbitrary constants.

2.2 Change of scale property:

If $M\{F(t)\} = R(v)$ then $M\{F(at)\} = aR\left(\frac{v}{a}\right)$

Proof: By the definition of Mohand transform, we have

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt$$

$\Rightarrow M\{F(at)\} = v^2 \int_0^\infty F(at)e^{-vt}dt$ …… (3)

Put $at = p \Rightarrow adt = dp$ in(3), we have

$$M\{F(at)\} = \frac{v^2}{a}\int_0^\infty F(p)e^{\frac{-vp}{a}}dp$$

$$\Rightarrow M\{F(at)\} = a\left[\frac{v^2}{a^2}\int_0^\infty F(p)e^{\frac{-vp}{a}}dp\right]$$

$\Rightarrow M\{F(at)\} = aR\left(\frac{v}{a}\right)$ ………………. (4)

2.3 Shifting theorem for Mohand transform:

If $M\{F(t)\} = R(v)$ then $M\{e^{at}F(t)\} = \frac{v^2}{(v-a)^2}R(v-a)$

Proof: By the definition of Mohand transform, we have

$$M\{F(t)\} = v^2 \int_0^\infty F(t)e^{-vt}dt$$

$$\Rightarrow M\{e^{at}F(t)\} = v^2 \int_0^\infty e^{at}F(t)e^{-vt}dt = v^2 \int_0^\infty F(t)e^{-(v-a)t}dt$$

$$\Rightarrow M\{e^{at}F(t)\} = \frac{(v-a)^2}{(v-a)^2}.v^2 \int_0^\infty F(t)e^{-(v-a)t}dt = \frac{v^2}{(v-a)^2}R(v-a)$$

2.4Mohand transform of some basic mathematical functions [7, 8]:

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = R(v)$
1.	1	v
2.	t	1
3.	t^2	$\frac{2!}{v}$
4.	$t^n, n \in N$	$\frac{n!}{v^{n-1}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$
6.	e^{at}	$\frac{v^2}{v-a}$
7.	$sinat$	$\frac{av^2}{(v^2+a^2)}$
8.	$cosat$	$\frac{v^3}{(v^2+a^2)}$
9.	$sinhat$	$\frac{av^2}{(v^2-a^2)}$
10.	$coshat$	$\frac{v^3}{(v^2-a^2)}$

2.5 Convolution theorem for Mohand transform [8]:

If $M\{F_1(t)\} = R_1(v)$ and $M\{F_2(t)\} = R_2(v)$ then $M\{F_1(t) * F_2(t)\} = \frac{1}{v^2}M\{F_1(t)\}M\{F_2(t)\}$

$\Rightarrow M\{F_1(t) * F_2(t)\} = \frac{1}{v^2}R_1(v)R_2(v)$, where $F_1(t) * F_2(t)$ is called the convolution of $F_1(t)$ and $F_2(t)$ and it is defined as $F_1(t) * F_2(t) = \int_0^t F_1(t-x)\, F_2(x)dx = \int_0^t F_1(x)\, F_2(t-x)dx$

3. INVERSE MOHAND TRANSFORM

If $M\{F(t)\} = R(v)$ exists then the function $F(t)$ is called inverse Mohand transform of $R(v)$ and mathematically it can be expressed as $F(t) = M^{-1}\{R(v)\}$, where M^{-1} is the inverse Mohand transform operators.

4. SOLUTION OF LINEAR VOLTERRA INTEGRAL EQUATIONS OF SECOND KIND USING MOHAND TRANSFORM:

In this paper, we have taken convolution type linear Volterra integral equations of second kind so the kernel $K(x,t)$ of (1) can be expressed by $K(x-t)$. Thus (1) can be expressed as

$$y(x) = f(x) + \lambda \int_0^x K(x-t)y(t)dt \quad \ldots. (5)$$

Now, applying Mohand transform on both sides of (5), we have

$$M\{y(x)\} = M\{f(x)\} + \lambda M\left\{\int_0^x K(x-t)y(t)dt\right\} \ldots\ldots (6)$$

Applying convolution theorem on (6), we have

$$M\{y(x)\} = M\{f(x)\} + \lambda.\frac{1}{v^2}M\{K(x)\}M\{y(x)\} \ldots\ldots (7)$$

After simplification (7), we have

$$M\{y(x)\} = \left[\frac{M\{f(x)\}}{1-\frac{\lambda}{v^2}M\{K(x)\}}\right] \ldots\ldots (8)$$

Now taking inverse Mohand transform both sides of (8), we get

$$y(x) = M^{-1}\left\{\left[\frac{M\{f(x)\}}{1-\frac{\lambda}{v^2}M\{K(x)\}}\right]\right\} \ldots\ldots (9)$$

which is the required solution of (5).

5. APPLICATIONS

In this section, some applications are given to explain the procedure of solution of linear Volterra integral equations of second kind using Mohand transform.

Application: 5.1 Consider the linear Volterra integral equations of second kind

$$y(x) = sinx - 2\int_0^x cos(x-t)y(t)dt \ldots (10)$$

Now, applying Mohand transform on both sides of (10), we have

$$M\{y(x)\} = M\{sinx\} - 2M\left\{\int_0^x cos(x-t)y(t)dt\right\} \ldots\ldots (11)$$

Applying convolution theorem on (11), we have

$$M\{y(x)\} = \frac{v^2}{(v^2+1)} - 2.\frac{1}{v^2}M\{cosx\}M\{y(x)\}$$

$$\Rightarrow M\{y(x)\} = \frac{v^2}{(v^2+1)} - \frac{2}{v^2}.\frac{v^3}{(v^2+1)}M\{y(x)\} \ldots\ldots (12)$$

After simplification (12), we have

$$M\{y(x)\} = \frac{v^2}{(v+1)^2} \ldots\ldots (13)$$

Now taking inverse Mohand transform both sides of (13), we get

$$y(x) = M^{-1}\left\{\frac{v^2}{(v+1)^2}\right\} = xe^{-x} \ldots\ldots (14)$$

which is the required solution of (10).

Application: 5.2 Consider the linear Volterra integral equations of second kind

$$y(x) = x^2 + \int_0^x sin(x-t)y(t)dt \ldots\ldots (15)$$

Now, applying Mohand transform on both sides of (15), we have

$$M\{y(x)\} = M\{x^2\} + M\left\{\int_0^x sin(x-t)y(t)dt\right\} \ldots\ldots (16)$$

Applying convolution theorem on (16), we have

$$M\{y(x)\} = \frac{2}{v} + \frac{1}{v^2}M\{sinx\}M\{y(x)\}$$

$$\Rightarrow M\{y(x)\} = \frac{2}{v} + \frac{1}{v^2}.\frac{v^2}{(v^2+1)}M\{y(x)\} \ldots\ldots (17)$$

After simplification (17), we have

$$M\{y(x)\} = \frac{2}{v^3} + \frac{2}{v} \ldots\ldots (18)$$

Now taking inverse Mohand transform both sides of (18), we get

$$y(x) = M^{-1}\left\{\frac{2}{v^3} + \frac{2}{v}\right\} = 2M^{-1}\left\{\frac{1}{v^3}\right\} + 2M^{-1}\left\{\frac{1}{v}\right\}$$

$\Rightarrow y(x) = \frac{x^4}{12} + x^2$……………. (19)

which is the required solution of (15).

Application: 5.3 Consider the linear Volterra integral equations of second kind

$y(x) = cosx + sinx - \int_0^x y(t)\, dt$ …… (20)

Now, applying Mohand transform on both sides of (20), we have

$M\{y(x)\} = M\{cosx\} + M\{sinx\} - M\{\int_0^x y(t)dt\}$
…………………….. (21)

Applying convolution theorem on (21), we have

$M\{y(x)\} = \frac{v^3}{(v^2+1)} + \frac{v^2}{(v^2+1)} - \frac{1}{v^2} M\{1\}M\{y(x)\}$

$\Rightarrow M\{y(x)\} = \frac{v^3}{(v^2+1)} + \frac{v^2}{(v^2+1)} - \frac{1}{v^2}. vM\{y(x)\}$
……………………… (22)

After simplification (22), we have

$M\{y(x)\} = \frac{v^3}{(v^2+1)}$ ……………… (23)

Now taking inverse Mohand transform both sides of (23), we get

$y(x) = M^{-1}\left\{\frac{v^3}{(v^2+1)}\right\} = cosx$ ……… (24)

which is the required solution of (20).

Application: 5.4 Consider the linear Volterra integral equations of second kind

$y(x) = x - \int_0^x (x-t)y(t)\, dt$……… (25)

Now, applying Mohand transform on both sides of (25), we have

$M\{y(x)\} = M\{x\} - M\{\int_0^x (x-t)y(t)dt\}$
…………….. (26)

Applying convolution theorem on (26), we have

$M\{y(x)\} = 1 - \frac{1}{v^2} M\{x\}M\{y(x)\}$

$\Rightarrow M\{y(x)\} = 1 - \frac{1}{v^2}. 1. M\{y(x)\}$
…………… (27)

After simplification (27), we have

$M\{y(x)\} = \frac{v^2}{(v^2+1)}$………… (28)

Now taking inverse Mohand transform both sides of (28), we get

$y(x) = M^{-1}\left\{\frac{v^2}{(v^2+1)}\right\} = sinx$ ……… (29)

which is the required solution of (25).

6. CONCLUSIONS

In this paper, we have successfully find the exact solution of linear Volterra integral equations of second kind using Mohand transform without large computational work and spending a very little time.

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