

Micro- α -open sets in Micro Topological Spaces

¹S.Chandrasekar, ²G.Swathi

¹Assistant Professor, Department of Mathematics,
Arignar Anna Government Arts College,
Namakkal(DT)Tamil Nadu, India

²M.Phil Scholar, Department of Mathematics,
Arignar Anna Government Arts College,
Namakkal(DT)Tamil Nadu, India

E-mail:chandrumat@gmail.com, glswathi5@gmail.com

Abstract- Micro topology is a simple extension of Nano topology. Nano topology was introduced by Lellis Thivagar. Nano topology provides wide range of interesting results and applications. But some time we want extend some open sets in Nano topology, by Using Micro Topology we can do it. Micro Topological spaces introduced by S.Chandraekar. In this paper we introduce Micro- α -open sets and Micro- α -continuity in Micro Topological Spaces

Index Terms- Micro Topological Spaces, Micro- α -closed sets, Micro- α -open sets, Micro- α -interior and Micro- α -Closure and Micro- α -continuous

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1. INTRODUCTION

The concept of Nano topology was introduced by Lellis Thivagar [3] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established the weak forms of open sets namely Nano open sets, Nano semi open sets and Nano pre open sets in a Nano topological space. Nano topologies have Minimum three and Maximum Five Nano open sets. But some time we want extend some open sets in Nano topology; by Using Micro Topology we can do it. Micro topology was introduced by S.Chandrasekar. Nano topology can be extent using Micro topology concept Minimum Four open sets to Maximum nine open sets. In this paper we introduce Micro- α -open sets and Micro- α -continuity in Micro Topological Spaces

2. PRELIMINARIES

Definition: 2.1.,[3]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is,

$$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$$

where $R(x)$ denotes the equivalence class determined by X .

(ii) The upper approximation of X with respect to R is

the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is,

$$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$$

(iii) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not- X with respect to R and it is denoted by $B_R(X)$

$$\text{That is, } B_R(X) = U_R(X) - L_R(X)$$

Definition: 2.2., [3]

- (i). $L_R(X) \subseteq X \subseteq U_R(X)$
- (ii). $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$
- (iii). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- (iv). $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- (v). $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- (vi). $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- (vii). $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- (viii). $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
- (ix). $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
- (x). $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition: 2.3 [3]

Let U be the universe, R be an equivalent relation on U and $\tau_R(X) = \{U, \emptyset, U_R(X), L_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 1.3, $\tau_R(X)$ satisfies the following axioms.

- (i). U and $\emptyset \in \tau_R(X)$.
- (ii). The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
- (iii). The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X .

$(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U . Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being are called dual Nano topology of $\tau_R(X)$.

Definition: 2.4.[3]

A space $(U, \tau_R(X))$ is called a locally indiscrete space if every Nano open set of U is Nano closed in U .

Definition: 2.5. [3]

$(U, \tau_R(X))$ is a Nano topological space here

$\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X)$
and called it Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$.

Definition 2.6.[3]

The Micro topology $\mu_R(X)$ satisfies the following axioms

- (i). $U, \phi \in \mu_R(X)$
- (ii). The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$
- (iii). The intersection of the elements of any finite sub collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then $\mu_R(X)$ is called the Micro topology on U with respect to X .

The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological spaces and .The elements of $\mu_R(X)$ are called Micro open sets. and its complement is called Micro closed sets

Example 2.7.

$U = \{1, 2, 3, 4\}$, with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{1, 2\} \subseteq U$

Nano topology $\tau_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$
Then $\mu = \{3\}$

Micro topology $\mu_R(X) = \{U, \phi, \{1\}, \{3\}, \{1, 3\}, \{2, 4\}, \{2, 3, 4\}, \{1, 2, 4\}\}$

Example 2.8.

Let $U = \{p, q, r, s, t\}$
 $U/R = \{\{p\}, \{q, r, s\}, \{t\}\}$.

Let $X = \{q, r\} \subseteq U$. Then
Nano topology $\tau_R(X) = \{U, \phi, \{q, r, s\}\}$.

Then $\mu = \{p\}$
Micro topology $\mu_R(X) = \{U, \phi, \{p\}, \{p, q, r, s\}, \{q, r, s\}\}$

Example 2.9

Let $U = \{a, b, c, d, e\}$ with $U/R = \{\{a, b\}, \{c\}, \{d, e\}\}$
and

$X = \{a, c\} \subseteq U$.
Then $\tau_R(X) = \{U, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$ and

Then $\mu = \{e\}$
Micro topology

$\mu_R(X) = \{U, \phi, \{c\}, \{e\}, \{a, b\}, \{c, e\}, \{a, b, c\}, \{a, b, e\}, \{a, b, c, e\}\}$

Definition 2.10. [3]

$\text{Mic-int}(A) = \cup \{G / G \text{ is an Mic-OS in } X \text{ and } G \subseteq A\}$,
 $\text{Mic-cl}(A) = \cap \{K / K \text{ is an Mic-CS in } X \text{ and } A \subseteq K\}$.

Definition 2.11.[2]

Let $(U, \tau_R(X), \mu_R(X))$. be a Micro-topological space. and $A \subseteq U$ Then A is said to be
Micro-Pre-open if $A \subseteq \text{Mic-int}(\text{Mic-cl}(A))$ and

Micro-Pre-closed set if $\text{Mic-cl}(\text{Mic-int}(A)) \subseteq A$.

Definition 2.11.[2]

Let $(U, \tau_R(X), \mu_R(X))$. be a Micro-topological space. and $A \subseteq U$ Then A is said to be

Micro-semi open ,if $A \subseteq \text{Mic-cl}(\text{Mic-int}A)$.

Micro-semiclosed, if $\text{Mic-int}(\text{Mic-cl}A) \subseteq A$.

3. MICRO- α OPEN SET.

In this section, we introduce the concept of Micro α -closed sets (Shortly Mic α -closed set) and some of their properties are discussed details.

Definition 3.1.

Let $(U, \tau_R(X), \mu_R(X))$ be an Micro topological space. An set A is called an Micro- α open set

(briefly, Mic- α OS)
if $A \subseteq \text{Mic-int}(\text{Mic-cl}(\text{Mic-int}(A)))$.

The complement of an Micro- α -open set is called an Micro- α closed set.

Example 3.2.

Let $U = \{a, b, c, d, e\}$ with $U/R = \{\{a, b\}, \{c\}, \{d, e\}\}$
and

$X = \{a, c\} \subseteq U$.

Then $\tau_R(X) = \{U, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$ and
Then $\mu = \{d\}$

Micro topology
 $\mu_R(X) = \{U, \phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\}$

Micro- α open sets
 $= \{U, \phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b, c, d\}\}$.

Theorem 3.3.

Every Micro open set is Micro- α -open.

Proof.

Let A be an Micro open set in $(U, \tau_R(X), \mu_R(X))$. Since $A \subseteq \text{Mic-cl}(A)$, we get $A \subseteq \text{Mic-cl}(\text{Mic-int}(A))$. Then $\text{Mic-int}(A) \subseteq \text{Mic-int}(\text{Mic-cl}(\text{Mic-int}(A)))$. Hence $A \subseteq \text{Mic-int}(\text{Mic-cl}(\text{Mic-int}(A)))$.

Theorem 3.4.

Every Micro- α open set is Micro semi-open.

Proof.

Let A be an Micro- α open set in $(U, \tau_R(X), \mu_R(X))$. Then, $A \subseteq \text{Mic-int}(\text{Mic-cl}(\text{Mic-int}(A)))$. It is obvious that $\text{Mic-int}(\text{Mic-cl}(\text{Mic-int}(A))) \subseteq \text{Mic-cl}(\text{Mic-int}(A))$ Hence $A \subseteq \text{Mic-cl}(\text{Mic-int}(A))$.

The converse of the above theorem need not be true as shown by the following example.

Example 3.5

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$.

Let $X = \{a\} \subseteq U$. Then

Nano topology $\tau_R(X) = \{U, \phi, \{a\}\}$.

Then $\mu = \{b\}$

Micro topology $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$

Micro- α open sets = $\{U, \phi, \{a\}, \{b\}, \{a, b\}\}$

Micro-Semi open sets
 $= \{U, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

Here $\{a,c\}, \{b,c\}$ are Micro-Semi open set and but not Micro- α open set

Theorem 3.6.

Every Micro- α open set is Micro pre-open.

Proof.

Let A be an Micro- α open set in $(U, \tau_R(X), \mu_R(X))$.

Then, $A \subseteq \text{Mic-int}(\text{Mic-cl}(\text{Mic-int}(A)))$.

It is obvious that $A \subseteq \text{Mic-int}(\text{Mic-cl}(A))$.

The converse of the above theorem need not be true as shown by the following

Example:3.7

$U = \{1, 2, 3, 4\}$, with $U/R = \{\{1\}, \{3\}, \{2, 4\}\}$ and $X = \{2, 4\} \subseteq U$

Nano topology $\tau_R(X) = \{U, \phi, \{2, 4\}\}$

Then $\mu = \{1\}$

Micro topology

$\mu_R(X) = \{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$

Micro- α open sets = $\{U, \phi, \{1\}, \{1, 2, 4\}, \{2, 4\}\}$

Micro pre-open sets = $\{U, \phi, \{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}\}$

Here $\{1, 3, 4\}, \{1, 2, 3\}$ are Micro-pre open and but not Micro- α open set

Theorem 3.8.

(i) Arbitrary union of Micro- α open set is always Micro- α open set.

(ii) Finite intersection of Micro- α open set may fail to be Micro- α open set.

Proof.

It is obvious

Theorem 3.9.

(i) Arbitrary intersection of Micro- α closed sets is always Micro- α closed set.

(ii) Finite union of Micro- α closed sets may fail to be Micro- α closed set.

Proof.

The proof follows immediately from Theorem 3.8.

Theorem 3.10.

(i) The intersection of an Micro-open set and an Micro- α -open set is Micro- α open.

(ii) The intersection of an Micro- α open set and an Micro-pre-open set is Micro pre-open.

Proof.

It is obvious.

Definiion 3.11.

The Micro α -closure of a set A is denoted by $\text{Mic-}\alpha\text{-cl}(A) = \cap \{G : G \text{ is an Mic-}\alpha \text{ closed set in } U \text{ and } G \subseteq A\}$ and

the Micro- α interior of a set A is denoted by $\text{Mic-}\alpha\text{-int}(A) = \cup \{K : K \text{ is an Mic-}\alpha \text{ open set in } U \text{ and } A \subseteq K\}$

Remark 3.12.

It is clear that $\text{Mic-}\alpha\text{-int}(A)$ is an Micro- α open set and $\text{Mic-}\alpha\text{-cl}(A)$ is an Micro- α closed set.

Theorem 3.13.

For any $x \in X$, $x \in \text{Mic-}\alpha\text{-cl}(A)$ if and only if $A \cap H \neq \phi$ for every Micro- α -open set V containing x.

Proof:

Let $x \in \text{Mic-}\alpha\text{-cl}(A)$. Suppose there exists an Micro- α -open set H containing x such that $H \cap A = \phi$. Then $A \subseteq U - H$. Since U-H is Micro- α closed, $\text{Mic-}\alpha\text{-cl}(A) \subseteq U - H$. This implies $x \notin \text{Mic-}\alpha\text{-cl}(A)$ which is a contradiction.

Hence Conversely, let $A \cap H \neq \phi$ for every Micro- α -open set H containing x. To prove that $x \in \text{Mic-}\alpha\text{-cl}(A)$. Suppose $x \notin \text{Mic-}\alpha\text{-cl}(A)$. Then there exists a Micro- α -closed set G containing A such that $x \notin G$. Then $x \in U - G$ and U-G is Micro- α -open. Also $(U - G) \cap A \neq \phi$ which is a contradiction to the hypothesis. Hence $x \in \text{Mic-}\alpha\text{-cl}(A)$.

Theorem 3.14

If $A \subseteq X$, then $A \subseteq \text{Mic-}\alpha\text{-cl}(A) \subseteq \text{Mic-cl}(A)$.

Proof:

Since every closed set is Micro- α -closed, the proof follows.

Remark 3.15

Both containment relations in the Theorem 3.14 may be proper as seen from the following example.

Example 3.16

Let $U = \{a, b, c\}$

$U/R = \{\{a\}, \{b, c\}\}$.

Let $X = \{a\} \subseteq U$. Then

Nano topology $\tau_R(X) = \{U, \phi, \{a\}\}$.

Then $\mu = \{b\}$

Micro topology $\mu_R(X) = \{U, \phi, \{a\}, \{b\}, \{a, b\}\}$

Micro- α open sets = $\{U, \phi, \{a\}, \{b\}, \{a, b\}\}$

Let $E = \{a\}$. Then $\text{Mic-}\alpha\text{-cl}(E) = \{a, c\}$

and so $E \subseteq \text{Mic-}\alpha\text{-cl}(E) \subseteq \text{Mic-cl}(E)$,

Theorem 3.17

Let A and B be subsets of $(U, \tau_R(X), \mu_R(X))$. Then

(i). $\text{Mic-}\alpha\text{-cl}(\phi) = \phi$

(ii). $\text{Mic-}\alpha\text{-cl}(U) = U$

(iii). $\text{Mic-}\alpha\text{-cl}(A)$ is Micro- α -closed set in $(U, \tau_R(X), \mu_R(X))$.

(iv). If $A \subseteq B$, then $\text{Mic-}\alpha\text{-cl}(A) \subseteq \text{Mic-}\alpha\text{-cl}(B)$.

(v). $\text{Mic-}\alpha\text{-cl}(A \cup B) = \text{Mic-}\alpha\text{-cl}(A) \cup \text{Mic-}\alpha\text{-cl}(B)$.

(vi). $\text{Mic-}\alpha\text{-cl}[\text{Mic-}\alpha\text{-cl}(A)] = \text{Mic-}\alpha\text{-cl}(A)$.

Proof:

The proof of (i), (ii), (iii) and (iv) follow from the Definition 3.11

(v). To prove that $\text{Mic-}\alpha\text{-cl}(A) \cup \text{Mic-}\alpha\text{-cl}(B) \subseteq \text{Mic-}\alpha\text{-cl}(A \cup B)$ We have $\text{Mic-}\alpha\text{-cl}(A) \subseteq \text{Mic-}\alpha\text{-cl}(A \cup B)$ and $\text{Mic-}\alpha\text{-cl}(B) \subseteq \text{Mic-}\alpha\text{-cl}(A \cup B)$. Therefore $\text{Mic-}\alpha\text{-cl}(A) \cup \text{Mic-}\alpha\text{-cl}(B) \subseteq \text{Mic-}\alpha\text{-cl}(A \cup B) \dots\dots\dots (1)$.

Now we prove $\text{Mic-}\alpha\text{-cl}(A \cup B) \subseteq \text{Mic-}\alpha\text{-cl}(A) \cup \text{Mic-}\alpha\text{-cl}(B)$ Let x be any point such that $x \in \text{Mic-}\alpha\text{-cl}(A \cup B)$. Then there exists Micro- α -closed sets A and B such that $A \subseteq E$ and $B \subseteq F$, $x \notin E$ and $x \notin F$. Then $x \notin E \cup F$, $A \cup B \subseteq E \cup F$ and $E \cup F$ is Micro- α -closed set. Thus $x \notin \text{Mic-}\alpha\text{-cl}(A \cup B)$. Therefore we have $\text{Mic-}\alpha\text{-cl}(A \cup B) \subseteq \text{Mic-}\alpha\text{-cl}(A) \cup \text{Mic-}\alpha\text{-cl}(B) \dots\dots\dots (2)$.

Hence from (1) and (2),

$\text{Mic-}\alpha\text{-cl}(A \cup B) = \text{Mic-}\alpha\text{-cl}(A) \cup \text{Mic-}\alpha\text{-cl}(B)$.

(vi). Let E be Micro- α -closed set containing A. Then by Definition $\text{Mic-}\alpha\text{-cl}(A) \subseteq E$. Since E is Micro- α -closed set and contains $\text{Mic-}\alpha\text{-cl}(A)$ and is contained in every

Mic- α -closed set containing A, it follows that $\text{Mic-}\alpha\text{-cl}[\text{Mic-}\alpha\text{-cl}(A)] \subseteq \text{Mic-}\alpha\text{-cl}(A)$. Therefore $\text{Mic-}\alpha\text{-cl}[\text{Mic-}\alpha\text{-cl}(A)] = \text{Mic-}\alpha\text{-cl}(A)$.

Theorem 3.18:

A subset A is Mic- α -closed if and only if

$$\text{Mic-}\alpha\text{-cl}(A) = A.$$

Proof:

Let A be Mic- α -closed set in $(U, \tau_R(X), \mu_R(X))$. Since $A \subseteq A$ and A is Mic- α -closed set, $A \in \{G: A \subseteq G, G \text{ is Mic-}\alpha\text{-closed set}\}$ which implies that $\bigcap \{G: A \subseteq G, G \text{ is Mic-}\alpha\text{-closed set}\} \subseteq A$. That is $\text{Mic-}\alpha\text{-cl}(A) \subseteq A$. Note that $A \subseteq \text{Mic-}\alpha\text{-cl}(A)$ is always true. Hence $A = \text{Mic-}\alpha\text{-cl}(A)$.

Conversely, suppose $\text{Mic-}\alpha\text{-cl}(A) = A$. Since $A \subseteq A$ and A is Mic- α -closed set. Therefore A must be a closed set. Hence A is Mic- α -closed.

Theorem 3.18:

If $A \subseteq B$, then $\text{Mic-}\alpha\text{-int}(A) \subseteq \text{Mic-}\alpha\text{-int}(B)$.

Proof:

Suppose $A \subseteq B$. We know that $\text{Mic-}\alpha\text{-int}(A) \subseteq A$. Also we have $A \subseteq B$, which implies $\text{Mic-}\alpha\text{-int}(A) \subseteq B$, $\text{Mic-}\alpha\text{-int}(A)$ is an open set which is contained in B. But $\text{Mic-}\alpha\text{-int}(B)$ is the largest open set contained in B. Therefore $\text{Mic-}\alpha\text{-int}(B)$ is larger than $\text{Mic-}\alpha\text{-int}(A)$. That is $\text{Mic-}\alpha\text{-int}(A) \subseteq \text{Mic-}\alpha\text{-int}(B)$.

Theorem 3.19

For any subset A of X, the following results are true:

- (i). $\text{Mic-}\alpha\text{-int}(\phi) = \phi$
- (ii). $\text{Mic-}\alpha\text{-int}(U) = U$
- (iii). If $A \subseteq B$ then $\text{Mic-}\alpha\text{-int}(A) \subseteq \text{Mic-}\alpha\text{-int}(B)$
- (iv). $\text{Mic-}\alpha\text{-int}(A)$ is the largest Mic- α -open set contained in A
- (v). $\text{Mic-}\alpha\text{-int}(A \cap B) = \text{Mic-}\alpha\text{-int}(A) \cap \text{Mic-}\alpha\text{-int}(B)$
- (vi). $\text{Mic-}\alpha\text{-int}(A \cup B) \supseteq \text{Mic-}\alpha\text{-int}(A) \cup \text{Mic-}\alpha\text{-int}(B)$
- (vii). $\text{Mic-}\alpha\text{-int}[\text{Mic-}\alpha\text{-int}(A)] = \text{Mic-}\alpha\text{-int}(A)$

Proof:

Proof follows from the Definition 3.17

4. MICRO- α CONTINUOUS MAP.

In this section, we introduce the concept of Micro α -continuous map, and some of their properties are discussed details.

Definition 4.1.

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro- α open sets and $\mu_R(X)$ be an associated Micro topology with $\mu_R(X)$. A map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is called Micro- α continuous map if the inverse image of each open set in Y is an Micro- α open set in U.

Theorem 4.2.

Every Micro continuous map is Micro- α continuous map.

Proof.

Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be an Micro continuous map and A is an open set in V. Then $f^{-1}(A)$ is an open set in U. Since $\mu_R(X)$ is associated

with $\tau_R(X)$, then $\tau_R(X) \subseteq \mu_R(X)$. Therefore, $f^{-1}(A)$ is an Micro open set in U which is an Micro open set in U. Hence f is an Micro- α continuous map.

Theorem 4.3.

Let $(U, \tau_R(X), \mu_R(X))$ and $(V, \tau_R(Y), \mu_R(Y))$ be two Micro topological spaces and μ be an associated Micro topology with $\tau_R(Y)$, Let f be a map from U into V.

Then the following are equivalent:

- (i) f is Micro- α continuous map
- (ii) The inverse image of a closed set in V is an Micro- α -closed set in U
- (iii) $\text{Mic-}\alpha\text{-cl}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}(A))$ for every set A in V.
- (iv) $f(\text{Mic-}\alpha\text{-cl}(A)) \subseteq \text{cl}(f(A))$ for every set A in U.
- (v) $f^{-1}(\text{int}(B)) \subseteq \text{Mic-}\alpha\text{-int}(f^{-1}(B))$ for every set B in V.

Proof.

(i) \rightarrow (ii) :

Let A be a closed set in V, then $V - A$ is open in V. Thus, $f^{-1}(U - A) = U - f^{-1}(A)$ is Mic- α -open in U. It follows that $f^{-1}(A)$ is a Mic- α -closed set of U.

(ii) \rightarrow (iii):

Let A be any subset of U. Since $\text{Mic-cl}(A)$ is Micro closed in V, then it follows that $f^{-1}(\text{Mic-cl}(A))$ is Mic- α -closed in U. Therefore, $f^{-1}(\text{Mic-cl}(A)) = \text{Mic-}\alpha\text{-cl}(f^{-1}(\text{Mic-cl}(A))) \supseteq \text{Mic-}\alpha\text{-cl}(f^{-1}(A))$.

(iii) \rightarrow (iv) :

Let A be any subset of U. By (iii) we obtain, $f^{-1}(\text{Mic-cl}(f(A))) \supseteq \text{Mic-}\alpha\text{-cl}(f^{-1}(f(A))) \supseteq \text{Mic-}\alpha\text{-cl}(A)$ and hence $f(\text{Mic-}\alpha\text{-cl}(A)) \subseteq \text{Mic-cl}(f(A))$.

(iv) \rightarrow (v) :

Let $f(\text{Mic-}\alpha\text{-cl}(A)) \subseteq \text{Mic-cl}(f(A))$ for every set A in U. Then $\text{Mic-}\alpha\text{-cl}(A) \subseteq f^{-1}(\text{Mic-cl}(f(A)))$, $U - \text{Mic-}\alpha\text{-cl}(A) \supseteq U - f^{-1}(\text{Mic-cl}(f(A)))$ and $\text{Mic-}\alpha\text{-int}(U - A) \supseteq f^{-1}(\text{Mic-int}(Y - f(A)))$. Then $\text{Mic-}\alpha\text{-int}(f^{-1}(B)) \supseteq f^{-1}(\text{Mic-int}(B))$. Therefore $f^{-1}(\text{Mic-int}(B)) \subseteq \text{Mic-int}(f^{-1}(B))$, for every B in V

(v) \rightarrow (i) :

Let A be a open set in V. Therefore, $f^{-1}(\text{int}(A)) \supseteq \text{Mic-}\alpha\text{-int}(f^{-1}(A))$, hence $f^{-1}(A) \subseteq \text{Mic-}\alpha\text{-int}(f^{-1}(A))$. But by other hand, we know that, $\text{Mic-}\alpha\text{-int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Then $f^{-1}(A) = \text{Mic-}\alpha\text{-int}(f^{-1}(A))$. Therefore, $f^{-1}(A)$ is a Mic- α -open set.

Theorem 4.4.

If a map $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ is a Mic- α -continuous and $g: (V, \tau_R(Y), \mu_R(Y)) \rightarrow (W, \tau_R(Z), \mu_R(Z))$ is continuous, then $(g \circ f)$ is Mic- α -continuous.

Proof.

Obvious.

Theorem 4.5.

Let $f: (U, \tau_R(X), \mu_R(X)) \rightarrow (V, \tau_R(Y), \mu_R(Y))$ be an Micro- α -continuous map, if one of the following holds:

- (i) $f^{-1}(\text{Mic-}\alpha\text{-int}(A)) \subseteq \text{int}(f^{-1}(A))$ for every set A in V
- (ii) $\text{Mic-cl}(f^{-1}(A)) \subseteq f^{-1}(\text{Mic-}\alpha\text{-cl}(A))$ for every set A in V.
- (iii) $f(\text{Mic-cl}(B)) \subseteq \text{Mic-}\alpha\text{-cl}(f(B))$ for every set B in U.

Proof.

Let A be any open set of V , if condition (i) is satisfied, then $f^{-1}(\text{Mic-aint}(A)) \subseteq \text{Mic-int}(f^{-1}(A))$. We get, $f^{-1}(A) \subseteq \text{Mic-int}(f^{-1}(A))$. Therefore $f^{-1}(A)$ is a Micro open set. Every Micro open set is Micro-open set. Hence f is a Micro- α continuous function. If condition (ii) is satisfied, then we can easily prove that f is a Micro- α -continuous function. If condition (iii) is satisfied, and A is any open set of V . Then $f^{-1}(A)$ is a set in U and $f(\text{Mic-cl}(f^{-1}(A))) \subseteq \text{Mic-}\alpha\text{-cl}(f(f^{-1}(A)))$. This implies $f(\text{Mic-cl}(f^{-1}(A))) \subseteq \text{Mic-}\alpha\text{-cl}(A)$. This is nothing but condition (ii). Hence f is a Micro- α -continuous function.

4. CONCLUSION

Many different forms of topological spaces have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, In this paper we introduce Micro- α -open sets and Micro- α -continuity in Micro Topological Spaces and investigate some of the basic properties. This shall be extended in the future Research with some applications.

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