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Application of Mohand Transform for Solving Linear Volterra Integro – Differential Equations

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Abstract-Many integral transforms like Laplace, Fourier are useful to solve physics, chemistry, biology and engineering applications with initial and boundary conditions. In this paper, Mohand transform is used to solve linear Volterra Integro-Differential equations of second kind. The technique is described and explained with some applications. The results are exact results with less computational work.

Keywords-Mohand Transform; Partial Differential Equations; Volterra Integro – Differential Equation.

1. INTRODUCTION

An integral equation is an equation in which the unknown function $u(x)$ to be determined appears under the integral sign **[1]**. Many of the engineering, physics, biology, chemistry applications, integral equations play a vital role.

In integro-differential equations, the unknown function $u(x)$ occurs in one side as a derivative and the same function appears in other side under the integral sign. Several phenomena in biology and physics give rise to this type of integro-differential equations. The new type of integral equation was called as Volterra integro-differential equations of second kind, given in the form

$$u^{(n)}(x) = f(x) + \lambda \int_0^x k(x,t)u(t)dt \quad ---- \quad (1)$$

where $u^{(n)}(x)$ is the nth derivative of the unknown function $u(x)$ with respect to x. It is very necessary to define initial conditions $u(0), u'(0), ..., u^{(n-1)}(0)$.

The Mohand transform **[2]** denoted by the operator M [.] defined by the integral equation

$$M[f(t)] = v^2 \int_0^\infty f(t)\, e^{-vt}\, dt\, , t \geq 0\, ,$$

$$k_1 \leq v \leq k_2 \quad ----- \quad (2)$$

Mohand et al. **[3]** gave the solution of ordinary differential equation with variable coefficients using Aboodh transform. Sudhanshu Aggarwal et al. **[4,5]** discussed the solution of linear Volteera integro-differential equations of second kind using Aboodh transform and Kamal transforms. Elzaki **[6]** discussed the solution of the systems of integro-differential equations by using Elzaki transform method. The aim of this paper is to give exact result for solving linear Volterra integro-differential equations of second kind with less computational work.

2. MOHAND TRANSFORM OF SOME STANDARD FUNCTIONS

Mohand transform of some standard functions are listed in table 1.

Table 1. Standard functions

S.No.	f(t)	M [f(t)]
1.	1	v
2.	t	1
3.	t^n	$\frac{n!}{v^{n-1}}$
4.	e^{at}	$\frac{v^2}{v-a}$
5.	e^{-at}	$\frac{v^2}{v+a}$
6.	$cosat$	$\frac{v^3}{v^2+a^2}$
7.	$sinat$	$\frac{v^2}{v^2+a^2}$
8.	$coshat$	$\frac{v^3}{v^2-a^2}$
9.	$sinhat$	$\frac{v^2}{v^2-a^2}$

3. MOHAND TRANSFORM OF DERIVATIVES

If M [f(t)] = R(v) then

a) $M[f'(t)] = vR(v) - v^2 f(0)$

b) $M[f''(t)] = v^2R(v) - v^3 f(0) - v^2 f'(0)$

c) $M[f'''(t)] = v^3R(v) - v^4 f(0) - v^3 f'(0) - v^2 f''(0)$

In general

$M[f^n(t)] = v^nR(v) - v^{n+1} f(0) - v^n f'(0) - v^{n-1} f''(0) - ...$

That is $M[f^n(t)] = v^nR(v) - \sum_{k=0}^{n-1} v^{n-k+1} f^{(k)}(0)$.

4. CONVOLUTION THEOREM FOR MOHAND TRANSFORM

Convolution of two functions $F(t)$ and $G(t)$ is denoted by $F(t)*G(t)$ and it is defined by

$$F(t)*G(t) = F*G = \int_0^t F(x)G(t-x)dx$$
$$= \int_0^t F(t-x)G(x)dx$$

Now,

M [$f(t) \, * \, g(t)$] = $\frac{1}{v^2}$ $M[f(t)].M\,[g(t)]$.

5. APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Mohand transform for solving linear Volterra integro-differential equation of second kind. In this work, we will assume that the kernel $k(x,t)$ of "Eq.(1)" can be expressed as $(x-t)$. Now (1) can be written as

$$u^{(n)}(x) = f(x) + \lambda \int_0^x k(x-t)u(t)dt \quad \text{-----} \quad (3)$$

with initial conditions $u(0), u'(0), ..., u^{(n-1)}(0)$.

5.1. *Application 1*

Consider linear Volterra integro-differential equation of second kind

$$u'(x) = 2 + \int_0^x u(t)dt\,,\; u(0) = 2.$$

Apply Mohand transform on both sides and using initial condition, we have

$$vM\left[u(x)\right] - v^2.2 = M(2) + M\left\{\int_0^x u(t)dx\right\}$$

Using convolution theorem of Mohand transform and simplify, we have

$$vM\left[u(x)\right] = 2v^2 + 2v + \frac{1}{v^2}.M\left[u(x)\right].u(1)$$
$$= 2v^2 + 2v + \frac{1}{v^2}M\left[u(x)\right].v$$

$$M\left[u(x)\right]\left(v - \frac{1}{v}\right) = 2v(v+1)$$

$$M\left[u(x)\right] = \frac{2v(v+1)v}{v^2-1} = \frac{2v^2}{v-1}$$

Taking inverse Mohand transform on both sides, we have

$$u(x) = M^{-1}\left[\frac{2v^2}{v-1}\right] = 2M^{-1}\left[\frac{v^2}{v-1}\right] = 2e^x.$$

5.2. *Application 2*

Consider linear Volterra integro-differential equation of second kind

$$u''(x) = 1 + \int_0^x (x-t)u(t)dt \text{ with } u(0) = 1, u'(0) = 0.$$

Apply the Mohand transform on both sides and using initial conditions, we have

$$M\left[u''(x)\right] = M(1) + M\left\{\int_0^x (x-t)u(t)dt\right\}$$

$$v^2M\left[u(x)\right] - v^3(1) - v^2(0) = v + M\left\{\int_0^x (x-t)u(t)dt\right\}$$

Using convolution theorem of Mohand transform and simplify, we have

$$v^2M\left[u(x)\right] - v^3 = v + \frac{1}{v^2}M\left[u(x)\right].M\left[x\right]$$

$$v^2M\left[u(x)\right] - \frac{M\left[u(x)\right]}{v} = v^3 + v = v\left(v^2+1\right)$$

$$M\left[u(x)\right]\left(v^2 - \frac{1}{v^2}\right) = v\left(v^2+1\right)$$

$$M\left[u(x)\right] = \frac{v\left(v^2+1\right)v^2}{v^4-1} = \frac{v^3}{v^2-1}$$

Taking inverse Mohand transform on both sides, we have $u(x) = M^{-1}\left[\frac{v^3}{v^2-1}\right] = \cosh x$.

5.3. *Application 3*

Consider linear Volterra integro-differential equation of second kind

$$u''(x) = -1 + \int_0^x u(t)dt \, with \; u(0) = u'(0) = 1, u''(0) = -1.$$

Apply Mohand transform on both sides and using initial condition, we have

$$M\left[u''(x)\right] = M(-1) + M\left\{\int_0^x u(t)\,dt\right\}$$

$$v^3 M\left[u(x)\right] - v^4(1) - v^3(1) - v^2(-1) = M(-1) + M\left\{\int_0^x u(t)\,dt\right\}$$

Using convolution theorem of Mohand transform and simplify, we have

$$v^3 M\left[u(x)\right] - v^4 - v^3 + v^2 = -v + \frac{1}{v^2} M\left[u(x)\right].M[1]$$

$$v^3 M\left[u(x)\right] = v^4 + v^3 - v^2 - v + \frac{1}{v} M\left[u(x)\right]$$

$$M\left[u(x)\right]\left(v^3 - \frac{1}{v}\right) = v^4 + v^3 - v^2 - v$$

$$M\left[u(x)\right] = \frac{v\left(v^4 + v^3 - v^2 - v\right)}{v^4 - 1}$$

$$= \frac{v^2\left(v^3 + v^2 - v - 1\right)}{v^4 - 1} = \frac{v^2\left(\left(v^3 - v\right) + \left(v^2 - 1\right)\right)}{v^4 - 1}$$

$$= \frac{v^2(v+1)\left(v^2 - 1\right)}{v^4 - 1} = \frac{v^2(v+1)}{v^2 + 1} = \frac{v^3}{v^2 + 1} + \frac{v^2}{v^2 + 1}$$

Taking inverse Mohand transform on both sides, we have

$$u(x) = M^{-1}\left[\frac{v^3}{v^2 + 1}\right] + M^{-1}\left[\frac{v^2}{v^2 + 1}\right] = \cos x + \sin x\ .$$

5.4. *Application 4*

Consider linear Volterra integro-differential equation of second kind

$$u''(x) = x + \int_0^x (x - t)u(t)\,dt,\ with\ u(0) = u'(0) = 1.$$

Applying the Mohand transform to both sides and using initial condition, we have

$$v^2 M\left[u(x)\right] - v^3(0) - v^2(1) = 1 + \frac{1}{v^2} M\left[u(x)\right].1$$

$$v^2 M\left[u(x)\right] = v^2 + 1 + \frac{1}{v^2} M\left[u(x)\right]$$

$$M\left[u(x)\right]\left(v^2 - \frac{1}{v^2}\right) = v^2 + 1$$

$$M\left[u(x)\right] = \frac{v^2\left(v^2 + 1\right)}{v^4 - 1} = \frac{v^2}{v^2 - 1}.$$

Taking inverse Mohand transform on both sides, we have $u(x) = M^{-1}\left[\frac{v^2}{v^2 - 1}\right] = \sinh x$.

6. CONCLUSION

In this paper, we have successfully applying the new integral transform Mohand transform for solving linear Volterra integro-differential equations of second kind. The results are exact and less number of computational work and no complexity for solve those problems by using this transform technique.

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