

(1, 2) * - α^* - Compact Spaces in Bitopological Spaces

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Abstract- In this paper, (1, 2)*- α^* - isolated point, (1, 2)*- α^* - compact spaces, (1, 2)*- α^* - countably compact spaces and sequentially (1, 2) * - α^* - compact spaces in bitopological spaces are introduced and its properties are investigated.

Keywords- (1, 2)*- α^* - isolated point, (1, 2) * - α^* - compact spaces, (1, 2)*- α^* - countably compact spaces and sequentially (1, 2) * - α^* - compact spaces.

1. INTRODUCTION

The study about a class of compact space called as S-closed space using semi open cover was first initiated by DI Maio and Noiri [4]. The notion of locally S-closed space was investigated by Noiri[7]. The class of compact space namely GO-compact space and GO-connected space using g-open cover was introduced by Balachandran, Sundaram and Maki[1]. Pauline Mary Helen, Ponnuthai Selvarani, Veronica Vijayan, Punitha Tharani[8] studied about g^{**} compact space and g^{**} compact space modulo I space Mohana and Arockiarani [6] introduced a class of (1, 2)*- $\pi g\alpha^{**}$ -compact spaces and (1, 2)*- $\pi g\alpha^{**}$ -connected spaces using (1, 2)*- $\pi g\alpha^{**}$ -open sets in bitopological spaces.

In this paper, we introduce a new class of (1, 2)*- α^* -compact space using (1, 2)*- α^* open sets and investigate their properties. Further, we study (1, 2)*- T_{α^*} -space, (1, 2)*- $T_{g\alpha^*}$ -space and their properties.

2. PRELIMINARIES

Throughout this paper, X and Y denote the bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) respectively, on which no separation axioms are assumed.

Definition 2.1.[5] A subset S of a bitopological space X is said to be $\tau_{1,2}$ -open if $S=A \cup B$ where $\tau_1 \in A$ and $\tau_2 \in B$. A subset S of X is said to be (i) $\tau_{1,2}$ -closed if the complement of S is $\tau_{1,2}$ -open. (ii) $\tau_{1,2}$ -clopen if S is both $\tau_{1,2}$ -open and $\tau_{1,2}$ -closed.

Definition 2.2.[5] Let S be a subset of the bitopological space X. Then the $\tau_{1,2}$ -interior of S denoted by $\tau_{1,2}$ -int(S) is defined by $\cup\{G: G \subseteq S \text{ and } G \text{ is } \tau_{1,2}\text{-open}\}$ and $\tau_{1,2}$ -closure of S denoted by

$\tau_{1,2}\text{-cl}(S)$ is defined by $\cap\{F: S \subseteq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\}$.

Definition 2.3. A subset A of a bitopological space X is said to be

i) (1, 2)*-regular open [5] if $A = \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$.

ii) (1, 2)*- α -open [5] if $A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)))$.

iii) (1, 2)*- α^* -closed [2] if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is (1, 2)*- α -open set in X.

Definition 2.4. A map $f: X \rightarrow Y$ is called

(i) (1, 2)*- α^* -continuous [3] if $f^{-1}(V)$ is (1, 2)*- α^* -closed in X for every $\sigma_{1,2}$ -closed set V of Y.

(ii) (1, 2)*- α^* -irresolute [3] if $f^{-1}(V)$ is (1, 2)*- α^* -closed in X for every (1, 2)*- α^* -closed V of Y.

Definition 2.5. [8] A map $f: X \rightarrow Y$ is called g^{**} -resolute if $f(V)$ is g^{**} -open in Y whenever U is g^{**} -open in X.

3. (1, 2) * - α^* - COMPACT SPACES

Definition 3.1. A collection $\{A_i : i \in I\}$ of (1, 2)*- α^* -open sets in a bitopological space X is called a (1, 2)*- α^* -open cover of a subset B if $B \subseteq \cup\{A_i : i \in I\}$.

Definition 3.2. A bitopological space X is called (1, 2)*- α^* -compact, if every (1, 2)*- α^* -open covering of X contains a finite subcollection that also covers X. A subset A of X is said to be (1, 2)*- α^* -compact if every (1, 2)*- α^* -open covering of A contains a finite subcollection that also covers A.

Definition 3.3. A subset B of a bitopological space X is said to be (1, 2)*- α^* -compact relative to X, if for

every collection $\{A_i : i \in I\}$ of $(1, 2)^*-\alpha^*$ -open subsets of X such that $B \subseteq \bigcup \{A_i : i \in I\}$ then, there exists a finite subset I_0 of I such that $B \subseteq \bigcup \{A_i : i \in I_0\}$.

Definition 3.4. A subset B of a bitopological space X is said to be $(1, 2)^*-\alpha^*$ -compact if B is $(1, 2)^*-\alpha^*$ -compact as the subset of X .

Remark 3.5. Any topological space having only finitely many points is necessarily $(1, 2)^*-\alpha^*$ -compact and $(1, 2)^*$ -compact.

Theorem 3.6. A $(1, 2)^*-\alpha^*$ -closed subset of $(1, 2)^*-\alpha^*$ -compact space is $(1, 2)^*-\alpha^*$ -compact.

Proof. Let A be a $(1, 2)^*-\alpha^*$ -closed subset of $(1, 2)^*-\alpha^*$ -compact space (X, τ_1, τ_2) and $\{U_\alpha\}_{\alpha \in \Delta}$ be a $(1, 2)^*-\alpha^*$ -open cover for A . Then, $\{\{U_\alpha\}_{\alpha \in \Delta}, (X - A)\}$ is a $(1, 2)^*-\alpha^*$ -open cover for X . Since X is $(1, 2)^*-\alpha^*$ -compact, there exists $\alpha_1, \alpha_2, \dots, \alpha_n \in \Delta$ such that $A \subseteq \bigcup_{i=1}^n U_{\alpha_i}$. Therefore, which proves A is $(1, 2)^*-\alpha^*$ -compact.

Theorem 3.7. A $(1, 2)^*-\alpha^*$ -closed subset of $(1, 2)^*-\alpha^*$ -compact space is $(1, 2)^*-\alpha^*$ -compact relative to X .

Proof. Let A be a $(1, 2)^*-\alpha^*$ -closed subset of a $(1, 2)^*-\alpha^*$ -compact space X . Then A^c is $(1, 2)^*-\alpha^*$ -open in X . Let S be a cover of A by $(1, 2)^*-\alpha^*$ -open sets in X . Then, $\{S, A^c\}$ is a $(1, 2)^*-\alpha^*$ -open cover of X . Since X is $(1, 2)^*-\alpha^*$ -compact, it has a finite subcover, say $\{G_1, G_2, \dots, G_n\}$. If this subcover contains A^c , we discard it. Otherwise leave the subcover as it is. Thus we have obtained a finite $(1, 2)^*-\alpha^*$ -open subcover of A and so A is $(1, 2)^*-\alpha^*$ -compact relative to X .

Theorem 3.8.

- (i) If $f : X \rightarrow Y$ is $(1, 2)^*-\alpha^*$ -continuous image of a $(1, 2)^*-\alpha^*$ -compact space, then $(1, 2)^*-\alpha^*$ -compact.
- (ii) If a map $f : X \rightarrow Y$ is $(1, 2)^*-\alpha^*$ -irresolute and a subset B is $(1, 2)^*-\alpha^*$ -compact relative to X , then the image $f(B)$ is $(1, 2)^*-\alpha^*$ -compact relative to Y .

Proof.

- (i) Let $f : X \rightarrow Y$ be a $(1, 2)^*-\alpha^*$ -continuous map from a $(1, 2)^*-\alpha^*$ -compact space X onto a bitopological space Y . Let $\{A_i : i \in I\}$ be an open cover of Y . Then $\{f^{-1}(A_i) : i \in I\}$ is a $(1, 2)^*-\alpha^*$ -open cover of X . Since X is $(1, 2)^*-\alpha^*$ -compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$. Since f is onto, $\{A_1, A_2, \dots, A_n\}$ is a $\sigma_{1,2}$ -open cover of Y and so Y is $(1, 2)^*$ -compact.
- (ii) Let $\{A_i : i \in I\}$ be any collection of $(1, 2)^*-\alpha^*$ -open subsets of Y such that $f(B) \subseteq \bigcup \{A_i : i \in I\}$.

Then, $B \subseteq \bigcup \{f^{-1}(A_i) : i \in I\}$. By using assumptions, there exists a finite subset I_0 of I such that $B \subseteq \bigcup \{f^{-1}(A_i) : i \in I_0\}$. Therefore, we have $f(B) \subseteq \bigcup \{A_i : i \in I_0\}$ which shows that $f(B)$ is $(1, 2)^*-\alpha^*$ -compact relative to Y .

Definition 3.9. A map $f : X \rightarrow Y$ is called $(1, 2)^*-\alpha^*$ -resolute if $f(V)$ is $(1, 2)^*-\alpha^*$ -open in Y whenever U is $(1, 2)^*-\alpha^*$ -open in X .

Definition: 3.10 A map $f : X \rightarrow Y$ is called strongly- $(1, 2)^*-\alpha^*$ -continuous if $f^{-1}(V)$ is $\tau_{1,2}$ -closed ($\tau_{1,2}$ -open) in X for every $(1, 2)^*-\alpha^*$ -closed ($(1, 2)^*-\alpha^*$ -open) set V of Y .

Definition 3.11. A topological space (X, τ_1, τ_2) is said to be a $(1, 2)^*-\alpha^*$ - T_1 -space if for every pair of distinct points x, y in X , there exists disjoint $(1, 2)^*-\alpha^*$ -open sets U and V in X such that $x \in U, y \notin U$ and $x \notin V, y \in V$.

Definition 3.12. A topological space (X, τ_1, τ_2) is said to be a $(1, 2)^*-\alpha^*$ - T_2 -space if for every pair of distinct points x, y in X , there exists disjoint $(1, 2)^*-\alpha^*$ -open sets U and V in X such that $x \in U$ and $y \in V$.

Definition: 3.13. A collection ζ of subsets of X is said to have finite intersection property if for every subcollection $\{C_1, C_2, \dots, C_n\}$ of ζ the intersection $C_1 \cap C_2 \cap \dots \cap C_n$ is non-empty.

Theorem: 3.14. If a map $f : X \rightarrow Y$ is strongly- $(1, 2)^*-\alpha^*$ -continuous from a $(1, 2)^*-\alpha^*$ -compact space X onto a bitopological space Y , then Y is $(1, 2)^*-\alpha^*$ -compact.

Proof: Let $\{A_i : i \in I\}$ be a $(1, 2)^*-\alpha^*$ -open cover of Y . Then $\{f^{-1}(A_i) : i \in I\}$ is a $\tau_{1,2}$ -open cover of X . As f is strongly- $(1, 2)^*-\alpha^*$ -continuous. Since X is $(1, 2)^*-\alpha^*$ -compact, it has a finite subcover, say $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$. Since f is onto, $\{A_1, A_2, \dots, A_n\}$ is a finite $(1, 2)^*-\alpha^*$ -open cover of Y and so Y is $(1, 2)^*-\alpha^*$ -compact.

Theorem: 3.15. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be two topological spaces and $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function. Then

- 1. f is $(1, 2)^*-\alpha^*$ -irresolute and A is a $(1, 2)^*-\alpha^*$ -compact subset of $X \Rightarrow f(A)$ is a $(1, 2)^*-\alpha^*$ -compact subset of Y .
- 2. f is one to one, $(1, 2)^*-\alpha^*$ -resolute and B is a $(1, 2)^*-\alpha^*$ -compact subset of $Y \Rightarrow f^{-1}(B)$ is a $(1, 2)^*-\alpha^*$ -compact subset of X .

Proof: (1) and (2) Obvious from the definitions.

Theorem 3.16 A topological space (X, τ_1, τ_2) is $(1, 2)^*-\alpha^*$ -compact if and only if for every collection ζ of $(1, 2)^*-\alpha^*$ -closed sets in X having finite intersection property, $\bigcap_{c \in C} C$ of all elements of ζ is non-empty.

Proof: Let (X, τ_1, τ_2) is $(1, 2)^*-\alpha^*$ -compact and ζ be a collection of $(1, 2)^*-\alpha^*$ -closed sets with finite intersection property. Suppose $\bigcap_{c \in C} C = \emptyset$, then $\bigcap_{c \in C} (X - C) = X$. Therefore, $\{X - C\}_{c \in C}$ is a $(1, 2)^*-\alpha^*$ -open cover for X . Then there exists $C_1, C_2, \dots, C_n \in \zeta$ such that $\bigcup_{i=1}^n (X - C_i) = X$.

Therefore, $\bigcap_{i=1}^n C_i = \emptyset$ which is a contradiction.

Therefore, $\bigcap_{c \in C} C \neq \emptyset$. Conversely, assume the hypothesis given in the statement. To prove X is $(1, 2)^*-\alpha^*$ -compact. Let $\{U_\alpha\}_{\alpha \in \Delta}$ be a $(1, 2)^*-\alpha^*$ -open cover for X . Then $\bigcup_{\alpha \in \Delta} U_\alpha = X \Rightarrow \bigcap_{\alpha \in \Delta} (X - U_\alpha) = \emptyset$. By the

hypothesis, there exists $\alpha_1, \alpha_2, \dots, \alpha_n$ such that $\bigcap_{i=1}^n X - U_{\alpha_i} = \emptyset$. Therefore, $\bigcup_{i=1}^n U_{\alpha_i} = X$. Therefore, X is $(1, 2)^*-\alpha^*$ -compact.

Corollary 3.17: Let (X, τ_1, τ_2) be a $(1, 2)^*-\alpha^*$ -compact space and let $C_1 \supseteq C_2 \supseteq \dots \supseteq C_n \supseteq C_{n+1} \supseteq \dots$ be a nested sequence of non-empty $(1, 2)^*-\alpha^*$ -closed sets in X . Then $\bigcap_{c \in Z^+} C_n$ is non-empty.

Proof: Obviously $\{C_n\}_{n \in Z^+}$ has finite intersection property. Therefore by Theorem 3.15, $\bigcap_{c \in Z^+} C_n$ is non-empty.

Theorem 3.18. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function, then

- 1) f is $(1, 2)^*-\alpha^*$ -continuous, onto and X is $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is compact.
- 2) f is continuous, onto and X is $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is compact.
- 3) f is $(1, 2)^*-\alpha^*$ -irresolute, onto and X is $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is $(1, 2)^*-\alpha^*$ -compact.
- 4) f is strongly $(1, 2)^*-\alpha^*$ -irresolute, onto and X is $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is $(1, 2)^*-\alpha^*$ -compact.

5) f is $(1, 2)^*-\alpha^*$ -open, bijection and Y is $(1, 2)^*-\alpha^*$ -compact $\Rightarrow X$ is compact.

6) f is open, bijection and Y is $(1, 2)^*-\alpha^*$ -compact $\Rightarrow X$ is compact.

7) f is $(1, 2)^*-\alpha^*$ -resolute, bijection and Y is $(1, 2)^*-\alpha^*$ -compact $\Rightarrow X$ is $(1, 2)^*-\alpha^*$ -compact.

Proof: (1): Let $\{U_\alpha\}_{\alpha \in \Delta}$ be an open cover for Y . Then $\{f^{-1}(U_\alpha)\}_{\alpha \in \Delta}$ is a $(1, 2)^*-\alpha^*$ -open cover for X . Since X is $(1, 2)^*-\alpha^*$ -compact, there exists $\alpha_1, \alpha_2, \dots, \alpha_n$ such that $X \subseteq \bigcup_{i=1}^n f^{-1}(U_{\alpha_i})$. Therefore, Y is compact. Proof for (2) to (7) are similar above.

4. $(1, 2)^*-\alpha^*$ -COUNTABLY COMPACT SPACE

Definition 4.1. A subset A of a topological space (X, τ_1, τ_2) is said to be $(1, 2)^*-\alpha^*$ -countably compact space if every countable $(1, 2)^*-\alpha^*$ -open covering of A has a finite subcover.

Remark 4.2. Every $(1, 2)^*-\alpha^*$ -compact space is $(1, 2)^*-\alpha^*$ -countably compact space.

Definition 4.3: Let (X, τ_1, τ_2) be a topological space and $x \in X$. Every $(1, 2)^*-\alpha^*$ -open set containing x is said to be a $(1, 2)^*-\alpha^*$ -neighbourhood of x .

Definition 4.4: Let A be a subset of a topological space (X, τ_1, τ_2) . A point $x \in X$ said to be $(1, 2)^*-\alpha^*$ -limit point of A if every $(1, 2)^*-\alpha^*$ -neighbourhood of x contains a point of A other than x .

Theorem 4.5: In a $(1, 2)^*-\alpha^*$ -countably compact topological space every infinite subset has a $(1, 2)^*-\alpha^*$ -limit point.

Proof : Let (X, τ_1, τ_2) be $(1, 2)^*-\alpha^*$ -countably compact. Suppose that there exists an infinite subset which has no $(1, 2)^*-\alpha^*$ -limit point. Let $B = \{a_n / n \in N\}$ be a countable subset of A . Since B has no $(1, 2)^*-\alpha^*$ -limit point of B , there exists a $(1, 2)^*-\alpha^*$ -neighbourhood U_n of a_n such that $B \cap U_n = \{a_n\}$. Now $\{U_n\}$ is a $(1, 2)^*-\alpha^*$ -open cover for B . Since B^c is $(1, 2)^*-\alpha^*$ -open, $\{B^c, \{U_n\}_{n \in Z^+}\}$ is a countable $(1, 2)^*-\alpha^*$ -open cover for X . But it has no finite subcover which is a contradiction, since X is $(1, 2)^*-\alpha^*$ -countably compact. Therefore every infinite subset of X has a $(1, 2)^*-\alpha^*$ -limit point.

Corollary 4.6:In a compact topological space every infinite subset has a limit point.

Proof follows from theorem 4.5,since every $(1, 2)^*-\alpha^*$ -compact is $(1, 2)^*-\alpha^*$ -countably compact.

Theorem 4.7:A $(1, 2)^*-\alpha^*$ -closed subset of $(1, 2)^*-\alpha^*$ -countably compact space is $(1, 2)^*-\alpha^*$ -countably compact. Proof is similar to theorem 3.6.

Definition: 4.8 In a topological space (X, τ_1, τ_2) , a point $x \in X$ is said to be $(1, 2)^*-\alpha^*$ -isolated point of A if every $(1, 2)^*-\alpha^*$ -open set containing x contains no point of A other than x.

Theorem 4.9:Let X be a non-empty $(1, 2)^*-\alpha^*$ -compact $(1, 2)^*-\alpha^*$ - T_2 space. If has no $(1, 2)^*-\alpha^*$ -isolated points, then X is uncountable.

Proof: Let $x_i \in X$. Choose a point y of X different from x. This is possible since $\{x_i\}$ is not a $(1, 2)^*-\alpha^*$ -isolated point. Since X is $(1, 2)^*-\alpha^*$ - T_2 , there exists $(1, 2)^*-\alpha^*$ -open sets such that U_1 and V_1 such that $U_1 \cap V_1 = \emptyset; x \in U_1, y \in V_1$. Therefore V_1 is $(1, 2)^*-\alpha^*$ -open and $x_1 \notin g^{**}cl(V_1)$. By repeating the same process with V_1 in the place of X and x_1 in the place of y we get a point $x \neq x_1$ and a $(1, 2)^*-\alpha^*$ -open V_2 set such that V_2 is $(1, 2)^*-\alpha^*$ -open and $x_2 \notin (1, 2)^*-\alpha^*cl(V_2)$. In general ,given V_{n-1} which is $(1, 2)^*-\alpha^*$ -open and non-empty ,choose V_n to be a non-empty $(1, 2)^*-\alpha^*$ -open set such that $V_n \subseteq V_{n-1}$ and $x_n \notin (1, 2)^*-\alpha^*cl(V_n)$. Hence we get a nested sequence of $(1, 2)^*-\alpha^*$ -closed sets such that $(1, 2)^*-\alpha^*cl(V_n) \supseteq (1, 2)^*-\alpha^*cl(V_{n+1}) \supseteq \dots$. Since X is $(1, 2)^*-\alpha^*$ -compact, $\bigcap (1, 2)^*-\alpha^*cl(V_n) \neq \emptyset$. Therefore, there exists $x \in \bigcap (1, 2)^*-\alpha^*cl(V_n)$. But $x \neq x_n$, for every n, since $x_n \notin (1, 2)^*-\alpha^*cl(V_n)$ and $x \in (1, 2)^*-\alpha^*cl(V_n)$. Define $f : Z_+ \rightarrow X$ such that $f(n) = x_n$. Then $x \in X$ has no preimage. Therefore, f is not onto and hence X is uncountable.

Note 4.10: The converse of Theorem 4.5 is true in a $(1, 2)^*-\alpha^*$ - T_1 space.

Theorem 4.11:In a $(1, 2)^*-\alpha^*$ - T_1 space ,if every infinite subset has a $(1, 2)^*-\alpha^*$ -limit point, the X is $(1, 2)^*-\alpha^*$ -countably compact.

Proof: Let every infinite subset has a $(1, 2)^*-\alpha^*$ -limit point. To prove X is a $(1, 2)^*-\alpha^*$ -countably compact. If not there exists a countable $(1, 2)^*-\alpha^*$ -open cover $\{U_n\}$ such that it has no finite sub cover. Since $U_1 \neq X$, there exists $x_1 \notin U_1$; since $X \neq U_1 \cup U_2$, there exists $x_2 \notin U_1 \cup U_2$. Proceeding like this there exists $x_2 \notin U_1 \cup U_2 \cup \dots \cup U_n$ for all n. $A = \{x_n\}$ is an infinite set. If $x \in X$, then $x \in U_n$ for some n. But $x_k \notin U_n$ for all $k \geq n$. $U_n - \{x_1, x_2, x_3, \dots, x_{n-1}\}$ is a $(1, 2)^*-\alpha^*$ -open set (since X is $(1, 2)^*-\alpha^*$ - T_1) containing x which does not have a point of A other than x. Therefore, x is not a limit point of A which is a contradiction.

Theorem 4.12 A topological space (X, τ_1, τ_2) is $(1, 2)^*-\alpha^*$ -countably compact if and only if for every countable collection ζ of $(1, 2)^*-\alpha^*$ -closed sets in X having finite intersection property, of all elements of ζ is non- empty.

Proof: Similar to the proof of theorem 3.15.

Corollary 4.13. X is $(1, 2)^*-\alpha^*$ -countably compact if and only if every nested sequence of $(1, 2)^*-\alpha^*$ -closed non-empty sets $C_1 \supseteq C_2 \supseteq \dots$ has a non-empty intersection.

Proof. Obviously $\{C_n\}_{n \in Z^+}$ has finite intersection property. Therefore, by theorem 4.10, $\bigcap_{c \in Z^+} C_n$ is non-empty.

5. SEQUENTIALLY $(1, 2)^*-\alpha^*$ -COMPACT SPACE

Definition: 5.1 A subset A of a topological space (X, τ_1, τ_2) is said to be sequentially $(1, 2)^*-\alpha^*$ -compact space if every sequence in A contains a subsequence which $(1, 2)^*-\alpha^*$ -converges to some point in A.

Theorem: 5.2 A finite subset A of a topological space (X, τ_1, τ_2) is sequentially $(1, 2)^*-\alpha^*$ -compact.

Proof: Let $\{x_n\}$ be an arbitrary sequence in X .Since A is finite, at least one element of the sequence say x_0 must be repeated infinite number of times .So the

constant subsequence x_0, x_0, \dots must $(1, 2)^*-\alpha^*$ - converges to x_0 .

Theorem 5.3 Every sequentially $(1, 2)^*-\alpha^*$ -compact space is $(1, 2)^*-\alpha^*$ -countably compact.

Proof: Let (X, τ_1, τ_2) be sequentially $(1, 2)^*-\alpha^*$ -compact. Since X is not $(1, 2)^*-\alpha^*$ -countably compact. Then there exists countable $(1, 2)^*-\alpha^*$ -open cover $\{U_n\}_{n \in \mathbb{Z}^+}$ which has no finite subcover. Then $X = \bigcup_{n \in \mathbb{Z}^+} U_n$. Choose

$$x_1 \in U_1, x_2 \in U_2 - U_1, x_3 \in U_3 - \bigcup_{i=1,2} U_i, \dots, x_n \in U_n - \bigcup_{i=1}^{n-1} U_i$$

This is possible since $\{U_n\}$ has no finite subcover. Now $\{x_n\}$ is a sequence in X . Let $x \in X$ be arbitrary. Then $x \in U_k$ for some k . By our choice of $\{x_n\}$, $x_i \notin U_k$ for all greater than k . Hence there is no subsequence of $\{x_n\}$ which can $(1, 2)^*-\alpha^*$ -converge to x . Since x is arbitrary, the sequence $\{x_n\}$ has no convergent subsequence which is a contradiction. Therefore X is $(1, 2)^*-\alpha^*$ -countably compact.

Theorem 5.4: Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function, then

- 1) f is $(1, 2)^*-\alpha^*$ -resolute, bijection and Y is sequentially $(1, 2)^*-\alpha^*$ -compact $\Rightarrow X$ is sequentially $(1, 2)^*-\alpha^*$ -compact.
- 2) f is onto, $(1, 2)^*-\alpha^*$ -irresolute and X is sequentially $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is sequentially $(1, 2)^*-\alpha^*$ -compact.
- 3) f is onto, $(1, 2)^*-\alpha^*$ -irresolute and X is sequentially $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is sequentially $(1, 2)^*-\alpha^*$ -compact.
- 4) f is onto, $(1, 2)^*$ -continuous and X is sequentially $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is sequentially compact.
- 5) f is onto, strongly $(1, 2)^*-\alpha^*$ -continuous and X is sequentially $(1, 2)^*-\alpha^*$ -compact $\Rightarrow Y$ is sequentially $(1, 2)^*-\alpha^*$ -compact.

Proof:(1) Let $\{x_n\}$ be a sequence in X . Then $\{f(x_n)\}$ is a sequence in Y . It has a $(1, 2)^*-\alpha^*$ -convergent subsequence $\{f(x_{n_k})\}$ such that $f(x_{n_k}) \xrightarrow{(1,2)^*-\alpha^*} y_0$ in Y . Then there exists $x_0 \in X$ such that $f(x_0) = y_0$. Let U be a $(1, 2)^*-\alpha^*$ -open set containing x_0 . Then $f(U)$ is a $(1, 2)^*-\alpha^*$ -open set containing y_0 . Then

there exists N such that $f(x_{n_k}) \in f(U)$ for all $k \geq N$. Therefore $f^{-1} \circ f(x_{n_k}) \in f^{-1} \circ f(U)$.

Therefore $x_{n_k} \in U$ for all $k \geq N$. This proves that X is sequentially $(1, 2)^*-\alpha^*$ -compact. Proof for (2) to (5) is similar to the above.

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