International Journal of Research in Advent Technology, Vol.6, No.10, October 2018
E-ISSN: 2321-9637
Available online at www.ijrat.org

Fixed Points and Coupled Fixed Points in Hausdorff Intuitionistic L-Fuzzy Metric Spaces

S.Yahya Mohamed[1], E.Naargees Begum[2]
[1]*PG & Research Department of Mathematics, Government Arts College, Trichy-22.*
Email: yahya_md@yahoo.com
[2]*Department of Mathematics, Sri Kailash Women's College, Thalaivasal,Tamilnadu.*
Email:.mathsnb@gmail.com

Abstract: Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets are introduced by Lotfi.A.Zadeh [27] as an extension of the classical notion of sets. Intuitionistic fuzzy set can be utilized as a proper tool for representing hesitancy concerning both membership and non-membership of an element to a set. Atanassov [1] introduced and studied the concept of Intuitionistic fuzzy sets. The idea of Hausdorff fuzzy metric space introduced by Rodriguez-Lopez and Romaguera [16]. In this paper, a new concept of intuitionistic fuzzy fixed point theorems in Hausdorff intuitionistic L-fuzzy metric spaces is introduced and some properties and theorems about fixed points in Hausdorff intuitionistic L-fuzzy metric space are discussed.

Keywords: Intuitionistic Fuzzy mapping, Intuitionistic fuzzy point, fuzzy metric spaces, Hausdorff intuitionistic fuzzy metric spaces, Hausdorff intuitionistic L- fuzzy metric spaces.

AMS Classification 2010: 03B47, 03G25**,** 03G10, 03E72, 06D99.

1. INTRODUCTION

In 1965, Zadeh [27] introduced the concept of fuzzy set as a method of finding uncertainty. In 1986, the idea of intuitionistic fuzzy set was introduced by Atanassov [1] which is a generalization of fuzzy set. In recent years there has been an increasing interest in mathematical aspects of intuitionistic fuzzy sets. To use this concept in topology and analysis, many authors have extensively developed the theory of intuitionistic fuzzy sets and applications. The study of fixed point theorems of maps satisfying contractive type conditions in fuzzy metric spaces has been a very active field of research activity recently. George and Veeramani [11] introduced the concept of fuzzy metric spaces in different ways. Grabiec [13] obtained the fuzzy version of Banach contraction principle. Mishra et al. [22] proved common fixed point theorems for compatible maps on fuzzy metric spaces. In such ways, Park [23], V. Gregori [9] has defined intuitionistic fuzzy metric spaces and obtained several classical theorems on this new structure. In 2008, R. Saadati et.al [24] introduced Modified intuitionistic fuzzy metric space.

In 2004, Rodriguez- Lopez and Romaguera [16] introduced Hausdorff fuzzy metric on the set of the non-empty compact subsets of a given fuzzy metric space. Later, several authors proved some fixed point theorems for in intuitionistic fuzzy metric spaces, it can be found in [2, 3, 4, 17, 18, 19, 20, 21]. In this paper, the notion of Hausdorff intuitionistic L-fuzzy metric space is introduce and also the theorems on intuitionistic fuzzy fixed points and coupled Fuzzy Fixed Points in Hausdorff Intuitionistic L- Fuzzy Metric space are stated and proved.

2. PRELIMINARIES

Definition: 2.1

An ***L*-fuzzy** set ϕ on U is a mapping ϕ:U → L, where L is a 'transitive partially ordered set'. In this work, we assume that $(L,\leq)$ is a preordered set. Notice that it is natural to assume that the relation ≤ is not anti-symmetric; if $x, y \in L$ are synonyms, that is, words or expressions that are used with the same meaning, then $x \leq y$ and $x \geq y$, but still x and y are distinct words.

Example 2.1

Suppose that U consists of a group of people. The L-fuzzy set, whose membership function ϕ, describes how well the persons in U can ski. For instance, there exist people who can ski very well, some ski badly, and some are moderate skiers.

Definition 2.2

If X is a topological space, X is said to be ***metrizable*** if there exist a metric d on the set X that induces the topology on X.

A metric space is a metrizable space X together with a specific metric d that gives the topology on X.

Definition 2.3

A given metric space (X, d) is known as a ***Hausdorff space*** if for every pair of distinct $x_1, x_2 \in$

X there exist a neighbourhoods N_1, N_2 of x_1 and x_2 respectively, such that $N_1 \cap N_2 = \emptyset$.

Definition 2.4

A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is called a t-norm if it satisfies the following condition.

(i) $*$ is associative and commutative.

(ii) $a * 1 = a$ for every $a \in [0,1]$

(iii) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, for $a, b, c, d \in [0,1]$.

In addition, $*$ is continuous, then $*$ is called a ***continuous t-norm.***

Definition 2.5

A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is a continuous *t*-conorm if $\diamond$ satisfies the following conditions,

1) $\diamond$ is associative and commutative,
2) $\diamond$ is continuous,
3) $a \diamond 0 = a$ for all $a \in [0,1]$,
4) $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$, for all $a, b, c, d \in [0,1]$.

Definition: 2.6

A 5-tuple(X, M, N, *, ◊) is said to be an ***intuitionistic fuzzy metric space*** if X is an arbitrary set, * is a continuous *t*-norm, ◊ is a continuous *t*- conorm and, M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the conditions:

1. $M(x,y,t) + N(x,y,t) \le 1$ $for\ all, x, y \epsilon X\ and\ ;\ t > 0$
2. $M(x,y,0) = 0\ for\ all, x, y \epsilon X,$
3. $M(x,y,t) = 1\ for\ all, x, y \epsilon X, and\ t > 0\ if\ and\ only\ if\ x = y;$
4. $M(x,y,t) = M(y,x,t)\ for\ all, x, y \epsilon X\ and\ t > 0;$
5. $M(x,y,t) * M(y,z,s) \le M(x,z,t+s), for\ all, x, y, z \epsilon X\ and\ s, t > 0;$
6. $M(x,y,.) : [0,\infty) \to [0,\infty] is\ left\ continuous, for\ all, x, y \epsilon X;$
7. $\lim_{n\to\infty} \mathrm{M(x,y,t)} = 1$ for all, x,y ϵ X and t > 0;
8. $N(x,y,0) = 1 for\ all, x, y \epsilon X$;
9. $N(x,y,t) = 0, for\ all\ x, y \epsilon X\ and\ t > 0\ if\ and\ only\ if\ x = y;$
10. $N(x,y,t) = N(y,x,t)\ for\ all, x, y \epsilon X, and\ t > 0;$
11. $N(x,y,t) \lozenge N(y,z,s) \ge N(x,z,t+s)\ for\ all\ x, y, z \epsilon X\ and\ s, t > 0;$
12. $N(x,y,.) : [0,\infty) \to [0,1] is\ right\ continuous, for\ all\ x, y \epsilon X;$
13. $\lim_{t\to\infty} \mathrm{N(x,y,t)} = 0$ for all x,y ϵ X.

The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between *x* and *y* w.r.t. *t* respectively.

Definition 2.7

Let $(X, M, N *)$ be a fuzzy metric space. ***The Hausdorff intuitionistic fuzzy metric*** $H_M : (k_M(X))^2 \times (0,\infty)$ $H_N : (k_N(X))^2 \times (0,\infty)$ is defined by

$$H_M(A,B,t) = \min\left\{ \begin{matrix} inf \\ x \in A \end{matrix} \left(\begin{matrix} sup \\ y \in B \end{matrix} M(x,y,t) \right), \begin{matrix} inf \\ y \in B \end{matrix} \left(\begin{matrix} sup \\ y \in A \end{matrix} M(x,y,t) \right) \right\}$$

$$H_N(A,B,t) = \max\left\{ \begin{matrix} inf \\ x \in A \end{matrix} \left(\begin{matrix} sup \\ y \in B \end{matrix} M(x,y,t) \right), \begin{matrix} inf \\ y \in B \end{matrix} \left(\begin{matrix} sup \\ y \in A \end{matrix} M(x,y,t) \right) \right\}$$

for all $A, B \in k_M(X)$ $C, D \in k_N(X)$ and $t > 0$.

Where $k_M(X)$, $k_N(X)$ denotes the set of its nonempty compact subsets.

Definition 2.8

A fuzzy point x_α in X is called a fixed point of a fuzzy mapping F,if $x_\alpha \subset Fx$,

i.e., $\alpha \le Fx(x)$ or $x \in [Fx]_\alpha$, i.e., the fixed degree of x in F is at least α.

If $x_1 \in Fx$, then x is called a ***fixed point of the fuzzy mapping F***.

Definition 2.9

Let X be an arbitrary nonempty set, T be fuzzy mapping from X into $\mathfrak{F}(X)$ and z$\in X$. If there exists $\alpha \epsilon [0,1]$ such that z$\in [Tz]_\alpha$, then a point z is called an ***α- fuzzy fixed point of T.***

Definition 2.10

An element $((x)_\alpha, (y)_\alpha) \in X \times X$ is called a ***coupled fixed fuzzy point*** of the mapping $F : X \times X \to W_\alpha(X)$, if

$(x)_\alpha \subset \mathrm{F(x,y)}, \mathrm{(y)}_\alpha \subset \mathrm{F(y,x)},$

That is, $x \in [F(x,y)]_\alpha , y \in [F(y,x)]_\alpha$.

.

3. FIXED POINTS AND COUPLED FIXED POINTS IN HAUSDORFF INTUITIONISTIC L-FUZZY METRIC SPACES

Definition 3.1

Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. ***The Hausdorff intuitionistic L- fuzzy metric*** $H_M : (k_M(X))^2 \times L$ and $H_N : (k_N(X))^2 \times L$ is defined by

$$H_M(A,B,t) = \min\left\{ \begin{matrix} inf \\ x \in A \end{matrix} \left(\begin{matrix} sup \\ y \in B \end{matrix} M(x,y,t) \right), \begin{matrix} inf \\ y \in B \end{matrix} \left(\begin{matrix} sup \\ y \in A \end{matrix} M(x,y,t) \right) \right\}$$

$$H_N(A,B,t) = \max\left\{ \begin{matrix} sup \\ x \in A \end{matrix} \left(\begin{matrix} inf \\ y \in B \end{matrix} N(x,y,t) \right), \begin{matrix} sup \\ y \in B \end{matrix} \left(\begin{matrix} inf \\ y \in A \end{matrix} N(x,y,t) \right) \right\}$$

for all $A, B \in k_M(X)$ and $t > 0$.

Where $k_M(X)$, $k_N(X)$ denotes the set of its nonempty compact subsets.

Lemma 3.1

Let $(X, M, N *)$ be a Intuitionistic L-fuzzy metric space and $\{x_n\}$ is a sequence in X such that for all $n \in \mathbb{N}$,

$M(x_n, x_{n+1}, Kt) \geq M(x_{n-1}, x_n, t)$, $N(x_n, x_{n+1}, Kt) \leq N(x_{n-1}, x_n, t)$, where $0 < K < 1$. Suppose that

$\lim_{n\to\infty} *_{i=n}^{\infty} M(x_0, x_1, th^i) = 1$ .... (1)

$\lim_{n\to\infty} *_{i=n}^{\infty} N(x_0, x_1, th^i) = 0$ ... . (2)

For all t > 0 and h > 1. Then $\{x_n\}$ is a cauchy sequence.

PROOF:

Given, For each $n \in \mathbb{N}$ and $t > 0$, We have $M(x_n, x_{n+1}, Kt) \geq M(x_{n-1}, x_n, t)$, $N(x_n, x_{n+1}, Kt) \leq N(x_{n-1}, x_n, t)$

$$M(x_n, x_{n-1}, t) \geq M\left(x_{n-1}, x_n, \frac{1}{k}t\right) \geq M\left(x_{n-2}, x_{n-1}, \frac{1}{k^2}t\right) \geq \cdots \geq M\left(x_0, x_1, \frac{1}{k^{n-1}}t\right)$$

$$N(x_n, x_{n-1}, t) \leq N\left(x_{n-1}, x_n, \frac{1}{k}t\right) \leq N\left(x_{n-2}, x_{n-1}, \frac{1}{k^2}t\right) \leq \cdots \leq N\left(x_0, x_1, \frac{1}{k^{n-1}}t\right)$$

Thus for each $n \in \mathbb{N}$, We get

$$M(x_n, x_{n+1}, t) \geq M\left(x_0, x_1, \frac{1}{k^{n-1}}t\right)$$

$$N(x_n, x_{n+1}, t) \leq N\left(x_0, x_1, \frac{1}{k^{n-1}}t\right)$$

Pick the constant $h > 1$ and $l \in \mathbb{N}$ such that hk < 1 and $\sum_{i=1}^{\infty} \frac{1}{h^i} = \frac{\frac{1}{h^l}}{1-\frac{1}{h}} < 1$

Hence, for $m \geq n$, We get

$$M(x_n, x_m, t) \geq M\left(x_n, x_m, \left(\frac{1}{h^l} + \frac{1}{h^{l+1}} + \cdots + \frac{1}{h^{l+m}}\right)t\right)$$

$$\geq M\left(x_n, x_{n+1}, \frac{1}{h^l}t\right) * M\left(x_{n+1}, x_{n+2}, \frac{1}{h^{l+1}}t\right) * \ldots * M\left(x_{m-1}, x_m, \frac{1}{h^{l+m}}t\right)$$

$$\geq M\left(x_0, x_1, \frac{1}{k^{n-1}h^l}t\right) * M\left(x_0, x_1, \frac{1}{k^n h^{l+1}}t\right) * \ldots * M\left(x_0, x_1, \frac{1}{k^{m-2}h^{l+m-n-2}}t\right)$$

$$\geq M\left(x_0, x_1, \frac{1}{(kh)^{n-1}}t\right) * M\left(x_0, x_1, \frac{1}{(kh)^n}t\right) * \ldots * M\left(x_0, x_1, \frac{1}{(kh)^{m-2}}\right)$$

$$\geq *_{i=n}^{\infty} M\left(x_0, x_1, \frac{1}{(kh)^{i-1}}t\right)$$

$$N(x_n, x_m, t) \leq N\left(x_n, x_m, \left(\frac{1}{h^l} + \frac{1}{h^{l+1}} + \cdots + \frac{1}{h^{l+m}}\right)t\right)$$

$$\leq N\left(x_n, x_{n+1}, \frac{1}{h^l}t\right) \diamond N\left(x_{n+1}, x_{n+2}, \frac{1}{h^{l+1}}t\right) \diamond \ldots \diamond N\left(x_{m-1}, x_m, \frac{1}{h^{l+m}}t\right)$$

$$\leq N\left(x_0, x_1, \frac{1}{k^{n-1}h^l}t\right) \diamond N\left(x_0, x_1, \frac{1}{k^n h^{l+1}}t\right) \diamond \ldots \diamond N\left(x_0, x_1, \frac{1}{k^{m-2}h^{l+m-n-2}}t\right)$$

$$\leq N\left(x_0, x_1, \frac{1}{(kh)^{n-1}}t\right) \diamond N\left(x_0, x_1, \frac{1}{(kh)^n}t\right) \diamond \ldots \diamond N\left(x_0, x_1, \frac{1}{(kh)^{m-2}}\right)$$

$$\leq \lim_{n\to\infty} *_{i=n}^{\infty} N\left(x_0, x_1, \frac{1}{(kh)^{i-1}}t\right)$$

Then, from the above, we have

$$\lim_{m,n\to\infty} M(x_n, x_m, t) \geq \lim_{n\to\infty} *_{i=n}^{\infty} \left(x_0, x_1, \frac{1}{(kh)^{i-1}}t\right) = 1,$$

$$\lim_{m,n\to\infty} N(x_n, x_m, t) \geq \lim_{n\to\infty} *_{i=n}^{\infty} \left(x_0, x_1, \frac{1}{(kh)^{i-1}}t\right) = 0$$

For each t > 0. Therefore, we get

$$\lim_{m,n\to\infty} M(x_n, x_m, t) = 1,$$

$$\lim_{m,n\to\infty} N(x_n, x_m, t) = 0.$$

For each t > 0 and so $\{x_n\}$ is a Cauchy sequence.

Hence the proof.

Theorem 3.1

Let $(X, M, N, *, \diamond)$ be a complete intuitionistic L-fuzzy metric space and $\alpha: X \to L$ be a mapping such that $[Tx]_{\alpha(x)}$ is a nonempty compact subset of X for all $x \in X$. Suppose that $T: X \to \mathfrak{F}(X)$ is an intuitionistic fuzzy mapping such that

$H_M([Tx]_{\alpha(x)}, [Tx]_{\alpha(y)}, kt) \geq M(x, y, t)$ .... (3)

$H_N([Tx]_{\alpha(x)}, [Tx]_{\alpha(y)}, kt) \leq N(x, y, t)$ .... (4)

For all t > 0, where $k \in (0,1)$. If there exist $x_0 \in X$ and $x_1 \in [Tx_0]_{\alpha(x_0)}$ such that

$\lim_{n\to\infty} *_{i=n}^{\infty} M(x_0, x_1, th^i) = 1$ .... (5)

$\lim_{n\to\infty} *_{i=n}^{\infty} N(x_0, x_1, th^i) = 0$ .... (6)

For all t > 0 and h > 1, then T has an α-intuitionistic fuzzy fixed point.

Proof :

We start from $x_0 \in X$ and $x_1 \in [Tx_0]_{\alpha(x_0)}$ and under the hypothesis.

From the assumption,

We have $[Tx_1]_{\alpha(x_1)}$ is a nonempty compact subset of X.

If $[Tx_0]_{\alpha(x_0)} = [Tx_1]_{\alpha(x_1)}$, then $x_1 \in [Tx_1]_{\alpha(x_1)}$ and so x_1 is an α-fuzzy fixed point of T and the proof is finished.

Therefore , We may assume that $[Tx_0]_{\alpha(x_0)} \neq [Tx_1]_{\alpha(x_1)}$.

Since $x_1 \in [Tx_1]_{\alpha(x_1)}$ satisfying

$$M(x_1, x_2, kt) = \sup_{x_2' \in [Tx_1]_{\alpha(x_1)}} M(x_1, x_2', kt)$$

$$\geq H_M([Tx_0]_{\alpha(x_0)}, [Tx_1]_{\alpha(x_2)}, kt)$$

$$M(x_1, x_2, kt) \geq M(x_0, x_1, t)$$

$$N(x_1, x_2, kt) = \sup_{x_2' \in [Tx_1]_{\alpha(x_1)}} N(x_1, x_2', kt)$$

$$\leq H_N([Tx_0]_{\alpha(x_0)}, [Tx_1]_{\alpha(x_2)}, kt)$$

$$N(x_1, x_2, kt) \leq N(x_0, x_1, t)$$

If $[Tx_1]_{\alpha(x_1)} = [Tx_2]_{\alpha(x_2)}$, then $x_2 \in [Tx_2]_{\alpha(x_2)}$

This implies that x_2 *is an* α-intuitionistic fuzzy fixed point of T and then the proof is finished.

Therefore, we may assume that $[Tx_1]_{\alpha(x_1)} \neq [Tx_2]_{\alpha(x_2)}$

Since $x_2 \in [Tx_1]_{\alpha(x_1)}$ and $[Tx_2]_{\alpha(x_2)}$ is a nonempty compact subset of X,

We know that from equation (3) and (4), there exists $x_3 \in [Tx_2]_{\alpha(x_2)}$ satisfying

$$M(x_2, x_3, kt) = \sup_{x_3' \in [Tx_2]_{\alpha(x_2)}} M(x_2, x_3', kt)$$

$$M(x_2, x_3, kt) \geq H_M([Tx_1]_{\alpha(x_1)}, [Tx_2]_{\alpha(x_2)}, kt)$$

$$M(x_2, x_3, kt) \geq M(x_1, x_2, t)$$

$$N(x_2, x_3, kt) = \sup_{x_3' \in [Tx_2]_{\alpha(x_2)}} N(x_2, x_3', kt)$$

$$N(x_2, x_3, kt) \leq H_N([Tx_1]_{\alpha(x_1)}, [Tx_2]_{\alpha(x_2)}, kt)$$

$$N(x_2, x_3, kt) \leq N(x_1, x_2, t)$$

By induction, We can construct the sequence $\{x_n\}$ in X such that $x_n \in [Tx_{n-1}]_{\alpha(x_{n-1})}$ and

$M(x_n, x_{n+1}, kt) \geq M(x_{n-1}, x_n, t)$, for all $n \in \mathbb{N}$.

$N(x_n, x_{n+1}, kt) \geq N(x_{n-1}, x_n, t)$, for all $n \in \mathbb{N}$.

From lemma 3.1, we get $\{x_n\}$ is a Cauchy sequence.

Since $(X, M, N, *, \diamond)$ is a complete L-intuitionistic fuzzy metric space, there exists $x \in X$ such that $\lim_{n\to\infty} x_n = x$,

Which means $\lim_{n\to\infty} M(x_n, x, t) = 1$,
$\lim_{n\to\infty} N(x_n, x, t) = 0$, for each t > 0.

Now we claim that $x \in [Tx]_{\alpha(x)}$.

Since $H_M([Tx_n]_{\alpha(x_n)}, [Tx]_{\alpha(x)}, kt) \geq M(x_n, x, t)$

$H_N([Tx_n]_{\alpha(x_n)}, [Tx]_{\alpha(x)}, kt) \leq N(x_n, x, t)$

and $\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$ then for each t > 0.

We get,

$$\lim_{n\to\infty} H_M([Tx_n]_{\alpha(x_n)}, [Tx]_{\alpha(x)}, kt) = 1 \quad \text{.........} (7)$$

$$\lim_{n\to\infty} H_N([Tx_n]_{\alpha(x_n)}, [Tx]_{\alpha(x)}, kt) = 0 \quad \text{.........} (8)$$

This implies that

$$\lim_{n\to\infty} \sup_{x' \in [Tx]_{\alpha(x)}} M(x_{n+1}, x', t) = 1$$

$$\lim_{n\to\infty} \sup_{x' \in [Tx]_{\alpha(x)}} N(x_{n+1}, x', t) = 0$$

And thus there exists a sequence $\{x_n'\}$ *in* $[Tx]_{\alpha(x)}$ such that

$$\lim_{n\to\infty} M(x_n, x_n', t) = 1,$$

$\lim_{n\to\infty} N(x_n, x_n', t) = 0$ for each t > 0 .

For each $n \in \mathbb{N}$, We have

$$M(x_n', x_n, t) \geq M\left(x_n', x_n, \frac{t}{2}\right) * M\left(x_n, x, \frac{t}{2}\right)$$

Since $\lim_{n\to\infty} M\left(x_n', x_n, \frac{t}{2}\right) = 1$ *and* $\lim_{n\to\infty} M\left(x_n, x, \frac{t}{2}\right) = 1$.

We get, $\lim_{n\to\infty} M(x_n', x, t) = 1$,

$$N(x_n', x_n, t) \geq N\left(x_n', x_n, \frac{t}{2}\right) \diamond N\left(x_n, x, \frac{t}{2}\right)$$

Since $\lim_{n\to\infty} N\left(x_n', x_n, \frac{t}{2}\right) = 0$ *and* $\lim_{n\to\infty} N\left(x_n, x, \frac{t}{2}\right) = 0$.

We get, $\lim_{n\to\infty} N(x_n', x, t) = 0$, ...(9)

That is, $\lim_{n\to\infty} x_n' = x$.

It follows from $[Tx]_{\alpha(x)}$ being a compact subset of X and $x_n' \in [Tx]_{\alpha(x)}$ that $x \in [Tx]_{\alpha(x)}$. Therefore, x is an α-intuitionistic fuzzy fixed point of T.

Corollary 3.2

Let (X,d) be a complete metric space and $\alpha: X \to L$ be a mapping such that $[Tx]_{\alpha(x)}$ is a nonempty compact subset of X for all $x \in X$.

Suppose that $T: X \to \mathfrak{F}(X)$ be an intuitionistic fuzzy mapping such that

$$H_M([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \leq kd(x, y)$$

$$H_N([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \geq kd(x, y)$$

For all t > 0, where $k \in (0,1)$. Then T has an α-intuitionistic fuzzy fixed point.

Proof:

Let $(X, M, N, *, \diamond)$ be standard Intuitionistic L-fuzzy metric space induced by the metric d with $a * b = ab$.

Now we show that the condition of Theorem 3.6 is satisfied.

Since (X,d) is a complete metric space then $(X, M, *)$ is complete. It is easy to see that $(X, M, *)$ satisfies equation (7) in Theorem 3.6.

For each nonempty compact subset of X, We have

$$H_M(A, B, t) = \frac{t}{t + H(A, B)} \quad \text{............} (10)$$

$$H_N(A,, B, t) = 1 - \frac{t}{t + H(A, B)} \quad \text{............} (11)$$

By the above equality, We have

$H_M([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) = \frac{kt}{kt+H([Tx]_{\alpha(x)},[Ty]_{\alpha(y)})}$

$$H_M([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) \geq \frac{kt}{k(t+d(x,y))}$$

$$H_M([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) \geq \frac{t}{t+d(x,y)}$$

$H_M([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) = M(x,y,t)$, for each t > 0 and each $x,y \in X$.

$H_N([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) = 1 - \frac{kt}{kt+H([Tx]_{\alpha(x)},[Ty]_{\alpha(y)})}$

$$H_N([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) \leq 1 - \frac{kt}{k(t+d(x,y))}$$

$$H_N([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) \leq 1 - \frac{t}{t+d(x,y)}$$

$H_N([Tx]_{\alpha(x)},[Ty]_{\alpha(y)},kt) = N(x,y,t)$, for each t > 0 and each $x,y \in X$.

Therefore, T has an α-intuitionistic fuzzy fixed point.

Theorem 3.2

Let $(X,M,N\,*)$ be a complete intuitionistic L-fuzzy metric space and $F: X \to L$ an intuitionistic fuzzy mapping. Suppose that, for all $x,y \in X$ and $t > 0$, the following condition holds:

$H_{M_\alpha}(F_x,F_y,\psi(M(x,y,t))t) \geq M(x,y,t)$, ……… (12)

$H_{N_\alpha}(F_x,F_y,\psi(N(x,y,t))t) \leq N(x,y,t)$, ……… (13)

Where $\psi \in s$. Then F has a fixed fuzzy point.

PROOF :

Let x_0 be a given point in X and $x_1 \in (F_0)_\alpha$. Since $(Fx_0)_\alpha$ is compact, We can choose $x_2 \in (Fx_1)_\alpha$ such that

$$M(x_2,x_1,t) \geq M_\alpha(x_2,x_1,\psi(M(x_1,x_0,t))t)$$

$$= \sup_{y \in (Fx_1)_\alpha} M(y,x_1,\psi(M(x_1,x_0,t))t)$$

$$\geq H_{M_\alpha}(Fx_1,Fx_0,\psi(M(x_1,x_0,t))t) \geq M(x_1,x_0,t)$$

$$N(x_2,x_1,t) \leq N_\alpha(x_2,x_1,\psi(N(x_1,x_0,t))t)$$

$$= \sup_{y \in (Fx_1)_\alpha} N(y,x_1,\psi(N(x_1,x_0,t))t)$$

$$\leq H_{N_\alpha}(Fx_1,Fx_0,\psi(N(x_1,x_0,t))t) \leq N(x_1,x_0,t)$$

Thus, We have

$$M(x_2,x_1,t) \geq M(x_1,x_0,t).$$
$$N(x_2,x_1,t) \leq N(x_1,x_0,t).$$

Continuing this way, We can obtain a sequence $\{x_n\}$ in X such that $x_{n+1} \in (Fx_n)_\alpha$ for all $n \geq 0$, and

$$M(x_{n+2},x_{n+1},t) \geq M(x_{n+1},x_n,t)$$
$$N(x_{n+2},x_{n+1},t) \geq N(x_{n+1},x_n,t)$$

Thus the sequence $\{M(x_{n+2},x_{n+1},t)\}$, $\{N(x_{n+2},x_{n+1},t)\}$ is non-decreasing.

If follows from an assumption on ψ that

$$\lim_{n\to+\infty}\sup\psi(M(x_{n+2},x_{n+1},t)) < 1$$
$$\lim_{n\to+\infty}\sup\psi(N(x_{n+2},x_{n+1},t)) < 1$$

And hence there exist $k < 1$ and $n_1 \in \mathbb{N}$ such that $\psi(M(x_{n+1},x_n,t)) < k$, $\psi(N(x_{n+1},x_n,t)) < k$ for all $n > n_1$. …………… (14)

As $M(x,y,.)\ N(x,y,.)$ *are* non-decreasing, then from equation (6) We have

$$M(x_{n+2},x_{n+1},t) \geq M(x_{n+2},x_{n+1},\psi(M(x_{n+1},x_n,t))t)$$

$$\geq H_{M_\alpha}(Fx_{n+1},Fx_n,\psi(M(x_{n+1},x_n,t))t) \geq M(x_{n+1},x_n,t)$$ for all $n \in \mathbb{N}$ with $n > n_1$.

$$N(x_{n+2},x_{n+1},t) \leq N(x_{n+2},x_{n+1},\psi(N(x_{n+1},x_n,t))t)$$

$$\leq H_{N_\alpha}(Fx_{n+1},Fx_n,\psi(N(x_{n+1},x_n,t))t) \leq N(x_{n+1},x_n,t)$$ for all $n \in \mathbb{N}$ with $n > n_1$.

Hence we have

$$M(x_{n+2},x_{n+1},t) \geq M(x_{n+1},x_n,\tfrac{t}{k}),$$

$N(x_{n+2},x_{n+1},t) \geq N(x_{n+1},x_n,\frac{t}{k})$ for all $n \in \mathbb{N}$ with $n > n_1$.

Continuing this way, for all $n > n_1$, We can obtain

$$M(x_n,x_{n+1},t) \geq M\left(x_{n-1},x_n,\frac{t}{k}\right) \geq M\left(x_{n-2},x_{n-1},\frac{t}{k^2}\right) \geq \ldots$$

$$\geq M\left(x_n,x_{n_1+1},\frac{t}{k^{n-n_1}}\right).$$

$$N(x_n,x_{n+1},t) \leq N\left(x_{n-1},x_n,\frac{t}{k}\right) \leq N\left(x_{n-2},x_{n-1},\frac{t}{k^2}\right) \leq \ldots \leq N\left(x_n,x_{n_1+1},\frac{t}{k^{n-n_1}}\right).$$

Now, we note that the sequence $t_n = \frac{t}{n(n+1)k^{n-n_1}}$ is an s-increasing sequence,

Since $\lim_{n\to+\infty}(t_{n+1}-t_n) = +\infty$, for all $t > 0$.

Next fix $\epsilon > 0$, then there exists $n_0 \in \mathbb{N}$ with $n_0 > n_1$ such that

$$\prod_{j\geq n}^{+\infty} M(x_{n_1},x_{n_1+1},t_j) \geq 1-\varepsilon,$$

$\prod_{j\geq n}^{+\infty} N(x_{n_1},x_{n_1+1},t_j) \geq 1-\varepsilon$ for all $n \geq n_0$.

Then, for all $m > n \geq n_0$ and $h_j = \frac{1}{j(j+1)}$ for $j \in \{n,n+1,\ldots,m-1\}$, Since $\sum_{j=n}^{m-1} h_j < 1$, We write

$$M(x_n, x_m, t) \geq M\left(x_n, x_m, \sum_{j=n}^{m-1} h_j t\right)$$

$$\geq M(x_n, x_{n+1}, h_n t) * M(x_{n+1}, x_{n+2}, h_{n+1} t) * \dots * M(x_{m-1}, x_m, h_{m-1} t)$$

$$\geq M\left(x_n, x_{n_1}, \frac{h_n}{k^{n-n_1}} t\right) * M\left(x_{n_1}, x_{n_1+1}, \frac{h_{n+1}}{k^{n-n_{1+1}}} t\right) * \dots * M\left(x_{n_1}, x_{n_1+1}, \frac{h_{m-1}}{k^{m-n_1-1}} t\right)$$

$$\geq \prod_{j=n}^{+\infty} M\left(x_{n_1}, x_{n_1+1}, \frac{h_j}{k^{j-n_1}} t\right) \geq 1 - \varepsilon.$$

$$N(x_n, x_m, t) \leq N\left(x_n, x_m, \sum_{j=n}^{m-1} h_j t\right)$$

$$\leq N(x_n, x_{n+1}, h_n t) \diamond N(x_{n+1}, x_{n+2}, h_{n+1} t) \diamond \dots \diamond N(x_{m-1}, x_m, h_{m-1} t)$$

$$\leq N\left(x_n, x_{n_1}, \frac{h_n}{k^{n-n_1}} t\right) \diamond N\left(x_{n_1}, x_{n_1+1}, \frac{h_{n+1}}{k^{n-n_{1+1}}} t\right) \diamond \dots \diamond N\left(x_{n_1}, x_{n_1+1}, \frac{h_{m-1}}{k^{m-n_1-1}} t\right)$$

$$\leq \prod_{j=n}^{+\infty} N\left(x_{n_1}, x_{n_1+1}, \frac{h_j}{k^{j-n_1}} t\right) \leq 1 - \varepsilon.$$

Hence $M(x_n, x_m, t) \geq 1 - \varepsilon$ $N(x_n, x_m, t) \leq 1 - \varepsilon$ for all $m > n \geq n_0$ and the sequence $\{x_n\}$ is a Cauchy sequence.

Since $(X, M, N, *, \diamond)$ is a complete intuitionistic L-fuzzy metric space, we have

$$\lim_{n\to+\infty} M(x_n, \bar{x}, t) = 1 \text{ for each } t > 0, \quad \dots\dots\dots\dots\dots (15)$$

$$\lim_{n\to+\infty} N(x_n, \bar{x}, t) = 0 \text{ for each } t > 0, \quad \dots\dots\dots\dots\dots (16)$$

For some $\bar{x}$ in x.

By given assumption, We have

$$\lim_{n\to+\infty} \sup \psi(\mathrm{M}(\mathrm{x_n}, \bar{\mathrm{x}}, \mathrm{t})) < 1\,.$$

$$\lim_{n\to+\infty} \sup \psi(\mathrm{N}(\mathrm{x_n}, \bar{\mathrm{x}}, \mathrm{t})) > 0\,.$$

Thus there exists λ with $k < \lambda < 1$ such that

$$\lim_{n\to+\infty} \sup \psi(M(x_n, \bar{x}, t)) < \lambda. \quad \dots\dots\dots\dots (17)$$

$$\lim_{n\to+\infty} \sup \psi(N(x_n, \bar{x}, t)) < \lambda. \quad \dots\dots\dots\dots\dots (18)$$

Now, by (1) and (4), We have

$$H_{M_\alpha}(Fx_n, F\bar{x}, \lambda t) \geq H_{M_\alpha}(Fx_n, F\bar{x}, \psi(M(x_n, \bar{x}, t))t)$$

$$H_{M_\alpha}(Fx_n, F\bar{x}, \lambda t) \geq \mathrm{M}(\mathrm{x_n}, \bar{\mathrm{x}}, \mathrm{t}).$$

$$H_{N_\alpha}(Fx_n, F\bar{x}, \lambda t) \leq H_{N_\alpha}(Fx_n, F\bar{x}, \psi(N(x_n, \bar{x}, t))t)$$

$$H_{N_\alpha}(Fx_n, F\bar{x}, \lambda t) \leq \mathrm{N}(\mathrm{x_n}, \bar{\mathrm{x}}, \mathrm{t}).$$

On taking limit as $n \to +\infty$, We obtain

$$\lim_{n\to+\infty} H_{M_\alpha}(Fx_n, F\bar{x}, t) = 1.$$

$$\lim_{n\to+\infty} H_{N_\alpha}(Fx_n, F\bar{x}, t) = 0$$

Since $x_{n+1} \in (Fx_n)_\alpha$, it follows that

$$\lim_{n\to+\infty} \sup_{v \in (Fx)_\alpha} M(x_{n+1}, v, t) = 1,$$

$$\lim_{n\to+\infty} \sup_{v \in (Fx)_\alpha} N(x_{n+1}, v, t) = 0.$$

And hence there exists a sequence $\{y_n\}$ with $y_n \in (F\bar{x})_\alpha$ such that

$$\lim_{n\to+\infty} M(x_n, y_n, t) = 1 \quad \dots\dots (19)$$

$$\lim_{n\to+\infty} N(x_n, y_n, t) = 0 \quad \dots\dots (20)$$

For each $t > 0$. Now, for each $n \in \mathbb{N}$,

$$M(y_n, \bar{x}, t) \geq M\left(y_n, x_n, \frac{t}{2}\right) * M\left(x_n, \bar{x}, \frac{t}{2}\right) \quad \dots\dots (21)$$

$$N(y_n, \bar{x}, t) \leq N\left(y_n, x_n, \frac{t}{2}\right) \diamond N\left(x_n, \bar{x}, \frac{t}{2}\right) \quad \dots\dots\dots (22)$$

On taking limit as $h \to +\infty$ in equation (10) and by equation (7) and (9),We have

$$\lim_{n\to+\infty} M(y_n, \bar{x}, t) = 1, \ \lim_{n\to+\infty} N(y_n, \bar{x}, t) = 0$$

That is, $\lim_{n\to+\infty} y_n = \bar{x}$.

Since $(F\bar{x})_\alpha$ is compact, $\bar{x} \in (F\bar{x})_\alpha$, that is, $(\bar{x})_\alpha \subset F\bar{x}$.

Hence $\bar{x}$ is a fixed intuitionistic fuzzy point of F.

4. CONCLUSION

Last three decades were very productive for fuzzy mathematics and the recent literature has observed the fuzzy application in almost every direction of mathematics. In this paper a general analysis has been done to reveal the link between the fixed point properties and the Hausdorff Intuitionistic L- fuzzy metric spaces. The analysis can be used to form some new fuzzy fixed point theorems by using different types. In future from this concept can able to extend fuzzy concept for find out various solution in various spaces.

REFERENCES

[1] Atanassov K.T, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1) (1986), 87-96.

[2] Ali A.M and I. Athal, Extended intuitionistic fuzzy metric and fixed point theorems, International J. of Pure and Engg. Mathematics, Vol. 3 No.2, (2015), 111-123.

[3] Ali A.M, N -Intuitionistic fuzzy metric spaces and various types of mappings, Annals of Fuzzy Mathematics and Informatics, Vol.13, No.2 (2017), 377-287.

[4] Ali A.M, some fixed point theorems in intuitionistic fuzzy 2-metric space using implicit relation, The Journal of Fuzzy Mathematics vol. 25, no. 4, 2017, 853-863.

[5] Bakry S. M, Common Fixed Theorem on Intuitionistic Fuzzy 2-Metric Spaces, Gen. Math. Notes, Vol. 27, No.2, April (2015), pp.69-84

[6] Erceg .M.A, "Metric spaces in fuzzy set theory," Journal of Mathematical Analysis and Applications,vol. 69, no. 1, pp. 205–230, 1979.

[7] Ferhan Sola Erduran and Cemil Yildiz, "Some Fixed Fuzzy Point Results Using Hausdroff

Metric in Fuzzy Metric Spaces", Annals of Fuzzy Mathematics and Informatics(2016).
[8] Grabeic.M, " Fixed points in Fuzzy Metric Space", Fuzzy Sets and Systems,27, 385-389 (1983).
[9] Gregori.V, and A. Sapena, "On fixed-point theorems in fuzzy metric spaces," Fuzzy Sets and Systems, vol. 125, no. 2, pp. 245–252, 2002.
[10] Gahler, 2-Metrische Raume and ihre topologische structure, Math.Nachr., 26 (1983), 115-148.
[11] George A and Veeramani P, On some results in fuzzy metric spaces, Fuzzy Sets Syst., 64 (1994), 395-399.
[12] Gregori V, Romaguera S and P. Veeramani, A note on intuitionistic fuzzy metric spaces, Chaos, Solitions and Fractals, 28 (2006), 902-905.
[13] Grabiec M, Fixed Points in Fuzzy Metric Spaces, Fuzzy Sets and Systems, vol. 27 (1988), 385-389.
[14] Kramosil and Michalek, Fuzzy metrics and statistical metric spaces, Kybernetika, 11 (1975), 326-334.
[15] Kaleva .O, and S. Seikkala, "On fuzzy metric spaces," Fuzzy Sets and Systems, vol. 12, no. 3, pp.215–229, 1984.
[16] Lopez.J.R. and S.Romaguera, "The Hausdroff Fuzzy Metric on Compact Sets", Fuzzy Sets and Systems 147 (2004) 273-283.
[17] Mohamed S.Y, Ali A.M., Intuitionistic fuzzy graph metric spaces, International Journal of Pure and Applied Mathematics, International Journal of Pure and Applied Mathematics, 118(6) (2018) 67-74.
[18] Mohamed S.Y, Ali A.M., Fixed point theorems in intuitionistic fuzzy graph metric space, Annals of Pure and Applied Mathematics, Vol. 17, No. 1, 2018, 57-65
[19] Mohamed S.Y, Begam E.N, "A study on Intuitionistic L-fuzzy metric Spaces" Annals of Pure and Applied Mathematics, Vol 15, No.1, 2017,67-75.
[20] Mohamed S.Y, Begam E.N, "A study on properties of connectedness in intuitionistic L-fuzzy special topological spaces" International Journal of Emerging Technologies and Innovative Research, Vol.5, Issue 7, 533-538, 2018.
[21] Mohamed S.Y, Begam E.N, "A study on fuzzy fixed points and coupled fuzzy fixed points in Hausdorff L- fuzzy metric spaces " International Journal of computer and Mathematical Sciences, Vol.9,1187-1200, 2018.
[22] Mishra S.N, Sharma N and Singh S.L, Common fixed points of maps on fuzzy metric spaces, Int.J.Math. Math. Sci., 17 (1994), 253-258.
[23] Park J.H., Intuitionistic fuzzy metric spaces, Chaos Solitons Fractals, 22 (5) (2004), 1039-1046.
[24] Saadati R, Sedghi S and Shobe N, Modified intuitionistic fuzzy metric spaces and some fixed point theorems, Chaos Solitons Fractals, 38 (2008), 36-47.
[25] Supak Phiangsungnoen, Wutiphol Sintunavarat and Poom Kumam, " Fuzzy Fixed Point Theorems in Hausdroff Fuzzy Metric Spaces", Journal of Inequalities and Applications (2014).
[26] Singh, B. and Chouhan, M.S., Common fixed points of compatible maps in Fuzzy metric spaces, Fuzzy sets and systems, 115 (2000), 471-475.
[27] Zadeh L A, "Fuzzy sets", Information and Control, vol. 8, no. 3, pp. 338-356, 1965.