Radio Odd Mean Number of Shadow graph of Star and Bistar

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Abstract - A radio odd mean labeling of a connected graph G is a one to one map from the vertex set V(G) to Z+ such that for two distinct vertices u and v of G, $d(u,v)+\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ *is odd* $\geq 1+diam(G)$. The radio odd mean number of *f*, romn (*f*) is the maximum number assigned to any vertex of G. The radio odd mean number of G, romn (G) is the minimum value of romn (*f*) taken over all radio odd mean labeling of G. In this paper we have determine radio odd mean number of Shadow graph of Star and Bistar.

Index Terms- Radio Odd Mean Labeling, Radio Odd Mean Graph, Shadow graph of Star, Shadow graph of Bistar

1. INTRODUCTION

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [9]. The symbols V (G) and E (G) will denote the vertex set and edge set of a graph G. Radio labeling [2] is motivated by the channel assignment problem introduced by Hale et al [8] in 1980.

To avoid interference, transmitters that are geographically close must be assigned channel with large frequency difference, transmitters that are further apart may receive channels with relatively close frequencies. The general solution is modeled by identifying transmitters with the vertices of a graph subject to a restriction conenering the distance between the vertices. The goal is to minimize the largest integer used. The radio labeling of a graph is most useful in FM radio channel restrictions to overcome from the effect of noise [3]. This problem turns out to find the minimum of maximum frequencies of all the radio stations considered under the network. Ponraj et al. [13] discussed the radio mean labeling.

Motivated by the notion of radio mean labeling we have introduced radio odd mean labeling [1]. In this paper we determine radio odd mean number of Shadow graph of Star and Bistar.

2. DEFINITIONS

2.1. *Definition*

Let G be a connected graph, the distance *d* (*u*, *v*) between any pair of vertices *u, v* is the length of the shortest path between them.

2.2. *Definition*

The diameter of a graph is denoted by *diam* (*G*) and defined as the maximum distance between any two vertices, that is, $diam(G) = \max\{d(u,v)\ ; u, v \in G\}$.

2.3. *Definition*

A radio labeling is one to one mapping f : V (G) →Z^+ satisfying the condition

$d(u,v)+|f(u)-f(v)| \geq 1+diam(G),$ for every vertices *u, v* in G.

2.4. *Definition*

A radio odd mean labeling of a connected graph G is a one to one map from the vertex set V of G to Z^+ such that for two distinct vertices u and v of G, $d(u,v)+\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ *is odd* $\geq 1+diam(G)$.

A graph that admits a radio odd mean labeling is called a radio odd mean graph.

2.5. ***Definition***

The span of a labeling f is the maximum integer that f maps to a vertex of graph G.

2.6. ***Definition***

Radio odd mean number of graph G denoted by *romn* (*G*) is defined as the lowest span taken over all radio labeling of graph G.

2.7. ***Definition Shadow graph of Bistar***

The shadow graph D_2 (G) of a connected graph G is constructing by taking two copies of G, say G and G'. Join each vertex u in G to the neighbors of the corresponding vertex u' in G'

3. MAIN RESULTS

Theorem 3.1

The radio odd mean number of the Shadow graph of Star $D_2(K_{1,n})$ is 8n+1.

Proof

Let the vertex set and edge set of the Shadow graph of star $D_2(K_{1,n})$.

$V[D_2(K_{1,n})] = \{v, v', v_i, v_i' : 1 \le i \le n\}$ and

$E[D_2(K_{1,n})] = \{vv_i, v'v_i, vv_i', v'v_i' : 1 \le i \le n\}$.

The general diameter of $[D_2(K_{1,n})]$ is 2.

Define the vertex labels as follows

For, $1 \le i \le n$, $f(v) = 4n+1$

$$f(v') = 4$$

$$f(v_i) = 4n+4i+1$$

$$f(v_i') = 4i-3$$

In order to satisfy the definitions of radio odd mean labeling

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd } \ge 1 + diam(G)$$

for every pair of vertices (*u*, *v*), $u \ne v$.

We have to show that

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd } \ge 3$$

Case (1) Consider the pair (v_i', v_j'), $i \ne j$, *for* $1 \le i, j \le n$

$$d(v_i', v_j') = 2$$

$$\Rightarrow 2 + \left\lceil \frac{4i-3+4j-3}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 2 + \lceil 2i+2j-3 \rceil \ge 3$$

Case (2) Verify the pair (v', v_i'), *for* $1 \le i \le n$

$$d(v', v_i') = 1$$

$$\Rightarrow 1 + \left\lceil \frac{4+4i-3}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 1 + \left\lceil \frac{4i+1}{2} \right\rceil \ge 3$$

Case (3) Verify the pair (v, v_i'), *for* $1 \le i \le n$

$$d(v, v_i') = 1$$

$$\Rightarrow 1 + \left\lceil \frac{4n+1+4i-3}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 1 + \lceil 2n+2i-1 \rceil \ge 3$$

Case (4) Verify the pair (v_i', v_j), *for* $1 \le i, j \le n$

Subcase (i) $i = j$, $d(v_i', v_j) = 2$,

$$\Rightarrow 2 + \left\lceil \frac{4i-3+4n+4i+1}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 2 + \lceil 2n+4i-1 \rceil \ge 3$$

Subcase (ii) $i \ne j$, $d(v_i', v_j) = 2$,

$$\Rightarrow 2 + \left\lceil \frac{4i-3+4n+4j+1}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 2 + \lceil 2n+2i+2j-1 \rceil \ge 3$$

Case (5) Verify the pair (v, v'),

$$d(v, v') = 2$$

$$\Rightarrow 2 + \left\lceil \frac{4n+1+4}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 2 + \left\lceil \frac{4n+5}{2} \right\rceil \ge 3$$

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Case (6) Verify the pair (v', v_i), *for* $1 \le i, j \le n$

$$d(v', v_i) = 1$$

$$\Rightarrow 1 + \left\lceil \frac{4+4n+4i+1}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 1 + \left\lceil \frac{4n+4i+5}{2} \right\rceil \ge 3$$

Case (7) Verify the pair (v, v_i), *for* $1 \le i, j \le n$

$$d(v, v_i) = 1$$

$$\Rightarrow 1 + \left\lceil \frac{4n+1+4n+4j+1}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 1 + \lceil 4n+2i+1 \rceil \ge 3$$

Case (8) Verify the pair (v_i, v_j), $i \ne j$, *for* $1 \le i, j \le n$

$$d(v_i, v_j) = 2$$

$$\Rightarrow 2 + \left\lceil \frac{4n+4i+1+4n+4j+1}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 2 + \lceil 4n+2i+2j+1 \rceil \ge 3$$

In all the cases satisfies the radio odd mean condition,

Hence, *romn* $[D_2(K_{1,n})] = 8n+1$.

Theorem 3.2

The radio odd mean number of the Shadow graph of Bistar graph $D_2(B_{n,n})$ is 16n+9.

Proof

Let the vertex set and edge set of the Shadow graph of Bistar $D_2(B_{n,n})$.

$$V(D_2(B_{n,n})) = \{u, v, u_i, v_i, u, v, u_i', v_i' : 1 \le i \le n\}$$

and $E(D_2(B_{n,n})) = \{u'u_i', uu_i', u'u_i, uu_i\} \cup$

$$\{uv, uv', u'v', u'v\} \cup \{vv_i, vv_i', v'v_i', v'v_i\}$$

The general diameter of $D_2(B_{n,n})$ is 3.

Define the vertex labels as follows

For, $1 \le i \le n$, $f(u) = 4n+1$

$$f(v) = 12n+9$$

$$f(v_i) = 12n+4i+9$$

$$f(u_i) = 4n+4i+1$$

$$f(u') = 4$$

$$f(u_i') = 4i-3$$

$$f(v_i') = 8n+4i+1$$

$$f(v') = 12n+5$$

In order to satisfy the definition of radio odd mean labeling

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \text{ is odd } \ge 1 + diam(G)$$

for every pair of vertices (u, v), $u \ne v$.

We have to show that

$$d(v, v_i) + \left\lceil \frac{f(v)+f(v_i)}{2} \right\rceil \text{ is odd } \ge 4$$

Case (1) Verify the pair (u_i', u_j'), $i \ne j$,

$$d(u_i', u_j') = 2$$

$$\Rightarrow 2 + \left\lceil \frac{4i-3+4j-3}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 2 + \lceil 2i+2j-3 \rceil \ge 5$$

Case (2) Verify the pair (u_i', u'), *for* $1 \le i \le n$,

$$d(u_i', u') = 1$$

$$\Rightarrow 1 + \left\lceil \frac{4i-3+4}{2} \right\rceil \ge 1+2$$

$$\Rightarrow 1 + \left\lceil \frac{4i+1}{2} \right\rceil \ge 4$$

Case (3) Verify the pair (u, u_i'), *for* $1 \le i \le n$,

$$d(u, u_i') = 1$$

$$\Rightarrow 1 + \left\lceil \frac{4n+1+4i-3}{2} \right\rceil \ge 1+3$$

$$\Rightarrow 1 + \lceil 2n+2i-1 \rceil \ge 6$$

Case (4) Verify the pair (u_i', u_j), *for* $1 \le i, j \le n$,

$$d(u_i', u_j) = 2$$

Subcase (i) $i = j$,

$$\Rightarrow 2 + \left\lceil \frac{4i-3+4n+4i+1}{2} \right\rceil \ge 1+3$$

$$\Rightarrow 2+\lceil 2n+4i-1\rceil \ge 9$$

Subcase (ii) *$i \neq j$, ,*

$$\Rightarrow 2+\left\lceil \frac{4i-3+4n+4j+1}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 2+\lceil 2n+2i+2j-1\rceil \ge 11$$

Case (5) Verify the pair (u', u), $i \neq j$, *for* $1 \le i, j \le n$, $d(u', u)=2$

$$\Rightarrow 2+\left\lceil \frac{4+4n+1}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 2+\left\lceil \frac{4n+5}{2}\right\rceil \ge 9$$

Case (6) Verify the pair (u', u_i), *for* $1 \le i \le n$, $d(u', u_i)=1$

$$\Rightarrow 1+\left\lceil \frac{4+4n+4i+1}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 1+\left\lceil \frac{4n+4i+5}{2}\right\rceil \ge 10$$

Case (7) Verify the pair (u, u_i), *for* $1 \le i \le n$, $d(u, u_i)=1$

$$\Rightarrow 1+\left\lceil \frac{4n+1+4n+4i+1}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 1+\lceil 4n+2i+1\rceil \ge 12$$

Case (8) Verify the pair (u_i, u_j), $i \neq j$, *for* $1 \le i, j \le n$, $d(u_i, u_j)=2$

$$\Rightarrow 2+\left\lceil \frac{4n+4i+1+4n+4j+1}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 2+\lceil 4n+2i+2j+1\rceil \ge 11$$

Case (9) Verify the pair (v_i', v_j'), $i \neq j$, *for* $1 \le i, j \le n$, $d(v_i', v_j')=2$

$$\Rightarrow 2+\left\lceil \frac{8n+4i+1+8n+4j+1}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 2+\lceil 8n+2i+2j+1\rceil \ge 14$$

Case (10) Verify the pair (v_i', v'), *for* $1 \le i \le n$, $d(v_i', v')=1$

$$\Rightarrow 1+\left\lceil \frac{8n+4i+1+12n+5}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 1+\lceil 10n+2i+3\rceil \ge 14$$

Case (11) Verify the pair (v, v_i'), *for* $1 \le i \le n$, $d(v, v_i')=1$

$$\Rightarrow 1+\left\lceil \frac{12n+9+8n+4i+1}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 1+\lceil 10n+2i+5\rceil \ge 14$$

Case (12) Verify the pair (v_i', v_j), *for* $1 \le i, j \le n$, $d(v_i', v_j)=2$

Subcase (i) *$i = j$,*

$$\Rightarrow 2+\left\lceil \frac{8n+4i+1+12n+4i+9}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 2+\lceil 10n+4i+5\rceil \ge 17$$

Subcase (ii) *$i \neq j$,*

$$\Rightarrow 2+\left\lceil \frac{8n+4i+1+12n+4j+9}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 2+\lceil 10n+2i+2j+5\rceil \ge 18$$

Case (13) Verify the pair (v, v'), $d(v, v')=2$

$$\Rightarrow 2+\left\lceil \frac{12n+9+12n+5}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 2+\lceil 12n+7\rceil \ge 19$$

Case (14) Verify the pair (v', v_i), *for* $1 \le i \le n$, $d(v', v_i)=1$

$$\Rightarrow 1+\left\lceil \frac{12n+5+12n+4i+9}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 1+\lceil 12n+2i+7\rceil \ge 18$$

Case (15) Verify the pair (v, v_i), *for* $1 \le i \le n$, $d(v, v_i)=1$

$$\Rightarrow 1+\left\lceil \frac{12n+9+12n+4i+9}{2}\right\rceil \ge 1+3$$

$$\Rightarrow 1+\lceil 12n+2i+9\rceil \ge 19$$

Case (16) Verify the pair (v_i, v_j), $i \neq j$, for $1 \le i, j \le n$, $d(v_i, v_j) = 2$

$$\Rightarrow 2 + \left\lceil \frac{12n + 4i + 9 + 12n + 4j + 9}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 2 + \lceil 12n + 2i + 2j + 9 \rceil \ge 22$$

Case (17) Verify the pair (u_i', v_j'), for $1 \le i, j \le n$, $d(u_i', v_j') = 3$

Subcase (i) $i = j$,

$$\Rightarrow 3 + \left\lceil \frac{4i - 3 + 8n + 4i + 1}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 3 + \lceil 4n + 4i - 1 \rceil \ge 14$$

Subcase (ii) $i \neq j$,

$$\Rightarrow 3 + \left\lceil \frac{4i - 3 + 8n + 4j + 1}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 3 + \lceil 4n + 2i + 2j - 1 \rceil \ge 16$$

Case (18) Verify the pair (u_i', v'), $i \neq j$, for $1 \le i, j \le n$, $d(u_i', v') = 2$

$$\Rightarrow 2 + \left\lceil \frac{4i - 3 + 12n + 5}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 2 + \lceil 6n + 2i + 1 \rceil \ge 10$$

Case (19) Verify the pair (u_i', v), for $1 \le i \le n$, $d(u_i', v) = 2$

$$\Rightarrow 2 + \left\lceil \frac{4i - 3 + 12n + 9}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 2 + \lceil 6n + 2i + 3 \rceil \ge 11$$

Case (20) Verify the pair (u_i', v_j), for $1 \le i, j \le n$, $d(u_i', v_j) = 3$

Subcase (i) $i = j$,

$$\Rightarrow 3 + \left\lceil \frac{4i - 3 + 12n + 4i + 9}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 3 + \lceil 6n + 4i + 3 \rceil \ge 22$$

Subcase (ii) $i \neq j$,

$$\Rightarrow 3 + \left\lceil \frac{4i - 3 + 12n + 4j + 9}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 3 + \lceil 6n + 2i + 2j + 3 \rceil \ge 28$$

Case (21) Verify the pair (u', v_i'), for $1 \le i \le n$, $d(u', v_i') = 2$

$$\Rightarrow 2 + \left\lceil \frac{4 + 8n + 4i + 1}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 2 + \left\lceil \frac{8n + 4i + 5}{2} \right\rceil \ge 15$$

Case (22) Verify the pair (u', v'), for $1 \le i \le n$, $d(u', v') = 3$

$$\Rightarrow 1 + \left\lceil \frac{4 + 12n + 5}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 1 + \left\lceil \frac{12n + 9}{2} \right\rceil \ge 18$$

Case (23) Verify the pair (u', v),

$d(u', v) = 1$

$$\Rightarrow 1 + \left\lceil \frac{4 + 12n + 9}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 1 + \left\lceil \frac{12n + 13}{2} \right\rceil \ge 20$$

Case (24) Verify the pair (u', v_i), for $1 \le i \le n$, $d(u', v_i) = 2$

$$\Rightarrow 2 + \left\lceil \frac{4 + 12n + 4i + 9}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 2 + \left\lceil \frac{12n + 4i + 13}{2} \right\rceil \ge 23$$

Case (25) Verify the pair (u, v_i'), for $1 \le i \le n$, $d(u, v_i') = 2$

$$\Rightarrow 2 + \left\lceil \frac{4n + 1 + 8n + 4i + 1}{2} \right\rceil \ge 1 + 3$$

$$\Rightarrow 2 + \lceil 6n + 2i + 1 \rceil \ge 17$$

Case (26) Verify the pair (u, v'),

$$d(u, v') = 1$$

$$\Rightarrow 1 + \left\lceil \frac{4n+1+12n+5}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 1 + \lceil 8n+3 \rceil \geq 20$$

Case (27) Verify the pair (u, v), $d(u, v)=1$

$$\Rightarrow 1 + \left\lceil \frac{4n+1+12n+9}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 1 + \lceil 8n+5 \rceil \geq 22$$

Case (28) Verify the pair (u, v_i), *for* $1 \leq i \leq n$,

$$d(u, v_i) = 2$$

$$\Rightarrow 2 + \left\lceil \frac{4n+1+12n+4i+9}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 2 + \lceil 8n+2i+5 \rceil \geq 25$$

Case (29) Verify the pair (u_i, v'_j), $i = j$,

for $1 \leq i, j \leq n$, $d(u_i, v'_j) = 3$

$$\Rightarrow 3 + \left\lceil \frac{4n+4i+1+8n+4j+1}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 3 + \lceil 6n+2i+2j+1 \rceil \geq 22$$

Case (30) Verify the pair (u_i, v'), *for* $1 \leq i \leq n$,

$$d(u_i, v') = 2$$

$$\Rightarrow 2 + \left\lceil \frac{4n+4i+1+12n+5}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 2 + \lceil 8n+2i+3 \rceil \geq 23$$

Case (31) Verify the pair (u_i, v), *for* $1 \leq i \leq n$,

$$d(u_i, v) = 2$$

$$\Rightarrow 2 + \left\lceil \frac{4n+4i+1+12n+9}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 2 + \lceil 8n+2i+5 \rceil \geq 25$$

Case (32) Verify the pair (u_i, v_j), *for* $1 \leq i \leq n$,

$$d(u_i, v_j) = 3$$

Subcase (i) $i = j$,

$$\Rightarrow 3 + \left\lceil \frac{4n+4i+1+12n+4i+9}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 3 + \lceil 8n+4i+5 \rceil \geq 28$$

Subcase (ii) $i \neq j$,

$$\Rightarrow 3 + \left\lceil \frac{4n+4i+1+12n+4j+9}{2} \right\rceil \geq 1+3$$

$$\Rightarrow 3 + \lceil 8n+2i+2j+5 \rceil \geq 30$$

In all the cases satisfies the radio odd mean condition , Hence, *romn* $[D_2(B_{n,n})] = 16n+9$.

CONCLUSION REMARKS

The establishment of radio transmitter's networks which is free of interference is the demand of the current time. It has also posed some new challenges we take up this problem in the context of Shadowgraph of Star and Bistar. In this paper we have determined the radio odd mean number of Shadow graph of Star and Bistar.

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