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Multiple Attribute Decision Making Method using Singular Perturbation Problem under Interval Valued Intuitionistic Fuzzy Sets

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Abstract-In this paper, we investigate a method of finding the unknown weights of the decision maker for the decision making process. To solve Multiple Attribute Decision Making (MADM) problem, the Interval Valued Intuitionistic Fuzzy Sets (IVIFSs) are considered. The unknown weights can be determined using the exact and numerical solution of Singularly Perturbed Convection-Diffusion problem. For decision making process, first we use the Interval Valued Intuitionistic Fuzzy Ordered Weighted Averaging (IVIFOWA) operator to aggregate all the IVIFS matrices into a single IVIFS matrix, and then we utilize the Interval Valued Intuitionistic Fuzzy Hybrid Averaging (IVIFHA) operator to aggregate the single matrix into a column matrix. Finally, we use the newly proposed correlation coefficient method and accuracy function for ranking the best alternative from the available alternatives. Numerical illustration is given to show the effectiveness of the proposed approach.

Keywords: MADM, Intuitionistic Fuzzy Sets, interval valued intuitionistic fuzzy sets, Singular perturbation problem, Interval Valued Intuitionistic Fuzzy Ordered Weighted Averaging operator, Interval Valued Intuitionistic Fuzzy Hybrid Averaging operator.

1. INTRODUCTION

Decision Making is used to choose the best alternative among a set of feasible alternatives. In real life decision situation Multiple Attribute Decision Making (MADM) problems are widely spread. In 1986, **Atanssov [1,2,4]** introduced the concept of intuitionistic fuzzy sets which is a generalization of fuzzy sets developed by **zadeh [25]**. Fuzzy sets contain only a membership function but Intuitionistic Fuzzy set (IFS) contains both membership and non-membership functions. Later **Atanssov & Gargov [3]** developed the new concept named as Interval Valued Intuitionistic Fuzzy Sets (IVIFSs) which can be represented by intervals rather than exact values. **Joshi & kumar [5]** proposed a new improved accuracy function for IVIFS and its applications to MAGDM problem. **Nayagam et al. [8]** and **Xu & Da [19]** proposed the accuracy function for MAGDM problems under IVIFS. **Robinson & Amirtharaj [10-14]** and **Robinson & Jeeva [15]** discussed the various decision making operators and correlation coefficients for different higher order intuitionistic fuzzy sets and utilized them in ranking the alternatives in MAGDM problems. **Robinson & Indhumathi [16]** determined the unknown weights using Singularly Perturbed Delay Differential Equations and proposed a new correlation coefficient for IFSs and utilized them in ranking the alternatives in MAGDM problems. To solve MAGDM problems **Xu [18] & Yager[21]** introduced several aggregation operators for intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set. **Xu & Chen [17**] **& Xu et al. [20]** Proposed some Score and accuracy functions to rank Interval Valued Intuitionistic Fuzzy Numbers (IVIFNs). Later, **Ye [22] and Wu & Chiclana [24]** proposed a different accuracy function that is used to solve some drawbacks associated to the accuracy function developed by Xu & Chen [17]. **Wei [23]** utilized Interval-Valued Intuitionistic Fuzzy Weighted Averaging (IVIFWA) operator to aggregate the interval-valued intuitionistic fuzzy information for interval-valued intuitionistic fuzzy multi-criteria decision-making with preference information on alternatives, and provided a method to rank the alternatives based on the score function and accuracy function of the interval-valued intuitionistic fuzzy number. **Malley & Nayfeh [6, 7]** gave an introduction to singular perturbation problems. **Miller et al. [9]** have devoted the last five chapters of their work for SPPs in two dimension. They have suggested both a finite difference operator and a piecewise uniform fitted mesh to achieve the parameter-uniform numerical method.

In this paper, the solution of singular perturbation problem are utilized to derive the decision maker weights in MAGDM problems under interval valued intuitionistic fuzzy sets. The feasibility and effectiveness of the proposed method are illustrated using numerical examples.

2. PRELIMINARIES

In this Section, some basic concepts about the IVIFSs and different classes of aggregation operators are presented.

DEFINITION: Intuitionistic Fuzzy Set, IFS [1]

An IFS A in X is given by $A=\{\langle x,\ \mu_A(x),\ \gamma_A(x)\rangle / x\in X\}$, where $\mu_A: X\to[0,1], \gamma_A: X\to[0,1]$ with the condition $0\le\mu_A(x)+\gamma_A(x)\le 1,\ \forall x\in X$. The numbers $\mu_A(x)$ and $\gamma_A(x)$ represent, the membership degree and non-membership degree of the element x to the set A, respectively.

DEFINITION: Interval Valued Intuitionistic Fuzzy Set (IVIFS) [3]

Let a set X be fixed. An IFS Ã in X is an object having the form $\tilde{A} = \{<x,\ [\mu_{\tilde{A}L}(x),\mu_{\tilde{A}U}(x)],[\gamma_{\tilde{A}L}(x),\gamma_{\tilde{A}U}(x)]>,\ x\in X\}$, where the $[\mu_{\tilde{A}L}(x),\mu_{\tilde{A}L}(x)]: X\to[0,\ 1]$ and $[\gamma_{\tilde{A}L}(x),\gamma_{\tilde{A}U}(x)]: E\to[0,\ 1]$ define the degree of membership and degree of non-membership respectively, of the element x∈X to the set Ã, which is a subset of X, for every element x∈ X, $0\ \le[\mu_{\tilde{A}L}(x),\mu_{\tilde{A}U}(x)]\ +[\gamma_{\tilde{A}L}(x),\gamma_{\tilde{A}U}(x)]\le\ 1.$

DIFFERENT TYPES OF AGGREGATION OPERATORS:

DEFINITION: Interval-Valued Intuitionistic Fuzzy Weighted Averaging (*IVIFWA*) operator

Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j])$ for all j = 1, 2… n be a collection of Interval-Valued intuitionistic fuzzy values. The Interval-Valued Intuitionistic Fuzzy Weighted Averaging (*IVIFWA*) operator, $IVIFWA:\ Q^n\to\ Q$ is defined as:

$$IVIFWA_\omega(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n)=\sum_{j=1}^{n}\omega_j\tilde{a}_j$$

$$=\begin{pmatrix}\left[1-\prod_{j=1}^{n}(1-a_j{}^{w_j}),1-\prod_{j=1}^{n}(1-b_j{}^{w_j})\right],\\ \left[\prod_{j=1}^{n}(c_j{}^{w_j}),\prod_{j=1}^{n}(d_j{}^{w_j})\right]\end{pmatrix}$$

Where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ be the weight vector of $\tilde{a}_j$, for all j = 1, 2… n such that $\omega_j>0$ and $\sum_{j=1}^{n}\omega_j=1.$

DEFINITION: Interval-Valued Intuitionistic Fuzzy Ordered Weighted Averaging Operator(*IVIFOWA*)

Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j])$, for all j = 1, 2… n be a collection of interval-valued intuitionistic fuzzy values. The Interval-Valued Intuitionistic Fuzzy Ordered Weighted Averaging (IVIFOWA) operator, $IVIFOWA: Q^n\to Q$ is defined as:

$$IVIFOWA_w(\tilde{a}_1,\tilde{a}_2,.....,\tilde{a}_n)$$

$$=\sum_{j=1}^{n}\tilde{a}_{\sigma(j)}^{w_j}$$

$$=\begin{pmatrix}\left[1-\prod_{j=1}^{n}(1-a_{\sigma_{(j)}}{}^{w_j}),1-\prod_{j=1}^{n}(1-b_{\sigma_{(j)}}{}^{w_j})\right],\\ \left[\prod_{j=1}^{n}(c_{\sigma(j)}{}^{w_j}),\prod_{j=1}^{n}(d_{\sigma(j)}{}^{w_j})\right]\end{pmatrix}$$

Where $w=(w_1,w_2,.....,w_n)^T$ is the associated Furthermore, $(\sigma(1),\sigma(2),.....,\sigma(n))$ is a permutation 0f (1, 2… n), such that $\tilde{a}_{\sigma(j-1)}\ge\tilde{a}_{\sigma(j)}$ for all j = 2… n.

DEFINITION: Interval-Valued Intuitionistic Fuzzy Hybrid Averaging Operator (*IVIFHA*)

Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j])$, for all j = 1, 2…n be a collection of interval-valued intuitionistic fuzzy values. The Interval-Valued Intuitionistic Fuzzy Hybrid Averaging (IVIFHA) operator, $IVIFHA: Q^n\to Q$ is defined as:

$$r_{ij}=IVIFHA_{\alpha,\lambda}(r_{ij}^{(1)},r_{ij}^{(2)},....,r_{ij}^{(1)})$$

$$=(\dot{r}_{ij}^{\sigma(1)})^{\alpha_1}\otimes(\dot{r}_{ij}^{\sigma(2)})^{\alpha_2}\otimes......\otimes(\dot{r}_{ij}^{\sigma(1)})^{\alpha_1}$$

$$=\left\langle\begin{matrix}\left[1-\prod_{k=1}^{n}(1-\dot{a}_{ij}^{\sigma(k)})^{\alpha_k},1-\prod_{k=1}^{n}(1-\dot{b}_{ij}^{\sigma(k)})^{\alpha_k}\right],\\ \left[\prod_{k=1}^{n}(\dot{c}_{ij}^{\sigma(k)})^{\alpha_k},\prod_{k=1}^{n}(\dot{d}_{ij}^{\sigma(k)})^{\alpha_k}\right]\end{matrix}\right\rangle$$

Where $\alpha=(\alpha_1,\alpha_2,....,\alpha_1)^T$ is a weight vector of *IVIFHA* operator with $\alpha_k>0$ (*k*= 1, 2,…, n) and $\sum_{k=1}^{n} w_k=1$ and $\dot{r}_{ij}=\left\langle \begin{matrix}[\dot{a}_{ij},\dot{b}_{ij}],\\ [\dot{c}_{ij},\dot{d}_{ij}]\end{matrix}\right\rangle$,

$\dot{r}_{ij}^{\sigma(k)}=\left\langle \begin{matrix}[\dot{a}_{ij}^{\sigma(k)},\dot{b}_{ij}^{\sigma(k)}],\\ [\dot{c}_{ij}^{\sigma(k)},\dot{d}_{ij}^{\sigma(k)}]\end{matrix}\right\rangle$ is the k[th] largest of the weighted IVIFHA $\dot{r}_{ij}^{(k)}=\left(r_{ij}^{(k)}\right)^{l\lambda_k}$, i=1,2,…,m, j=1,2,…,n.

3. CORRRELATION COEFFICIENT OF INTERVAL-VALUED INTUITIONISTIC FUZZY SETS (IVIFSs)

In this paper, we propose a new method for calculating correlation coefficient of IVIFSs based on the method proposed by Robinson & Amirtharaj [10] for calculating correlation coefficient of vague sets, taking not only the membership and non-membership grades into account but also the negation of non-membership degree and hesitancy degree also. Let $X=\{x_1,x_2,....,x_n\}$ be the finite universal set and let $A,B\in IVIFS(X)$, be given by

$$A=\left\{\left\langle \begin{matrix} x,(\mu_{AL}(x_i),\\ \mu_{AU}(x_i))((1-\gamma_{AL}(x_i)),\\ (1-\gamma_{AU}(x_i))),\\ (\pi_{AL}(x_i),\pi_{AU}(x_i))\end{matrix}\right\rangle / x\in X\right\},$$

$$B=\left\{\left\langle \begin{matrix} x,(\mu_{BL}(x_i),\mu_{BU}(x_i))((1-\gamma_{BL}(x_i)),\\ (1-\gamma_{BU}(x_i))),(\pi_{BL}(x_i),\pi_{BU}(x_i))\end{matrix}\right\rangle / x\in X\right\}$$

.

The correlation of $A,B\in IVIFS(X)$ is defined as follows:

$$E_{IVIFS}(A)=\frac{1}{2}\sum_{i=1}^{n}\left(\begin{matrix}\mu_{AL}^2(x)+\left(1-\gamma_{AL}^2(x)\right)+\pi_{AL}^2(x)+\\ \mu_{AU}^2(x)+\left(1-\gamma_{AU}^2(x)\right)+\pi_{AU}^2(x)\end{matrix}\right),$$

$$C_{IVIFS}(A,B)=\frac{1}{2}\sum_{i=1}^{n}\left[\begin{matrix}u_{AL}(x_i)u_{BL}(x_i)+u_{AU}(x_i)u_{BU}(x_i)+\\ \left(1-\gamma_{AL}(x_i)\right)\left(1-\gamma_{BL}(x_i)\right)+\left(1-\gamma_{AU}(x_i)\right)\\ \left(1-\gamma_{BU}(x_i)\right)+\pi_{AL}(x_i)\pi_{BL}(x_i)+\\ \pi_{AU}(x_i)\pi_{BU}(x_i)\end{matrix}\right]$$

And the correlation coefficient of $A,B\in IVIFS(X)$ is defined as follows:

$$K_{IVIFS}(A,B)=\frac{C_{IVIFS}(A,B)}{\sqrt{E_{IVIFS}(A)E_{IVIFS}(B)}}.$$

The following proposition and theorems are true for the above defined correlation coefficient.

Proposition:

For $A,B\in IVIFS(X)$, we have

i) $0\le C_{IVIFS}(A,B)\le 1$.

ii) $C_{IVIFS}(A,B)=C_{IVIFS}(B,A)$.

iii) $K_{IVIFS}(A,B)=K_{IVIFS}(B,A)$.

Theorem 3.1.

For $A,B\in IVIFS(X)$,then $0\le K_{IVIFS}(A,B)\le 1$.

Theorem 3.2.

$K_{IVIFS}(A,B)=1\Leftrightarrow A=B$.

Theorem 3.3.

$C_{IVIFS}(A,B)=0\Leftrightarrow$ A and B are non-fuzzy sets and satisfy the condition $\mu_A(x_i)+\mu_B(x_i)=1$ or $\gamma_A(x_i)+\gamma_B(x_i)=1$ or $\pi_A(x_i)+\pi_B(x_i)=1,\ \forall x_i\in X$.

Theorem 3.4.

$C_{IVIFS}(A,A)=1\Leftrightarrow$ A is a non-fuzzy set.

4. DIFFERENT TYPES OF ACCURACY FUNCTIONS

DEFINITION: (Ye, 2009) [22]

Let $A=([a,b],[c,d])$ be an interval valued intuitionistic fuzzy number, a novel accuracy function M of an interval valued intuitionistic fuzzy value based on the unknown degree is given by the formula:

$$M(A)=a+b-1+\frac{c+d}{2},\text{where } M(A)\in[-1,1].$$

DEFINITION: (Xu, 2007) [18]

Let $A=([a,b],[c,d])$ be an interval valued intuitionistic fuzzy number, an accuracy function H of an interval valued intuitionistic fuzzy value can be represented as follows

$$H(A)=\frac{a+b+c+d}{2},\text{ where}$$

$H(A)\in[0,1]$.

DEFINITION: (Nayagam et.al. 2011) [8]

Let $A = ([a,b],[c,d])$ be an interval valued intuitionistic fuzzy number, a novel accuracy function L of A, based on the unknown degree is given by the formula:

$$L(A) = \frac{a+b-d(1-b)-c(1-a)}{2},$$ where $L(A) \in [-1,1]$.

DEFINITION: (Deepa Joshi & Sanjay Kumar 2018) [5]

Let $A = ([a,b],[c,d])$ be an interval valued intuitionistic fuzzy number. The proposed accuracy function T of A, is defined as follows:

$$T(A) = \frac{a(1-c)+b(1-d)}{2},$$ where $T(A) \in [0,1]$.

5. AN APPROACH TO MAGDM PROBLEM WITH INTERVAL-VALUED INTUITIONISTIC FUZZY INFORMATION

Step: 1 Utilize the IVIFOWA operator to aggregate all individual interval-valued intuitionistic fuzzy decision matrices $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ $(k = 1,2,3,4)$ into a collective interval-valued intuitionistic fuzzy decision matrix $R=(r_{ij})_{mxn}$.

Step: 2 Utilize the IVIFHA operator ,

$$\tilde{r}_i = \left(\left[a_i, b_j\right],\left[c_i, d_j\right]\right) = IVIFHA_{\alpha,\gamma}$$
$$= \left(\tilde{r}_i^{(1)}, \tilde{r}_i^{(2)}, ..., \tilde{r}_i^{(t)}\right),\ i = 1,2,...m.$$

To derive the collective overall preference intuitionistic fuzzy values $\tilde{r}_i\left(i = 1,2,\ldots..m\right)$ of the alternative A_i where $v = \left(v_1, v_2 \ldots v_n\right)$ be the weighting vector of decision makers, with: $V_k \in [0,1]$, $\sum_{k=1}^{t} V_k = 1$; $w = \left(w_1, w_2 \ldots w_n\right)$ is the associated weighting vector of the IVIFHA Operator with $w_j \in [0,1]$, $\sum_{j=-1}^{n} w_j = 1$.

Step: 3 Calculate the correlation coefficient between the collective overall preference values r_i and the positive ideal value $\tilde{r}_i$, where $\tilde{r}_i = \left[(1,1),(0,0)\right]$.

Step: 4 The correlation coefficient of the IVIFSs, $A, B \in IVIFS(x)$ is given by the formula:

$$K_{IVIFS}(A,B) = \frac{C_{IVIFS}(A,B)}{\sqrt{E_{IVIFS}(A)E_{IVIFS}(B)}}.$$

Step: 5 Rank all the alternatives $A_i\left(i = 1,2,\ldots, m\right)$ and select one(s) in accordance with $K_{IVIFS}(A,B)$.

6. SINGULAR PERTURBATION PROBLEMS

In recent years Singular perturbation problems (SPPs) received much attention and arise in various fields such as fluid mechanics, aerodynamics, optimal control. Geophysics and so on. The word perturbation means a small disturbance in a physical system. In SPPs a small parameter ε is multiplying in the highest order derivative of the differential equations. We generally denote ε for the effect of small disturbance in physical system and ε is significantly less than unity. If $\varepsilon = 0$, then its order reduced by two is known as a convection-diffusion type problem.

In this paper, we consider the second order boundary value problem is of the form

$$-\varepsilon u_\varepsilon''(x) + a(x)u_\varepsilon'(x) = f(x),\ x \in [0,1]$$

with boundary condition $u_\varepsilon(0) = u_0, u_\varepsilon(1) = u_1$.

Where ε is a small parameter 0 <ε≤ 1; a(x) and f(x) are continuous on [0,1].

6.1. The Shishkin Mesh

In comparison with that of uniform mesh, in piecewise-uniform mesh we give more mesh points in the inner domains so that the layer information can be obtained. A piecewise-uniform Shishkin mesh with N mesh intervals is now constructed on $\bar{\Omega} = [0,1]$. The interval [0,1] is divided into two subintervals as follows:

$$[0,1-\tau] \cup [1-\tau,1].$$

The parameter τ, which determines the points separating the uniform meshes, is defined by

$$\tau = \min\left\{\frac{1}{2}, \frac{\varepsilon}{\alpha}\ln N\right\}.$$

Then, on the sub-interval $[0,\tau]$ a uniform mesh with N/2 mesh points is placed and on each of the subintervals $[1-\tau,1]$ a uniform mesh of N/2 mesh points is placed. Then we choose the boundary layer which occurs in the neighborhood of $x=(2-\tau)/2$. Hence we can determine the weights in the neighborhood of $x=(2-\tau)/2,$ in the sub-interval $[1-\tau,1]$.

6.2. Finite difference Methods

The classical finite difference operator with an appropriate piecewise uniform mesh is used to discrete the above boundary value problem is as follows:

$$L_\varepsilon^N = -\varepsilon\delta^2U(x_i)+a(x_i)D^+U(x_i)$$
$$= f(x_i), \forall x\in(0,1)$$

Where

$$\delta^2U(x_i)=\frac{(D^+-D^-)}{\bar{h}}U(x_i), \quad \bar{h}=\frac{h_i+h_{i+1}}{2},$$

$$D^+U(x_i)=\frac{U(x_{i+1})-U(x_i)}{h_{i+1}};$$

$$D^-U(x_i)=\frac{U(x_i)-U(x_{i-1})}{h_i}.$$

7. DETERMINING EXPERTS WEIGHTS FOR MAGDM PROBLEMS USING SINGULAR PERTURBATION PROBLEM

Problem proposed by Decision Maker.
The decision maker represents weighting vector in the form of the following second order singularly perturbed differential equation,

$-\varepsilon u''(x)+u'(x)=e^x$ with boundary conditions $u(0)=0,\ u(1)=1$.

Its exact solution is given by

$$u(x)=\frac{1}{1-\varepsilon}\left[e^x-\frac{1-e^{1-(1/\varepsilon)}+(e-1)e^{x-(1/\varepsilon)}}{1-e^{(-1/\varepsilon)}}\right].$$

The numerical solution can be calculated using the classical finite difference operators and is given as above:

Table 1. Exact Solution of $-\varepsilon u''(x)+u'(x)=e^x$

X	U(X)	NORMALIZATION
0.9953565	1.703983	0.199997136
0.9953610	1.703995	0.199998545
0.9953655	1.704007	0.199999953
0.9953700	1.704020	0.200001479
0.9953745	1.704032	0.200002887

Table 2. Numerical Solution of $-\varepsilon u''(x)+u'(x)=e^x$

X	U(X)	NORMALIZATION
0.9953835	1.690658	0.250014529
0.9953880	1.690593	0.250004917
0.9953926	1.690527	0.249995157
0.9953971	1.690461	0.249985397

Table 3. Exact Solution of $-\varepsilon u''(x)+u'(x)=e^x$

X	U(X)	NORMALIZATION
0.9953565	1.704056	0.249997286
0.9953610	1.704068	0.249999046
0.9953655	1.704081	0.250000954
0.9953700	1.704093	0.250002714

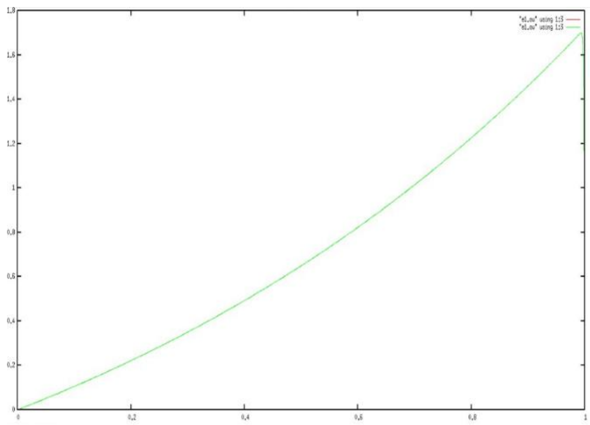
Hence, the weighting vectors given by the decision maker is calculated as

$\omega=(0.199997136, 0.199998545, 0.199999953, 0.200001479, 0.200002887)^T$,

$w=(0.250014529, 0., 0.250004917, 0.249995157, 0.249985397)^T$, and

$\gamma=(0.249997286, 0.249999046, 0.250000954, 0.250002714)^T$.

Figure 1.



8. NUMERICAL ILLUSTRATION

Due to the limited technology and capital, an enterprise itself may be unable to build the cloud platform and tries to seek a cloud service to realize its CRM. After the market research and preliminary screening, there are four potential cloud services for future evaluation including SAP sales on demand (A_1), sales force sales cloud (A_2), Microsoft dynamic CRM (A_3) and oracle cloud CRM (A_4). Five experts

(e_1, e_2, e_3, e_4, e_5) are invited to evaluate these cloud services on the indicators (attributes), including performance (u_1), payment (u_2), reputation (u_3), scalability (u_4), and security (u_5). In terms of each attribute, each expert has presented his (her) normalized information for four cloud services in Table 1-4.

$$R_1=\begin{pmatrix} \langle [0.55,0.65],[0.15,0.25] \rangle & \langle [0.35,0.55],[0.35,0.45] \rangle & \langle [0.65,0.75],[0.15,0.25] \rangle & \langle [0.55,0.75],[0.05,0.15] \rangle & \langle [0.10,0.40],[0.30,0.50] \rangle \\ \langle [0.35,0.45],[0.25,0.35] \rangle & \langle [0.15,0.35],[0.15,0.35] \rangle & \langle [0.35,0.45],[0.45,0.55] \rangle & \langle [0.25,0.45],[0.45,0.55] \rangle & \langle [0.70,0.80],[0.10,0.20] \rangle \\ \langle [0.55,0.65],[0.15,0.25] \rangle & \langle [0.75,0.85],[0.05,0.15] \rangle & \langle [0.55,0.85],[0.15,0.15] \rangle & \langle [0.45,0.65],[0.25,0.35] \rangle & \langle [0.50,0.60],[0.20,0.30] \rangle \\ \langle [0.35,0.55],[0.35,0.45] \rangle & \langle [0.15,0.25],[0.65,0.75] \rangle & \langle [0.15,0.25],[0.55,0.75] \rangle & \langle [0.35,0.45],[0.35,0.55] \rangle & \langle [0.20,0.30],[0.50,0.60] \rangle \\ \langle [0.62,0.32],[0.25,0.45] \rangle & \langle [0.35,0.45],[0.30,0.50] \rangle & \langle [0.85,0.05],[0.15,0.55] \rangle & \langle [0.75,0.15],[0.45,0.35] \rangle & \langle [0.10,0.65],[0.55,0.40] \rangle \end{pmatrix}$$

$$R_2=\begin{pmatrix} \langle [0.45,0.55],[0.25,0.45] \rangle & \langle [0.30,0.40],[0.40,0.60] \rangle & \langle [0.55,0.65],[0.10,0.15] \rangle & \langle [0.55,0.65],[0.05,0.25] \rangle & \langle [0.15,0.35],[0.25,0.45] \rangle \\ \langle [0.35,0.55],[0.30,0.50] \rangle & \langle [0.10,0.30],[0.30,0.70] \rangle & \langle [0.25,0.35],[0.35,0.45] \rangle & \langle [0.25,0.35],[0.55,0.65] \rangle & \langle [0.65,0.85],[0.05,0.15] \rangle \\ \langle [0.45,0.65],[0.25,0.35] \rangle & \langle [0.60,0.80],[0.10,0.20] \rangle & \langle [0.65,0.75],[0.05,0.15] \rangle & \langle [0.45,0.65],[0.25,0.35] \rangle & \langle [0.55,0.65],[0.15,0.35] \rangle \\ \langle [0.35,0.45],[0.35,0.55] \rangle & \langle [0.10,0.20],[0.60,0.80] \rangle & \langle [0.05,0.15],[0.65,0.75] \rangle & \langle [0.35,0.45],[0.35,0.55] \rangle & \langle [0.25,0.45],[0.45,0.55] \rangle \\ \langle [0.25,0.35],[0.45,0.55] \rangle & \langle [0.05,0.25],[0.45,0.65] \rangle & \langle [0.15,0.25],[0.45,0.35] \rangle & \langle [0.15,0.45],[0.45,0.65] \rangle & \langle [0.25,0.35],[0.45,0.05] \rangle \end{pmatrix}$$

$$R_3=\begin{pmatrix} \langle [0.45,0.75],[0.15,0.25] \rangle & \langle [0.35,0.55],[0.25,0.35] \rangle & \langle [0.60,0.70],[0.10,0.20] \rangle & \langle [0.55,0.65],[0.05,0.25] \rangle & \langle [0.35,0.55],[0.25,0.45] \rangle \\ \langle [0.45,0.55],[0.25,0.45] \rangle & \langle [0.25,0.45],[0.35,0.45] \rangle & \langle [0.40,0.50],[0.30,0.40] \rangle & \langle [0.15,0.25],[0.65,0.75] \rangle & \langle [0.65,0.75],[0.15,0.25] \rangle \\ \langle [0.25,0.45],[0.35,0.45] \rangle & \langle [0.65,0.85],[0.05,0.15] \rangle & \langle [0.50,0.70],[0.10,0.30] \rangle & \langle [0.55,0.75],[0.15,0.25] \rangle & \langle [0.65,0.85],[0.05,0.15] \rangle \\ \langle [0.35,0.45],[0.25,0.45] \rangle & \langle [0.15,0.25],[0.55,0.75] \rangle & \langle [0.10,0.30],[0.50,0.70] \rangle & \langle [0.25,0.35],[0.45,0.65] \rangle & \langle [0.15,0.25],[0.55,0.75] \rangle \\ \langle [0.25,0.45],[0.55,0.55] \rangle & \langle [0.65,0.25],[0.55,0.15] \rangle & \langle [0.15,0.25],[0.55,0.35] \rangle & \langle [0.10,0.55],[0.25,0.75] \rangle & \langle [0.05,0.15],[0.25,0.75] \rangle \end{pmatrix}$$

$$R_4=\begin{pmatrix} \langle [0.65,0.75],[0.15,0.25] \rangle & \langle [0.30,0.40],[0.30,0.40] \rangle & \langle [0.75,0.85],[0.05,0.15] \rangle & \langle [0.50,0.60],[0.10,0.30] \rangle & \langle [0.15,0.25],[0.45,0.65] \rangle \\ \langle [0.40,0.50],[0.40,0.50] \rangle & \langle [0.10,0.20],[0.20,0.30] \rangle & \langle [0.35,0.45],[0.45,0.55] \rangle & \langle [0.20,0.30],[0.40,0.60] \rangle & \langle [0.65,0.75],[0.05,0.15] \rangle \\ \langle [0.40,0.50],[0.30,0.40] \rangle & \langle [0.60,0.70],[0.10,0.30] \rangle & \langle [0.55,0.85],[0.15,0.15] \rangle & \langle [0.40,0.50],[0.20,0.30] \rangle & \langle [0.55,0.65],[0.25,0.35] \rangle \\ \langle [0.30,0.40],[0.40,0.50] \rangle & \langle [0.10,0.30],[0.60,0.70] \rangle & \langle [0.15,0.25],[0.55,0.75] \rangle & \langle [0.20,0.30],[0.40,0.50] \rangle & \langle [0.35,0.45],[0.45,0.55] \rangle \\ \langle [0.50,0.40],[0.35,0.65] \rangle & \langle [0.15,0.35],[0.45,0.05] \rangle & \langle [0.15,0.35],[0.45,0.05] \rangle & \langle [0.15,0.25],[0.45,0.35] \rangle & \langle [0.05,0.45],[0.45,0.25] \rangle \end{pmatrix}$$

By using, step1and step2 of the proposed algorithm, we get the overall values as:

$$\tilde{r}_1 = [0.389979, 0.528468, 0.097800, 0.228529];$$
$$\tilde{r}_2 = [0.29982, 0.43674, 0.17998, 0.32120];$$
$$\tilde{r}_3 = [0.465896, 0.663599, 0.085252, 0.168343];$$
$$\tilde{r}_4 = [0.15654, 0.26810, 0.38017, 0.54961];$$
$$\tilde{r}_5 = [0.27742, 0.26665, 0.31102, 0.24447].$$

By using step 4, we obtain:

$K(\tilde{r}_1, \tilde{r}^+)$ = 0.88148,
$K(\tilde{r}_2, \tilde{r}^+)$ = 0.84659,
$K(\tilde{r}_3, \tilde{r}^+)$ = 0.92516,
$K(\tilde{r}_4, \tilde{r}^+)$ = 0.77451,
$K(\tilde{r}_5, \tilde{r}^+)$ =0.78557,

Ranking the best alternative according to the correlation coefficient, we get:

$$A_3 > A_1 > A_2 > A_5 > A_4.$$

Using step 3 of the same algorithm proposed in the paper with different accuracy functions, we obtain the ranking of best alternatives and the results are tabulated as follows:

Table 4. Comparison of the Ranking with Accuracy Functions

Accurancy Functions	**Ranking the Alternatives**
Ye, 2009	$A_3 > A_4 > A_1 > A_2 > A_5.$
Xu, 2007	$A_3 > A_4 > A_1 > A_2 > A_5.$
Nayagam.et.al., 2011	$A_3 > A_1 > A_2 > A_5 > A_4.$
Deepa Joshi & Sanjay Kumar, 2018	$A_3 > A_1 > A_2 > A_5 > A_4.$

Hence the best alternative is A_3

9. CONCLUSION

In this paper, the unknown weights of the decision maker can be calculated using the exact and numerical solution of singular perturbation problem for decision making process. we can use the IVIFOWA AND IVIFHA operators to aggregate all the individual interval valued intuitionistic fuzzy decision matrices into a collective matrix. For ranking process we can utilize the proposed correlation coefficient method and accuracy function methods, using this two methods we can choose the most desirable alternatives.

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