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New Preservation Theorems on Nano Minimal Structure Spaces

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Abstract- In this paper, we introduce new concepts of NMg^*-continuous map, NMg^*-closed map and new preservation theorems on nano minimal structure spaces.

Keyword- NMg^*-continuous map, NMg^*-closed map, NMg^*-closed sets

1. INTRODUCTION

In 1970, Levine [6] introduced the notion of generalized closed (briefly g-closed) sets in topological spaces. M. K. R. S. Veera kumar [12] introduced a new class of sets, namely $g^\#$-closed sets in topological spaces. Veerakumar [11] introduced the notion of g^*-closed sets in topological spaces.

Bhuvaneswari et.al [3] introduced and investigated nano g-closed sets in nano topological spaces. Rajendran et.al [10] introduced the notion of nano g^*-closed sets and further properties of nano g^*-closed sets are investigated.

Recently, A. Pandi et.al [7] introduced the notion of Nmg^*-closed sets and properties of Nmg^*-closed sets were investigated.

V. BA. Vijeyrani and A.Pandi [13] introduced the notion of $Nmg^\#$-closed sets and properties of $Nmg^\#$-closed sets were investigated. In this paper, we introduce new concepts of NMg^*-continuous map, NMg^*-closed map and new preservation theorems on nano minimal structure spaces.

2. PRELIMINARIES

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space $(U, \tau_R(X))$, $Ncl(A)$ and $Nint(A)$ denote the nano closure of A and the nano interior of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1 *[8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U} \{R(x): R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U} \{R(x): R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2 *[4] If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)$;
2. $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;
9. $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
10. $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3 *[4] Let U be the universe, R be an equivalence relation on U and*

$\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$,

2. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

3. The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 *[4] If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5 *[4] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $Nint(A)$.*
That is, $Nint(A)$ is the largest nano open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $Ncl(A)$.

That is, $Ncl(A)$ is the smallest nano closed set containing A.

Definition 2.6 *[9] A subfamily m_X of the power set ℘(X) of a nonempty set X is called a minimal structure (briefly m-structure) on X if $\phi \in m_X$ and U $\in m_X$.*

By (X, m_X), we denote a nonempty set X with a minimal structure m_X on X and call it a m-space. Each member of m_X is said to be m_X-open (briefly m-open) and the complement of an m_X-open set is said to be m_X-closed (briefly m-closed).

Definition 2.7 *A subset A of a nano topological space $(U, \tau_R(X))$ is called*

1. nano α-open [4] if $A \subseteq Nint(Ncl(Nint(A)))$.

2. nano regular-open [4] if $A = Nint(Ncl(A))$.

3. nano π-open [1] if the finite union of nano regular-open sets.

The family of nano α-open (resp. nano regular-open, nano π-open, nano open) sets is denoted by $N\alpha O(U, \tau_R(X))$ (resp. $NRO(U, \tau_R(X))$, $N\pi O(U, \tau_R(X))$, $NO(U, \tau_R(X))$).

The complements of the above mentioned sets is called their respective closed sets.

The nano α-closure of a subset A of U is, denoted by $\alpha cl(A)$, defined to be the intersection of all nano α-closed sets containing A.

Definition 2.8 *A subset A of a nano topological space $(U, \tau_R(X))$ is called nano g-closed [2] if $Ncl(A) \subseteq G$, whenever $A \subseteq G$ and G is nano open.*

The family of all nano g-open sets of U is denoted by $NgO(U, \tau_R(X))$.

The complements of the nano g-closed sets is called nano g-open sets.

The nano g-closure of a subset A of U is, denoted by $Ngcl(A)$, defined to be the intersection of all nano g-closed sets containing A.

Definition 2.9 *[7] A nano subfamily Nm_U of the power set ℘(U) of a nonempty set U is called a nano minimal structure (briefly Nm-structure) on U if $\phi \in Nm_U$ and $U \in Nm_U$.*

By (U, Nm_U), we denote a nonempty set U with a nano minimal structure Nm_U on U and call it a nano m-space(briefly Nm-space). Each member of Nm_U is said to be nano m_U-open (briefly Nm-open) and the complement of an nano m_U-open set is said to be nano m_U-closed (briefly Nm-closed).

Definition 2.10 *[7] A nano topological space $(U, \tau_R(X))$ with a nano minimal structure Nm_U on U is called a nano minimal structure space $(U, \tau_R(X), Nm_U)$.*

Remark 2.11 *[7] Let $(U, \tau_R(X))$ be a nano topological space. Then the families $\tau_R(X)$, $N\alpha O(U, \tau_R(X))$, $NRO(U, \tau_R(X))$, $N\pi O(U, \tau_R(X))$ and $NgO(U, \tau_R(X))$ are all nano minimal structure space $(U, \tau_R(X), Nm_U)$.*

Definition 2.12 *[7] Let (U, m_U) be a Nm-space. For a subset A of U, the Nm_U-closure of A and the Nm_U-interior of A are defined in as follows:*

1. $Nm_U\text{-}cl(A) = \cap\{F : A \subseteq F, F^c \in Nm_U\}$.

2. $Nm_U\text{-}int(A) = \cup\{V : V \subseteq A, V \in Nm_U\}$.

Remark 2.13 *[7] Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space and A be a subset of U. If $Nm_U = \tau_R(X)$ (resp. $N\alpha O(U, \tau_R(X))$, $NgO(U, \tau_R(X))$), then we have Nm_U-$cl(A) = Ncl(A)$ (resp. $N\alpha cl(A)$, $Ngcl(A)$).*

Definition 2.14 *[7] Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be*

1. nano minimal generalized closed(briefly Nmg-closed) if $Ncl(A) \subseteq$ V whenever A ⊆ V and V is Nm_U-open.

2. nano minimal generalized open(briefly Nmg-open) if its complement is called Nmg-closed.

The family of all Nmg-open sets in U is an Nm-structure on U and denoted by $NmgO(U, \tau_R(X), Nm_U)$.

Definition 2.15 *[7] Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be*

1. nano minimal g^*-closed(briefly Nmg^*-closed) if $Ncl(A) \subseteq$ V whenever A ⊆ V and V is Nmg-open.

2. nano minimal g^*-open(briefly Nmg^*-open) if its complement is Nmg^*-closed.

The family of all Nmg^*open sets in U is an Nm-structure on U and denoted by $Nmg^*O(U, \tau_R(X), Nm_U)$.

Definition 2.16 *[7] Let $(U, NmgO(U, \tau_R(X), Nm_U))$ be a nano minimal structure space. For a subset A of U, the Nmg-closure of A and the Nmg-interior of A are defined as follows:*

1. Nmg-$cl(A) = \cap\{F : A \subseteq F, U - F \in NmgO(U, \tau_R(X), Nm_U)\}$.

2. Nmg-$int(A) = \cup\{V : V \subseteq A, V \in NmgO(U, \tau_R(X), Nm_U)\}$.

Definition 2.17 *[13] Let $(U, \tau_R(X), Nm_U)$ be a nano minimal structure space. A subset A of U is said to be*

1. nano minimal $g^\#$-closed(briefly $Nmg^\#$-closed) if $Ncl(A) \subseteq$ V whenever A ⊆ V and V is Nmg^*-open.

2. nano minimal $g^\#$-open(briefly $Nmg^\#$-open) if its complement is $Nmg^\#$-closed.

Definition 2.18 *[5] A function f : $(U, \tau_R(X)) \to (V, \tau_R(Y))$ is said to be nano continuous map if the inverse image of every nano open set in $(V, \tau_R(Y))$ is nano open in $(U, \tau_R(X))$.*

3. NEW PRESERVATION THEOREMS ON NANO MINIMAL STRUCTURE SPACES

Definition 3.1 *A function f : $(U, \tau_R(X), Nm_U) \to (V, \tau_R(Y), Nm_V)$ is said to be*

1. NMg^*-continuous map if $f^{-1}(Z)$ is Nmg^*-closed in $(U, \tau_R(X), Nm_U)$ for every Nmg^*-closed set Z in $(V, \tau_R(Y), Nm_V)$,

2. NMg^*-closed map if for each Nmg^*-closed set F of $(U, \tau_R(X), Nm_U)$, f(F) is Nmg^*-closed in $(V, \tau_R(Y), Nm_V)$.

Example 3.2 *Let U = {a, b, c} with $U/R = \{\{b\}, \{a, c\}\}$ and $X = \{a, b\}$. Then $\tau_R(X)$ = {∅, U, {b}, {a, c}} and Nm_U = {∅, U, {c}}. Then Nmg^*-closed sets are ∅, U, {b}, {a, b}, {a, c}. Let V = {a, b, c} with $V/R = \{\{b\}, \{a, c\}\}$ and $Y = \{c\}$. Then $\tau_R(Y)$ = {∅, V, {a, c}} and Nm_V = {∅, V}. Then Nmg^*-closed sets are ∅, V, {b}. Define f : $(U, \tau_R(X), Nm_U) \to (V, \tau_R(Y), Nm_V)$ is defined by f(a) = a, f(b) = b, f(c) = c. Then f^{-1}({b}) = {b} is Nmg^*-closed in $(U, \tau_R(X), Nm_U)$. That is, the inverse image of every Nmg^*-closed set in $(V, \tau_R(Y), Nm_V)$ is Nmg^*-closed set in $(U, \tau_R(X), Nm_U)$. Therefore, f is NMg^*-continuous map.*

Example 3.3 *Let U = {a, b, c} with $U/R = \{\{b\}, \{a, c\}\}$ and $X = \{c\}$. Then $\tau_R(X)$ = {∅, U, {a, c}} and Nm_U = {∅, U}. Then Nmg^*-closed sets are ∅, U, {b}. Let V = {a, b, c} with $V/R = \{\{b\}, \{a, c\}\}$ and $Y = \{a, b\}$. Then $\tau_R(Y)$ = {∅, V, {b}, {a, c}} and Nm_V = {∅, V, {c}}. Then Nmg^*-closed sets are ∅, V, {b}, {a, b}, {a, c}. Define f : $(U, \tau_R(X), Nm_U) \to (V, \tau_R(Y), Nm_V)$ is defined by f(a) = a, f(b) = b, f(c) = c. Then f({b}) = {b} is Nmg^*-closed in $(V, \tau_R(Y), Nm_V)$. That is, the image of every Nmg^*-closed set in $(U, \tau_R(X), Nm_U)$ is Nmg^*-closed set in $(V, \tau_R(Y), Nm_V)$. Therefore, f is NMg^*-closed map.*

Theorem 3.4 *Let $Nmg^*O(X)(U, \tau_R(X), Nm_U)$ be an Nm-structure with property $\mathcal{J}_\mathcal{N}$. Let f : $(U, \tau_R(X), Nm_U) \to (V, \tau_R(Y), Nm_V)$ be a function from a nano minimal space $(U, \tau_R(X), Nm_U)$ into a nano minimal space $(V, \tau_R(Y), Nm_V)$. Then the following are equivalent:*
1. f is NMg^*-continuous map,
2. $f^{-1}(Z) \in Nmg^*O(U, \tau_R(X), Nm_U)$ for every $Z \in Nmg^*O(V, \tau_R(Y), Nm_V)$.

Proof. Assume that f : $(U, \tau_R(X), Nm_U) \to (V, \tau_R(Y), Nm_V)$ is NMg^*-continuous map. Let $Z \in Nmg^*O(V, \tau_R(Y), Nm_V)$. Then Z^c is Nmg^*-closed in $(V, \tau_R(Y), Nm_V)$. Since f is NMg^*-continuous map, $f^{-1}(Z^c)$ is Nmg^*-closed in $(U, \tau_R(X), Nm_U)$. But

$f^{-1}(Z^c)$ = U−f^{-1}(Z). Thus U−f^{-1}(Z) is Nmg^*-closed in $(U,\tau_R(X),Nm_U)$ and so f^{-1}(Z) is Nmg^*-open in $(U,\tau_R(X),Nm_U)$.

Conversely, let for each Z ∈ Nmg^*O$(V,\tau_R(Y),Nm_V)$, f^{-1}(Z) ∈ Nmg^*O$(U,\tau_R(X),Nm_U)$. Let F be any Nmg^*-closed in $(V,\tau_R(Y),Nm_V)$. By assumption, $f^{-1}(F^c)$ is Nmg^*-open in $(U,\tau_R(X),Nm_U)$. But $f^{-1}(F^c)$= U−f^{-1}(F). Thus U−f^{-1}(F) is Nmg^*-open in $(U,\tau_R(X),Nm_U)$ and so f^{-1}(F) is Nmg^*-closed in $(U,\tau_R(X),Nm_U)$. Hence f is NMg^*-continuous map.

Lemma 3.5 *A function f : $(U,\tau_R(X),Nm_U) \to (V,\tau_R(Y),Nm_V)$ is NMg^*-closed if and only if for each subset B of Y and each U ∈ $Nmg^*O(U,\tau_R(X),Nm_U)$ containing f^{-1}(B), there exists Z ∈ $Nmg^*O(V,\tau_R(Y),Nm_V)$ such that B ⊆ Z and f^{-1}(Z) ⊆ H.*

Proof. Suppose that f is NMg^*-closed. Let B ⊆ V and H ∈ Nmg^*O$(U,\tau_R(X),Nm_U)$ containing f^{-1}(B). Put Z = V−f(U−H). Then Z is Nmg^*-open in $(V,\tau_R(Y),Nm_V)$ and f^{-1}(Z) ⊆ f^{-1}(V)−(U−H) = U−(U−H) = H. Also, since f^{-1}(B) ⊆ H, then U−H ⊆ f^{-1}(V−B) which implies f(U−H) ⊆ V−B and hence B ⊆ Z. Hence we obtain Z ∈ Nmg^*O$(V,\tau_R(Y),Nm_V)$ such that B ⊆ Z and f^{-1}(Z) ⊆ H.

Conversely, let F be any Nmg^*-closed set of $(U,\tau_R(X),Nm_U)$. Set f(F) = B, then F ⊆ f^{-1}(B) and f^{-1}(V−B) ⊆ U−F ∈ Nmg^*O$(U,\tau_R(X),Nm_U)$. By the hypothesis, there exists Z ∈ Nmg^*O$(V,\tau_R(Y),Nm_V)$ such that V−B ⊆ Z and f^{-1}(Z) ⊆ U−F and so F ⊆ f^{-1}(V−Z). Therefore f(F) ⊆ V−Z. Hence, we obtain V−Z ⊆ B = f(F) ⊆ V−Z. Therefore f(F) = V−Z is Nmg^*-closed in $(V,\tau_R(Y),Nm_V)$. Hence f is NMg^*-closed.

Theorem 3.6 *If f : (U, $\tau_R(X)$) → (V, $\tau_R(V)$) is nano closed and f : $(U,\tau_R(X),Nm_U) \to (V,\tau_R(Y),Nm_V)$ is NMg^*-continuous map, where $Nmg^*O(U,\tau_R(X),Nm_U)$ has property $\mathcal{J}_\mathcal{N}$, then f(A) is $Nmg^\#$-closed in $(V,\tau_R(Y),Nm_V)$ for each $Nmg^\#$-closed set A of $(U,\tau_R(X),Nm_U)$.*

Proof. Let A be any $Nmg^\#$-closed set of $(U,\tau_R(X),Nm_U)$ and f(A) ⊆ Z ∈ Nmg^*O$(V,\tau_R(Y),Nm_V)$. Then, by Theorem 3.4, A ⊆ f^{-1} (Z) ∈ Nmg^*O$(U,\tau_R(X),Nm_U)$. Since A is $Nmg^\#$-closed, Ncl(A) ⊆ f^{-1}(Z) and f(Ncl(A)) ⊆ Z. Since f is nano closed, Ncl(f(A)) ⊆ f(Ncl(A)) ⊆ Z. Hence f(A) is $Nmg^\#$-closed in $(V,\tau_R(Y),Nm_V)$.

Theorem 3.7 *If f : $(U,\tau_R(X),Nm_U) \to (V,\tau_R(Y),Nm_V)$ is nano continuous and f : $(U,\tau_R(X),Nm_U) \to (V,\tau_R(Y),Nm_V)$ is NMg^*-closed, then f^{-1}(B) is $Nmg^\#$-closed in $(U,\tau_R(X),Nm_U)$ for each $Nmg^\#$-closed set B of $(V,\tau_R(Y),Nm_V)$.*

Proof. Let B be any $Nmg^\#$-closed set of $(V,\tau_R(Y),Nm_V)$ and f^{-1}(B) ⊆ H ∈ Nmg^*O$(U,\tau_R(X),Nm_U)$. Since f is NMg^*-closed, by Lemma 3.5, there exists Z ∈ Nmg^*O$(V,\tau_R(Y),Nm_V)$ such that B ⊆ Z and f^{-1}(Z) ⊆ H. Since B is $Nmg^\#$-closed, Ncl(B) ⊆ Z and since f is nano continuous, Ncl(f^{-1}(B)) ⊆ f^{-1}(Ncl(B)) ⊆ f^{-1}(Z) ⊆ H. Hence f^{-1}(B) is $Nmg^\#$-closed in $(U,\tau_R(X),Nm_U)$.

4. CONCLUSION

In this paper, we introduced the notion of NMg^*-continuous map, NMg^*-closed map and new preservation theorems on nano minimal structure spaces. In future, it motivates to apply this concepts in nano minimal structure spaces.

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