International Journal of Research in Advent Technology, Vol.6, No.10, October 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Solution of Population Growth and Decay Problems by Using Aboodh Transform Method

Sudhanshu Aggarwal[1], Nirbhaya Asthana[2], D.P. Singh[3]
*[*1]Assistant Professor, Department of Mathematics, National P.G. College Barhalganj, Gorakhpur-273402, U.P., India*
[2,3]Assistant Professor, Department of Botany, National P.G. College Barhalganj, Gorakhpur-273402, U.P., India
sudhanshu30187@gmail.com, nirbhayaasthana@gmail.com, shaurya971@gmail.com

Abstract: The population growth and decay problems arise in the field of physics, chemistry, social science, biology, zoology etc. In this paper, we used Aboodh transform method for solving population growth and decay problems and some applications are given in order to demonstrate the effectiveness of Aboodh transform method for solving population growth and decay problems.

Keywords: Aboodh transform, Inverse Aboodh transform, Population growth problem, Decay problem, Half-life.

1. INTRODUCTION

Mathematically the population growth (growth of a cell, or a plant, or an organ, or a species) is governed by the linear first order ordinary differential equation [1-10]

$$\frac{dN}{dt} = kN \ldots\ldots\ldots\ldots\ldots (1)$$

with initial condition as

$$N(t_0) = N_0 \ldots\ldots\ldots\ldots (2)$$

where $k > 0$ is a real number, N and N_0 are the amount of populations at time t and initial time t_0.

Equation (1) represents the Malthusian law of population growth.

On the other hand the decay problem of the material is mathematically defined by the linear first order ordinary differential equation [7, 9-10]

$$\frac{dN}{dt} = -kN \ldots\ldots\ldots\ldots\ldots (3)$$

with initial condition as

$$N(t_0) = N_0 \ldots\ldots\ldots\ldots (4)$$

where N is the amount of the material at time $t, k > 0$ is a real number and N_0 is the amount of the material at initial time t_0.

The negative sign in the R.H.S. of equation (3) indicates the mass of the substance decreases with time and so the time derivative of N denoted by $\frac{dN}{dt}$ must be negative.

The Aboodh transform of the function $F(t)$ is defined as [11]:

$$A\{F(t)\} = \frac{1}{v}\int_0^\infty F(t)e^{-vt}dt$$
$$= K(v), t \geq 0, 0 < k_1 \leq v \leq k_2 \ldots\ldots (6)$$

where A is Aboodh transform operator.

The Aboodh transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Aboodh transform of the function $F(t)$.

The application of new transform "Aboodh Transform" to partial differential equations was given by Aboodh [12]. Aboodh et al. [13] gave the connection of Aboodh transform with some famous integral transforms. Aboodh et al. [14] used Aboodh transformation method for solving delay differential equations. Aboodh et al. [15] applied Aboodh transform to solve ordinary differential equation with variable coefficients. Solution of partial integro-differential equations by using Aboodh and double Aboodh transforms methods was given by Aboodh et al [16]. Aggarwal et al. [17] discussed the application of Aboodh transform for solving linear Volterra integro-differential equations of second kind. Aggarwal et al. [18] gave the Aboodh transform of Bessel's functions.

The aim of this work is to finding the solution of population growth and decay problems by using Aboodh transform method without large computational work.

2. LINEARITY PROPERTY OF ABOODH TRANSFORM [17-18]

If $A\{F(t)\} = H(v)$ and $A\{G(t)\} = I(v)$ then $A\{aF(t) + bG(t)\} = aA\{F(t)\} + bA\{G(t)\}$

$\Rightarrow A\{aF(t) + bG(t)\} = aH(v) + bI(v)$,

where a, b are arbitrary constants.

3. ABOODH TRANSFORM OF SOME ELEMENTARY FUNCTIONS [11, 14, 17-18]

S.N.	$F(t)$	$A\{F(t)\} = K(v)$
1.	1	$\frac{1}{v^2}$
2.	t	$\frac{1}{v^3}$
3.	t^2	$\frac{2!}{v^4}$
4.	$t^n, n\epsilon N$	$\frac{n!}{v^{n+2}}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n+2}}$
6.	e^{at}	$\frac{1}{v^2 - av}$
7.	$sinat$	$\frac{a}{v(v^2+a^2)}$
8.	$cosat$	$\frac{1}{v^2+a^2}$
9.	$sinhat$	$\frac{a}{v(v^2-a^2)}$
10.	$coshat$	$\frac{1}{v^2-a^2}$

4. INVERSE ABOODH TRANSFORM [17-18]

If $A\{F(t)\} = K(v)$ then inverse Aboodh transform of $K(v)$is given by $F(t)$and mathematically it is defined as $F(t) = A^{-1}\{K(v)\}$
where A^{-1} is the inverse Aboodh transform operator.

5. LINEARITY PROPERTY OF INVERSE ABOODH TRANSFORM

If $A^{-1}\{H(v)\} = F(t)$ and $A^{-1}\{I(v)\} = G(t)$ then
$A^{-1}\{aH(v) + bI(v)\} = aA^{-1}\{H(v)\} + bA^{-1}\{I(v)\}$
$\Rightarrow A^{-1}\{aH(v) + bI(v)\} = aF(t) + bG(t),$
where a, b are arbitrary constants.

6. INVERSE ABOODH TRANSFORM OF SOME ELEMENTARY FUNCTIONS [17-18]

S.N.	$K(v)$	$F(t) = A^{-1}\{K(v)\}$
1.	$\frac{1}{v^2}$	1
2.	$\frac{1}{v^3}$	t
3.	$\frac{1}{v^4}$	$\frac{t^2}{2!}$
4.	$\frac{1}{v^{n+2}}, n \geq 0$	$\frac{t^n}{n!}$
5.	$\frac{1}{v^{n+2}}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{v^2 - av}$	e^{at}
7.	$\frac{1}{v(v^2+a^2)}$	$\frac{sinat}{a}$
8.	$\frac{1}{v^2+a^2}$	$cosat$
9.	$\frac{1}{v(v^2-a^2)}$	$\frac{sinhat}{a}$
10.	$\frac{1}{v^2-a^2}$	$coshat$

7. ABOODH TRNANSFORM OF THE DERIVATIVES OF THE FUNCTION $F(t)$ [11, 14]

If $A\{F(t)\} = K(v)$ then

a) $A\{F'(t)\} = vK(v) - \frac{F(0)}{v}$

b) $A\{F''(t)\} = v^2K(v) - \frac{F'(0)}{v} - F(0)$

c) $A\{F^{(n)}(t)\} = v^nK(v) - \frac{F(0)}{v^{2-n}} - \frac{F'(0)}{v^{3-n}} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v}$

8. ABOODH TRANSFORM METHOD FOR POPULATION GROWTH PROBLEM

In this section, we present Aboodh transform method for population growth problem given by (1) and (2).
Applying the Aboodh transform on both sides of (1), we have

$$A\left\{\frac{dN}{dt}\right\} = kA\{N(t)\} \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

Now applying the property, Aboodh transform of derivative of function, on (5), we have

$$vA\{N(t)\} - \frac{N(0)}{v} = kA\{N(t)\} \ldots \ldots (6)$$

Using (2) in (6) and on simplification, we have

$$(v-k)\, A\{N(t)\} = \frac{N_0}{v}$$

$$\Rightarrow A\{N(t)\} = \frac{N_0}{v(v-k)} \ldots \ldots \ldots \ldots \ldots (7)$$

Operating inverse Aboodh transform on both sides of (7), we have

$$N(t) = N_0A^{-1}\left\{\frac{1}{v(v-k)}\right\} = N_0e^{kt} \ldots \ldots (8)$$

which gives the required amount of the population at time t.

9. ABOODH TRANSFORM METHOD FOR DECAY PROBLEM

In this section, we present Aboodh transform method for decay problem which is mathematically given by (3) and (4).
Applying the Aboodh transform on both sides of (3), we have

$$A\left\{\frac{dN}{dt}\right\} = -kA\{N(t)\} \dots \dots \dots \dots \dots \dots (9)$$

Now applying the property, Aboodh transform of derivative of function, on (9), we have

$$vA\{N(t)\} - \frac{N(0)}{v} = -kA\{N(t)\} \dots (10)$$

Using (4) in (10) and on simplification, we have

$$(v+k)\ A\{N(t)\} = \frac{N_0}{v}$$

$$\Rightarrow A\{N(t)\} = \frac{N_0}{v(v+k)} \dots \dots \dots \dots \dots (11)$$

Operating inverse Aboodh transform on both sides of (11), we have

$$N(t) = N_0 A^{-1}\left\{\frac{1}{v(v+k)}\right\}$$

$$\Rightarrow N(t) = N_0 e^{-kt} \dots \dots \dots \dots \dots \dots (12)$$

which gives the required amount of substance at time t.

10. APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Aboodh transform for solving population growth and decay problems.

Application: 10.1 The population of a country grows at a rate proportional to the number of people presently living in the country. If after three years, the population has doubled, and after five years the population is 10,000, then estimate the number of people initially living in the country.
Mathematically the above problem can be written as:

$$\frac{dN(t)}{dt} = kN(t) \dots \dots \dots \dots \dots (13)$$

where N denote the number of people living in the country at any time t and k is the constant of proportionality. Consider N_0 is the number of people initially living in the country at $t = 0$.
Applying the Aboodh transform on both sides of (13), we have

$$A\left\{\frac{dN(t)}{dt}\right\} = kA\{N(t)\} \dots \dots \dots \dots \dots \dots (14)$$

Now applying the property, Aboodh transform of derivative of function, on (14), we have

$$vA\{N(t)\} - \frac{N(0)}{v} = kA\{N(t)\} \dots \dots (15)$$

Since at $t = 0, N = N_0$, so using this in (15), we have

$$(v-k)\ A\{N(t)\} = \frac{N_0}{v}$$

$$\Rightarrow (v-k)\ A\{N(t)\} = \frac{N_0}{v}$$

$$\Rightarrow A\{N(t)\} = \frac{N_0}{v(v-k)} \dots \dots \dots \dots (16)$$

Operating inverse Aboodh transform on both sides of (16), we have

$$N(t) = N_0 A^{-1}\left\{\frac{1}{v(v-k)}\right\} = N_0 e^{kt} \dots (17)$$

Now at $= 3$, $N = 2N_0$, so using this in (17), we have

$$2N_0 = N_0 e^{3k}$$

$$\Rightarrow e^{3k} = 2$$

$$\Rightarrow k = \frac{1}{3} log_e 2 = 0.231 \dots \dots \dots \dots \dots (18)$$

Now using the condition at$t = 5, N = 10{,}000$, in (17), we have

$$10{,}000 = N_0 e^{5k} \dots \dots \dots \dots (19)$$

Now substituting the value of k from (18) to (19), we have

$$10{,}000 = N_0 e^{5\times 0.231}$$

$$\Rightarrow 10{,}000 = 3.174 N_0$$

$$\Rightarrow N_0 \simeq 3151 \dots \dots \dots \dots \dots \dots (20)$$

which are the required number of people initially living in the country.

Application: 10.2 A radioactive material is known to decay at a rate proportional to the amount present. If initially there is 100 milligrams of the radioactive material present and after three hours it is observed that the radioactive material has lost 20 percent of its original mass, find the half life of the radioactive material.
Mathematically the above problem can be written as:

$$\frac{dN(t)}{dt} = -kN(t) \dots \dots \dots \dots \dots (21)$$

where N denote the amount of radioactive material at time t and k is the constant of proportionality. Consider N_0 is the initial amount of the radioactive material at time $t = 0$.
Applying the Aboodh transform on both sides of (21), we have

$$A\left\{\frac{dN(t)}{dt}\right\} = -kA\{N(t)\} \dots \dots \dots \dots \dots \dots (22)$$

Now applying the property, Aboodh transform of derivative of function, on (22), we have

$$vA\{N(t)\} - \frac{N(0)}{v} = -kA\{N(t)\} \dots (23)$$

Since at $t = 0, N = N_0 = 100$, so using this in (23), we have

$$vA\{N(t)\} - \frac{100}{v} = -kA\{N(t)\}$$

$$\Rightarrow (v+k)\ A\{N(t)\} = \frac{100}{v}$$

$$\Rightarrow A\{N(t)\} = \frac{100}{v(v+k)} \ldots\ldots\ldots\ldots..(24)$$

Operating inverse Aboodh l transform on both sides of (24), we have

$$N(t) = 100A^{-1}\left\{\frac{1}{v(v+k)}\right\}$$

$$\Rightarrow N(t) = 100e^{-kt} \ldots\ldots..(25)$$

Now at $t = 3$, the radioactive material has lost 20 percent of its original mass 100 mg so $N = 100 - 20 = 80$, using this in (25), we have

$$80 = 100e^{-3k}$$

$$\Rightarrow e^{-3k} = 0.80$$

$$\Rightarrow k = -\frac{1}{3}log_e 0.80 = 0.07438 \ldots\ldots\ldots(26)$$

We required t when $N = \frac{N_0}{2} = \frac{100}{2} = 50$ so from (25), we have

$$50 = 100e^{-kt} \ldots\ldots\ldots\ldots.(27)$$

Now substituting the value of k from (26) to (27), we have

$$50 = 100e^{-0.07438t}$$

$$\Rightarrow e^{-0.07438t} = 0.50$$

$$\Rightarrow t = -\frac{1}{0.07438}log_e 0.50$$

$$\Rightarrow t = 9.32\ hours \ldots\ldots\ldots\ldots..(28)$$

which is the required half-time of the radioactive material.

11. CONCLUSION

In this paper, we have successfully developed the Aboodh transform method for solving the population growth and decay problems. The given applications show that the effectiveness of Aboodh transform method for solving population growth and decay problems. In the future, the proposed scheme can be applied for solving the problems of heat conduction and electrical circuit problems.

REFERENCES

[1] Weigelhofer, W.S.; Lindsay, K.A. (1999): Ordinary Differential Equations & Applications: Mathematical Methods for Applied Mathematicians, Physicists, Engineers and Bioscientists, Woodhead.

[2] Roberts, C. (2010): Ordinary Differential Equations: Applications, Models and Computing, Chapman and Hall/ CRC.

[3] Ahsan, Z. (2006): Differential Equations and Their Applications, PHI.

[4] Braun, M. (1975): Differential Equations and Their Applications, Springer.

[5] Abell, M.L.; Braselton, J.P. (2014): Introductory Differential Equations, Academic Press.

[6] Ang, W.T.; Park, Y.S. (2008): Ordinary Differential Equations: Methods and Applications, Universal Publishers.

[7] Gorain, G.C. (2014): Introductory Course on Differential Equations, Narosa.

[8] Zill, D.G.; Cullen, M.R. (1996): Differential Equations with Boundary Value Problems, Thomson Brooks/ Cole.

[9] Bronson, R.; Costa, G.B. (2006): Schaum's Outline of Differential Equations, McGraw-Hill.

[10] Kapur, J.N. (2005): Mathematical Modelling, New-Age.

[11] Aboodh, K.S. (2013): The New Integral Transform "Aboodh Transform", Global Journal of Pure and Applied Mathematics, 9(1), pp. 35-43.

[12] Aboodh, K.S. (2014): Application of New Transform "Aboodh Transform" to Partial Differential Equations, Global Journal of Pure and Applied Mathematics, 10(2), pp. 249-254.

[13] Aboodh, K.S. (2017): Idris Abdullahi and Nuruddeen, R.I., On the Aboodh Transform Connections with Some Famous Integral Transforms, International Journal of Engineering and Information Systems, 1(9), pp. 143-151.

[14] Aboodh, K.S.; Farah, R.A.; Almardy, I.A.; Osman, A.K. (2018): Solving Delay Differential Equations by Aboodh Transformation Method, International Journal of Applied Mathematics & Statistical Sciences, 7(2), pp. 55-64.

[15] Mohand, D.; Aboodh, K.S.; Abdelbagy, A. (2016): On the Solution of Ordinary Differential Equation with Variable Coefficients Using Aboodh Transform, Advances in Theoretical and Applied Mathematics, 11(4), pp. 383-389.

[16] Aboodh, K.S.; Farah, R.A.; Almardy, I.A.; Almostafa, F.A. (2016): Solution of Partial Integro-Differential Equations by Using Aboodh and Double Aboodh Transforms Methods, Global Journal of Pure and Applied Mathematics, 13(8), pp. 4347-4360.

[17] Aggarwal, S.; Sharma, N.; Chauhan, R. (2018): Application of Aboodh Transform for Solving Linear Volterra Integro-Differential Equations of Second Kind, International Journal of Research in Advent Technology, 6(6), pp. 1186-1190.

[18] Aggarwal, S.; Gupta, A.R.; Agrawal, D. (2018): Aboodh Transform of Bessel's Functions,

Journal of Advanced Research in Applied Mathematics and Statistics, 3(3), pp. 1-5.

Sudhanshu Aggarwal received his M.Sc. degree from M.S. College, Saharanpur in 2007. He has also qualified CSIR NET examination (June-2010, June-2012, June-2013, June-2014 & June-2015) in Mathematical Sciences. He is working as an Assistant Professor in National P.G. College Barhalganj Gorakhpur. He is equipped with an extraordinary caliber and appreciable academic potency. He has around eleven years of teaching experience at various engineering colleges affiliated to AKTU. His fields of interest include Integral Transform Methods, Differential and Partial Differential Equations, Integral Equations and Number Theory.

Nirbhaya Asthana received his M.Sc. degree in Botany from Shibli National College Azamgarh. He has also qualified CSIR NET examination (June-2002) in Life Sciences. He is working as an Assistant Professor in Department of Botany at National P.G. College Barhalganj Gorakhpur.

Dr. D.P. Singh received his M.Sc. and Ph.D degree from Allahabad University. He has also qualified CSIR NET examination (June-2000) in Life Sciences. He is working as an Assistant Professor in Department of Botany at National P.G. College Barhalganj Gorakhpur.