International Journal of Research in Advent Technology, Vol.7, No.4, April 2019
E-ISSN: 2321-9637
Available online at www.ijrat.org

Neutrosophic Pre-open Set in Simple Extended Neutrosophic Topology

F. Nirmala Irudayam[1], T. Madhumathi[2]
[1]*Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, India.*
[2]*Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, India,*
Email: nirmalairudayam@ymail.com[1], madhumanoj1822@gmail.com[2]

Abstract- The purpose of this article is to introduce and study concept of various open sets in simple extension topology to the neutrosophic setting. This paper aims at studying the basic characterizations of these newly developed sets in the much flourishing area of recent research introduced by Smarandache. Here the relationship between these sets are also proposed.
Index Terms- Neutrosophic pre^+-open set, Neutrosophic pre^+-closed set.

1. INTRODUCTION

Many real life problems in Business, Finance, Medical Sciences, Engineering and Social Sciences deal with uncertainties. There are difficulties in solving the uncertainties in these data by traditional mathematical models. To overcome these difficulties many authors have introduced many sets which deals with inconsistent data. Some of these approaches are fuzzy sets [17], Intuitionistic fuzzy sets [2], Neutrosophic sets [13] and so on which can be treated as mathematical tools to avert obstacles dealing with ambiguous data. The introduction of the idea of fuzzy set was introduced in the year 1965 by Zadeh[17]. He proposed that each element in a fuzzy set has a degree of membership. Thereafter the paper of Chang(1968)[4] paved way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Following this idea K.Atanassov[1,2,3] in 1983 introduced the idea of intuitionistic fuzzy set on a universe X as a generalization of fuzzy set. Here besides the degree of membership a degree of non-membership for each element is also defined. The topological framework of intuitionistic fuzzy set introduced by D. Coker[5]. The neutrosophic set was initiated by Smarandache and he explained that neutrosophic set is a generalization of intuitionistic fuzzy set. Smarandache[12,13] originally gave the definition of a neutrosophic set and neutrosophic logic. The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy and falsehood. In 2012 Salama,Alblowi [14,15] introduced the concept of neutrosophic topological space. In 2016 concept of neutrosophic semi-open sets in neutrosophic topological space by introduced P.Ishwarya and K.Bageerathi[6]. In 2017 V.Venkateswara Rao and Y. Srinivasa Rao[16] introduced the concept of neutrosophic pre-open sets and neutrosophic per-closed in neutrosophic topological spaces. The concept of extending a topology by a non-open set was proposed by Levine[8] in 1963. A simple extension of a topology τ is defined as $\tau(B) = \{(B \cap O) \cup O' / O, O' \in \tau\}$ by Levine. F.Nirmala Irudayam [10] and Sr.I.Arockiarani introduced the concept of b^+-open sets in extended topological spaces. T. Noiri, Sr. I. Arockiarani and F. Nirmala Irudayam [11] coined the idea of Ω_{gb}^{+*}, $\mho_{gb}^{+*}$ sets in simple extended topological spaces. B. Kanchana and F. Nirmala Irudayam [7] defined a new class of contra continuous functions via b-open sets in simple extended topological spaces. T. Madhumathi and F.Nirmala Irudayam [9] introduced the idea of Ω_{gb}^{+}-closed sets in simple extension topological spaces. Currently, in this paper we introduced the notion of neutrosophic pre^+-open sets and neutrosophic pre^+-closed in simple extension neutrosophic topological spaces.

2. PRELIMINARIES

Definition 2.1 ([13]). Let X be a non empty set. A neutrosophic set (NS for short) A is an object having the form A = <x,A^1,A^2,A^3> where A^1,A^2, A^3 represent the degree of membership, the degree of indeterminacy and the degree of non-membership respectively of each element x∈X of the set A.

Definition 2.2 ([13]). Let X be a non empty set, A = <x,A^1, A^2, A^3> and B = <x,B^1,B^2,B^3> be neutrosophic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X, where A^i = <x,A^1, A^2, A^3>

(i) A ⊆ B if and only if $A^1 \leq B^1$, $A^2 \leq B^2$ and $A^3 \geq B^3$
(ii) A = B if and only if A ⊆ B and B ⊆ A.
(iii) $\overline{A}$ = <x, A^3, A^2, A^1>
(iv)A∩B=<x, $A^1 \wedge B^1$, $A^2 \wedge B^2$, $A^3 \vee B^3$>
(v) A∪B=<x, $A^1 \vee B^1$, $A^2 \vee B^2$, $A^3 \wedge B^3$>
(vi) $\cup A_i$ = <x, $\vee A_i^1$, $\vee A_i^2$, $\wedge A_i^3$>
(vii) $\cap A_i$ = <x, $\wedge A_i^1$, $\wedge A_i^2$, $\vee A_i^3$>
(viii)A \ B = A ∩ $\overline{B}$.
(ix)0_N = <x,0,0,1>; 1_N = <x,1, 1, 0 >.

Definition 2.3 ([14]). A neutrosophic topology (NT for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:
(i) $0_N, 1_N \in \tau$.
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.
(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.
In this case the pair (X, τ) is called a Neutrosophic topological space (NTS for short) and any Neutrosophic

set in τ is called a Neutrosophic open set (NOS for short) in X. The complement A of a Neutrosophic open set A is called a Neutrosophic closed set (NCS for short) in X.

Definition 2.4 ([14]). Let (X, τ) be a neutrosophic topological space and A = <X,A_1,A_2,A_3> be a set in X. Then the closure and interior of A are defined by

Ncl(A) = ∩{K : K is a neutrosophic closed set in X and A ⊆ K},

Nint(A) = ∪{G : G is a neutrosophic open set in X and G ⊆ A}.

It can be also shown that Ncl(A) is a neutrosophic closed set and Nint(A) is a neutrosophic open set in X, and A is a neutrosophic closed set in X iff Ncl(A) = A; and A is a neutrosophic open set in X iff Nint(A) = A.

3. NEUTROSOPHIC PRE$^+$-OPEN

This section contributes to the study of the newly developed concept of neutrosophic pre$^+$-open set of X.Futher its characterizations and its relationship with the other sets are dealt with.

Note: For any subset A of X, the interior of A is same as the interior in usual neutrosophic topology and the closure of A is newly defined in simple extension neutrosophic topological space(SENTS). The complement of A is denoted by A^c or X-A respectively.

Definition 3.1. Let A be a neutrosophic set of a simple extension neutrosophic topology. Then A is said to be Neutrosophic pre$^+$ open [NP$^+$ O]set of X if there exists a neutrosophic open set NO such that NO ⊆ A ⊆ NO(Ncl$^+$ (A)).

Definition 3.2. A subset A of a topological space (X,τ$^+$**)** is said to be,

(i) A neutrosophic pre$^+$-open set if A ⊆ Nint(Ncl$^+$(A)) and neutrosophic pre$^+$-closed set if Ncl$^+$(Nint(A)) ⊆ A.

(ii) A neutrosophic α$^+$-open set if A ⊆ Nint(Ncl$^+$(Nint(A))) and neutrosophic α$^+$-closed set if Ncl$^+$(Nint(Ncl$^+$(A))) ⊆ A.

(iii) neutrosophic semi$^+$-open set A ⊆ Ncl$^+$(Nint(A)) and neutrosophic semi$^+$-closed set if Nint(Ncl$^+$(A)) ⊆ A.

(iv) neutrosophic b$^+$-open set if A ⊆ Ncl$^+$(Nint(A)) ∪ Nint(Ncl$^+$(A)) and neutrosophic b$^+$-closed set Ncl$^+$(Nint(A)) ∪ Nint(Ncl$^+$(A)) ⊆ A.

(v) a neutrosophic β$^+$-open set,if A ⊆ Ncl$^+$(Nint(Ncl$^+$(A))) and neutrosophic β$^+$-closed set if Nint(Ncl$^+$(Nint(A))) ⊆ A.

(vi) neutrosophic regular$^+$ open set if A =Nint(Ncl$^+$(A)) and neutrosophic regular$^+$ closed set, if A = Ncl$^+$(Nint(A)).

Theorem 3.3. A subset A of a simple extension neutrosophic topological space X is a neutrosophic pre$^+$ open set if and only if A ⊆ Nint(Ncl$^+$ (A)).

Proof: Let us consider that A ⊆ Nint(Ncl$^+$(A))

Now we have to prove that A is a neutrosophic pre$^+$-open set in simple extension neutrosophic topological space

We know that Neutrosophic open=Nint(A). Clearly NO ⊆ A ⊆ NO(Ncl$^+$ (A)).

Therefore A is NP$^+$O set

Conversely suppose that Let A is a neutrosophic pre$^+$-open set in simple extension neutrosophic topological space. i.e. NO ⊆ A ⊆ NO(Ncl$^+$ (A)) for some NO

But NO ⊆ Nint(A), thus NO(Ncl$^+$(A)) ⊆ Nint(Ncl$^+$ (A))

Hence A ⊆ NO(Ncl$^+$ (A)) ⊆ Nint(Ncl$^+$ (A))

Therefore A ⊆ Nint(Ncl$^+$ (A))

Hence proved the theorem.

Theorem 3.4. In simple extension neutrosophic topology the union of two neutrosophic pre$^+$-open sets again a neutrosophic pre$^+$- open set.

Proof: Let A and B are neutrosophic pre$^+$-open sets in X

A ⊆ Nint(Ncl$^+$ (A))

B ⊆ Nint(Ncl$^+$ (B))

Therefore A ∪ B ⊆ Nint(Ncl$^+$ (A)) ∪ Nint(Ncl$^+$ (B))

A ∪ B ⊆ Nint(Ncl$^+$ (A) ∪ Ncl$^+$ (B))

A ∪ B ⊆ Nint(Ncl$^+$ (A ∪ B))

Therefore by the above definition A ∪ B is also a neutrosophic pre$^+$ open set in X.

Theorem 3.5. Let (X, τ^+) be an SENTS. If $\{A_\alpha\}_{\alpha\in\Delta}$ is a collection of a neutrosophic pre$^+$-open sets in a SENTS X then $\cup_{\alpha\in\Delta} A_\alpha$ is NPO set in X.

Proof: Let us assume that for each $\alpha \in \Delta$,we have a neutrosophic open set NO_α such that

$NO_\alpha \subseteq A_\alpha \subseteq NO_\alpha(Ncl^+(A))$, then

$\cup_{\alpha\in\Delta} NO_\alpha \subseteq \cup_{\alpha\in\Delta} A_\alpha \subseteq \cup_{\alpha\in\Delta} NO_\alpha(Ncl^+(A))$

$\cup_{\alpha\in\Delta} A_\alpha \subseteq \cup_{\alpha\in\Delta} N\operatorname{int}_\alpha(Ncl^+(A))$

Hence the theorem is proved.

Remark 3.6. The intersection of any two neutrosophic pre$^+$-open sets need not be a neutrosophic pre$^+$-open set in X as shown by the following example.

Example 3.7. Let X={x}

P={<0.5, 0.5, 0.4>}

Q={<0.4, 0.6, 0.8>}

R={<0.5, 0.6, 0.4>}

S={<0.4, 0.5, 0.8>}

τ ={1_N, 0_N, P, Q, R, S}, B={<0.3, 0.2, 0.9>}

τ^+ ={1_N, 0_N, P, Q, R, S, B} is simple extension neutrosophic topological spaces

A_1={<0.5, 0.4, 0.5>}

A_2={<0.5, 0.3, 0.4>}

From this example $A_1 \cap A_2$ is not NP$^+$O set .

Theorem 3.8.

(i) Every neutrosophic open set in the simple extension neutrosophic topological space in X is neutrosophic pre$^+$-open set in X.

(ii) Every neutrosophic pre$^+$-open set in the simple extension neutrosophic topological spaces (X, τ^+) is neutrosophic b^+-open set in (X, τ^+).

(iii) Every neutrosophic $semi^+$-open set in the simple extension neutrosophic topological spaces (X, τ^+) is neutrosophic b^+-open set in (X, τ^+).

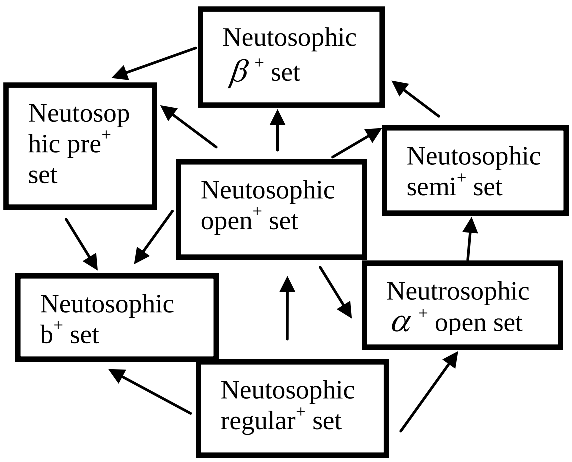
(iv) Every neutrosophic α^+-open set in the simple extension neutrosophic topological spaces (X, τ^+) is neutrosophic b^+-open set in (X, τ^+).

(v) Every neutrosophic $regular^+$-open set in the simple extension neutrosophic topological spaces (X, τ^+) is neutrosophic b^+-open set in (X, τ^+).

(vi) Every neutrosophic β^+-open set in the simple extension neutrosophic topological spaces (X, τ^+) is neutrosophic b^+-open set in (X, τ^+).

Proof: (i) Let A be NO set in NTS.
Then A=Nint(A)
Clearly $A \subseteq Ncl^+(A)$
$Nint(A) \subseteq Nint(Ncl^+(A))$
$A \subseteq Nint(Ncl^+(A))$
A is a neutrosophic pre^+-open set in X.
(ii) Let A be neutrosophic pre^+-open set in a SENTS.
Then $A \subseteq Nint(Ncl^+(A))$ which implies
$A \subseteq Nint(Ncl^+(A)) \cup Nint(A) \subseteq Nint(Ncl^+(A)) \cup Ncl^+(NintA)$.
Hence A is a neutrosophic b^+-closed sets.
(iii) Let A be neutrosophic $semi^+$-open set in a SENTS.
Then $A \subseteq Ncl^+(Nint(A))$ which implies
$A \subseteq Ncl^+(Nint(A)) \cup Nint(A) \subseteq Ncl^+(Nint(A)) \cup Nint(Ncl^+A)$.
Hence A is a neutrosophic b^+-closed sets.
(iv) (v) and (vi) Proof is obvious from above Definition.

Remark 3.11. From above the following implication and none of these implications is reversible as shown by examples given below



Example 3.12. Let X={x}
P={<0.5, 0.6, 0.4>}
Q={<0.4, 0.5, 0.8>}
τ ={1_N, 0_N, P, Q}, B={<0.3, 0.2, 0.9>}
τ^+ ={1_N, 0_N, P, Q,B} is simple extension neutrosophic topological spaces
A={<0.1, 0.3, 0.8>}
Then the set A is $neutrosophic^+$ open set but not neutrosophic $regular^+$ open set. Since A = $Ncl^+(Nint(A)) \neq 0_N$.

Example 3.13. Let X={x}
P={<0.3, 0.5, 0.8>}
Q={<0.4, 0.6, 0.7>}
τ ={1_N, 0_N, P, Q}, B={<0.3, 0.2, 0.9>}
τ^+ ={1_N, 0_N, P, Q,B} is simple extension neutrosophic topological spaces
A={<0.1, 0.3, 0.5>}
Then the set A is neutrosophic b^+ open set $A \subseteq Ncl^+(Nint(A)) \cup Nint(Ncl^+(A)) \subseteq 1_N$. but not neutrosophic $semi^+$ open set. Since $A \not\subseteq Nintcl^+ (N\ cl^+ (A)) \not\subseteq 0_N$.

Example 3.14. Let X={x}
P={<0.5, 0.6, 0.5>}
Q={<0.4, 0.7, 0.8>}
R={<0.5, 0.7, 0.5>}
S={<0.4, 0.6, 0.8>}
τ ={1_N, 0_N, P, Q, R, S}, B={<0.3, 0.2, 0.9>}
τ^+ ={1_N, 0_N, P, Q, R, S, B} is simple extension neutrosophic topological spaces
A={<0.6, 0.4, 0.5>}
Then the set A is neutrosophic b^+ open set but not neutrosophic pre^+ open set. Since $A \not\subseteq Nintcl^+ (N\ cl^+ (A)) \not\subseteq$ <0.5, 0.6, 0.5>

Theorem 3.15. Let A be a neutrosophic pre^+-open set in the simple extension neutrosophic topological space X and suppose $A \subseteq B \subseteq Ncl^+(A)$ then B is a neutrosophic pre^+-open set in X .

Proof: Let A be NO set in simple extension neutrosophic topological space X.
Then A=Nint(A)
Also $A \subseteq Ncl^+(A)$
$Nint(A) \subseteq Nint(Ncl^+(A))$
$A \subseteq Nint(Ncl^+(A))$
Hence the theorem is proved.

Lemma 3.16. Let A be an NO set in X and B a neutrosophic pre^+-open set in X then there exists an NO set G in X such that $B \subseteq G \subseteq Ncl^+(B)$ it follows that

$$A \cap B \subseteq A \cap G \subseteq A \cap Ncl^+(B) \subseteq Ncl^+(A \cap B)$$

Now, since $A \cap G$ is open, from the above (theorem 3.15) lemma, $A \cap B$ is a neutrosophic pre^+-open set in X .

Proposition 3.17. Let X and Y be two simple extended neutrosophic topological spaces such that X is a neutrosophic product related to Y then the neutrosophic product $A\times B$ of a neutrosophic pre$^+$ open set A of X and a neutrosophic pre$^+$ open set B of Y is a neutrosophic pre$^+$ open set of the neutrosophic product topological space $X\times Y$

Proof: Let $O_1 \subseteq A \subseteq Ncl^+(O_1)$ and

$$O_2 \subseteq B \subseteq Ncl^+(O_2)$$

$$O_1 \times O_2 \subseteq A \times B \subseteq Ncl^+(O_1) \times Ncl^+(O_2)$$

$$O_1 \times O_2 \subseteq A \times B \subseteq Ncl^+(O_1 \times O_2)$$

$$N\operatorname{int}(O_1 \times O_2) \subseteq N\operatorname{int}(A \times B) \subseteq N\operatorname{int}(Ncl^+(O_1 \times O_2))$$

$$O_1 \times O_2 \subseteq A \times B \subseteq Ncl^+(O_1 \times O_2)$$

Hence $A\times B$ is neutrosophic pre$^+$-open set in $X\times Y$.

Definition 3.18. Let (X, τ^+) be a SENTS. Then for a neutrosophic subset A of X, the neutrosophic pre$^+$ interior of A [NP$^+$Int (A) for short] is the union of all neutrosophic pre$^+$-open sets of X contained in A. That is, NP$^+$Int (A) = $\cup$ { G : G is a NP$^+$O set in X and G $\subseteq$ A}.

Theorem 3.19. Let (X, τ^+) be a SENTS. Then for any neutrosophic subsets A and B of a SENTS X we have

(i) NP$^+$int(A) $\subseteq$ A
(ii) A is NP$^+$O set in X $\Leftrightarrow$ NP$^+$int (A) = A
(iii) NP$^+$int (NP$^+$int (A)) = NP$^+$int (A)
(iv) If A $\subseteq$ B then NP$^+$int (A) $\subseteq$ NP$^+$int (B)

Proof : (i) follows from Definition 3.18.
(ii)Let A be NP$^+$O set in X.
Then A $\subseteq$ NP$^+$int (A).
By using (i) we get A = NP$^+$int (A).
Conversely assume that A = NP$^+$int (A).
By using Definition 3.18. A is NP$^+$O set in X.
Thus (ii) is proved.
(iii)By using (ii), NP$^+$int (NP$^+$int (A)) = NP$^+$int (A). This proves (iii).
(iv) Since A $\subseteq$ B, by using (i), NP$^+$int (A) $\subseteq$ A $\subseteq$ B.
That is NP$^+$int (A) $\subseteq$ B.
By (iii), NP$^+$int (NP$^+$int (A)) $\subseteq$ NP$^+$int (B). Thus NP$^+$int (A) $\subseteq$ NP$^+$int (B). This proves (iv).

Theorem 3.20. Let (X, τ^+) be a SENTS. Then for any neutrosophic subset A and B of a SENTS, we have

(i) NP$^+$int (A $\cap$ B) = NP$^+$int (A) $\cap$ NP$^+$int (B)
(ii) NP$^+$int (A $\cup$ B) $\supseteq$ NP$^+$int (A) $\cup$ NP$^+$int (B).

Proof : (i) Since A $\cap$ B $\subseteq$ A and A $\cap$ B $\subseteq$ B,
By using Theorem 3.19 (iv), NP$^+$int (A $\cap$ B) $\subseteq$ NP$^+$int (A) and
NP$^+$int (A $\cap$ B) $\subseteq$ NP$^+$int (B).
This implies that NP$^+$int (A $\cap$ B) $\subseteq$ NP$^+$int (A) $\cap$ NP$^+$int (B) -----(1).
By using Theorem 3.19 (i), NP$^+$int (A) $\subseteq$ A and NP$^+$int (B) $\subseteq$ B.
This implies that NP$^+$int (A) $\cap$ NP$^+$int (B) $\subseteq$ A $\cap$ B.
Now applying Theorem 3.19 (iv), NP$^+$int ((NP$^+$int (A) $\cap$ NP$^+$int (B)) $\subseteq$ NP$^+$int (A $\cap$ B).
By (1), NP$^+$int (NP$^+$int (A)) $\cap$ NP$^+$int (NP$^+$int (B)) $\subseteq$ NP$^+$int (A $\cap$ B).
By Theorem 3.19 (iii), NP$^+$int (A) $\cap$ NP$^+$int (B) $\subseteq$ NP$^+$int (A $\cap$ B) -----(2).
From (1) and (2), NP$^+$int (A $\cap$ B) = NP$^+$int (A) $\cap$ NP$^+$int (B).
This implies (i).
(ii) Since A $\subseteq$ A $\cup$ B and B $\subseteq$ A $\cup$ B,
By using Theorem 3.19 (iv), NP$^+$int (A$\subseteq$ NP$^+$int (A $\cup$ B) and
NP$^+$int (B) $\subseteq$ NP$^+$int (A $\cup$ B).
This implies that NP$^+$int (A) $\cup$ NP$^+$int (B) $\subseteq$ NP$^+$int (A $\cup$ B). Hence (ii).

4. NEUTROSOPHIC PRE$^+$-CLOSED

Definition 4.1. Let A be neutrosophic set of a simple extension neutrosophic topology spaces X . Then A is said to be neutrosophic pre$^+$-closed sets of X if there exists a NC set such that Ncl$^+$(NC) $\subseteq$A$\subseteq$NC.

Theorem 4.2. A subset A in a simple extension neutrosophic topological spaces X is neutrosophic pre$^+$ closed set if and only if Ncl$^+$(Nint(A)) $\subseteq$A.
Proof: Consider Ncl$^+$(Nint(A)) $\subseteq$A.
Then NC = Ncl$^+$(A) clearly Ncl$^+$(Nint(A)) $\subseteq$A$\subseteq$NC
Therefore A is NP$^+$C set
Conversely, suppose that Let A be NP$^+$C set in X
Then NC(Nint(A)) $\subseteq$A$\subseteq$NC for some NS closed set NC .but Ncl$^+$(A) $\subseteq$NC
Hence the theorem is proved.

Theorem 4.3. Let (X, τ^+) be simple extension neutrosophic topological space and A be a neutrosophic subset of X then A is a neutrosophic pre$^+$-closed sets if and only C(A) is neutrosophic pre$^+$-open set in X.
Proof: Let A be a neutrosophic pre-closed set subset of X.
Clearly Ncl$^+$(Nint(A)) $\subseteq$A.
Taking complement on both sides
C(A)$\subseteq$C(Ncl$^+$(Nint(A)))
C(A)$\subseteq$Nint(Ncl$^+$(C(A)))
Hence C(A) is a neutrosophic pre$^+$-open set
Conversely suppose that C(A) is a neutrosophic pre$^+$-open set i.e. C(A)$\subseteq$Nint(Ncl$^+$(C(A)))
Taking complement on both sides we get Ncl$^+$(Nint(A)) $\subseteq$A, A is a neutrosophic pre$^+$-closed set
Hence the theorem is proved.

Theorem 4.4. Let (X, τ^+) be a simple extension neutrosophic topological spaces. Then intersection of two neutrosophic pre$^+$-closed set is also a neutrosophic pre$^+$-closed set.
Proof: Let A and B are neutrosophic pre$^+$-closed sets on (X, τ^+)
Then Ncl$^+$(Nint(A)) $\subseteq$A, Ncl$^+$(Nint(B)) $\subseteq$B.

Consider

$$A \cap B \supseteq Ncl^{+}(N\operatorname{int}(A)) \cap Ncl^{+}(N\operatorname{int}(B))$$
$$\supseteq Ncl^{+}(N\operatorname{int}(A) \cap N\operatorname{int}(B))$$
$$\supseteq Ncl^{+}(N\operatorname{int}(A \cap B))$$

$Ncl^{+}(Nint(A \cap B)) \subseteq A \cap B$

Hence $A \cap B$ is a neutrosophic pre^{+}-closed set.

Remark 4.5. The union of any two neutrosophic pre^{+}-closed sets need not be a neutrosophic pre^{+}-closed set on (X, τ^{+}).

Theorem 4.6. Let $\{A_{\alpha}\}_{\alpha\in\Delta}$ be a collection of neutrosophic pre^{+}-closed sets on (X, τ^{+}).

then $\cap_{\alpha\in\Delta} A_{\alpha}$ is a neutrosophic pre^{+}-closed on (X, τ^{+}).

.Proof: We have a neutrosophic set NC_{α} such that $NC_{\alpha}(Nint(A)) \subseteq A_{\alpha} \subseteq NC_{\alpha}$ for all $\alpha \in \Delta$ Then

$$\cap_{\alpha\in\Delta} NC_{\alpha}(N\operatorname{int}(A)) \subseteq \cap_{\alpha\in\Delta} A_{\alpha} \subseteq \cap_{\alpha\in\Delta} NC_{\alpha}$$
$$\cap_{\alpha\in\Delta} Ncl_{\alpha}(N\operatorname{int}(A)) \subseteq \cap_{\alpha\in\Delta} A_{\alpha}$$

Hence $\cap_{\alpha\in\Delta} A_{\alpha}$ is a neutrosophic pre^{+}-closed sets on (X, τ^{+}).

Theorem 4.7. Every neutrosophic closed set in the simple extension neutrosophic topological spaces (X, τ^{+}) is neutrosophic pre^{+}-closed set in (X, τ^{+}).

Proof: Let A be simple extension neutrosophic closed set means $A=Ncl^{+}(A)$ and also $Nint(A) \subseteq A$

From that, $Ncl^{+}(Nint(A)) \subseteq Ncl^{+}(A)$, $Ncl^{+}(Nint(A)) \subseteq A$, Since $A=Ncl^{+}(A)$

Hence A is a neutrosophic pre^{+}-closed sets.

Theorem 4.8. Let A be a neutrosophic closed set in simple extension neutrosophic topological spaces (X, τ^{+}) and suppose $Nint(A) \subseteq B \subseteq A$ then B is neutrosophic pre^{+}-closed set on (X, τ^{+}).

Proof: Let A be a neutrosophic set in neutrosophic topological spaces (X, τ^{+})

Suppose $Nint(A) \subseteq B \subseteq A$

There exist a neutrosophic closed set NC, such that $NC(Nint(A)) \subseteq B \subseteq A \subseteq NC$.

Then $B \subseteq NC$ and also $Nint(B) \subseteq B \subseteq NC$

Thus, $Ncl^{+}(Nint(B)) \subseteq B$

Hence B is neutrosophic pre^{+}-closed set on (X, τ^{+}).

Theorem 4.9. Let X and Y are simple extension neutrosophic topological space such that X is neutrosophic product related to Y then the neutrosophic product $A \times B$ is a neutrosophic pre^{+}-closed set of the neutrosophic product topological space $X \times Y$. Where neutrosophic pre^{+}-closed set A of X and a neutrosophic pre^{+}-closed set B of Y.

Proof: Let A and B are neutrosophic pre^{+}-closed set.

$C_1(Nint(A)) \subseteq A \subseteq C_1$ and $C_2(Nint(B)) \subseteq B \subseteq C_2$

From the above,

$C_1(Nint(A)) \times C_2(Nint(B)) \subseteq A \times B \subseteq C_1 \times C_2$

$(C_1 \times C_2)(Nint(A \times B)) \subseteq A \times B \subseteq C_1 \times C_2$

Hence $A \times B$ is neutrosophic pre^{+}-closed set in simple extension neutrosophic topological space $X \times Y$.

Definition 4.10. Let (X, τ^{+}) be a SENTS. Then for a neutrosophic subset A of X, the neutrosophic pre^{+}-closure of A [$NPcl^{+}(A)$ for short] is the intersection of all neutrosophic pre^{+}-closed sets of X contained in A. That is, $NPcl^{+}(A) = \cap\{ K : K$ is a $NP^{+}C$ set in X and $K \supseteq A \}$.

Theorem 4.11. Let (X, τ^{+}) be a SENTS. Then for any simple extension neutrosophic subsets A of X,

(i) $C(NP^{+}int(A)) = NP^{+}cl(C(A))$,

(ii) $C(NP^{+}cl(A)) = NP^{+}int(C(A))$.

Proof : By using Definition 3.18, $NP^{+}int(A) = \cup\{ G : G$ is a $NP^{+}O$ set in X and $G \subseteq A \}$.

Taking complement on both sides,

$C(NP^{+}int(A)) = C(\cup\{ G : G$ is a $NP^{+}O$ set in X and $G \subseteq A \})$

$C(NP^{+}int(A)) = \cap\{ C(G) : C(G)$ is a $NP^{+}C$ set in X and $C(A) \subseteq C(G) \}$.

Replacing C(G) by K, we get $C(NP^{+}int(A)) = \cap\{ K : K$ is a $NP^{+}C$ set in X and $K \supseteq C(A) \}$.

By Definition 4.10, $C(NP^{+}int(A)) = NP^{+}cl(C(A))$.

This proves (i). By using (i), $C(NP^{+}int(C(A))) = NP^{+}cl(C(C(A))) = NP^{+}cl(A)$.

Taking complement on both sides, we get $NP^{+}int(C(A)) = C(NP^{+}cl(A))$. Hence proved (ii).

Theorem 4.12. Let (X, τ^{+}) be a SENTS. Then for any simple extension neutrosophic subsets A and B of a SENTS X we have

(i) $A \subseteq NP^{+}cl(A)$

(ii) A is $NP^{+}C$ set in X $\Leftrightarrow NP^{+}cl(A) = A$

(iii) $NP^{+}cl(NP^{+}cl(A)) = NP^{+}cl(A)$

(iv) If $A \subseteq B$ then $NP^{+}cl(A) \subseteq NP^{+}cl(B)$

Proof : (i) follows from Definition 4.10.

(ii)Let A be $NP^{+}C$ set in X.

By using Theorem 4.3, C(A) is $NP^{+}O$ set in X.

By Theorem 4.11 (ii), $NP^{+}int(C(A)) = C(A) \Leftrightarrow C(NP^{+}cl(A)) = C(A) \Leftrightarrow NP^{+}cl(A) = A$.

Thus proved (ii).

(iii)By using (ii), $NP^{+}cl(NP^{+}cl(A)) = NP^{+}cl(A)$.This proves (iii).

(iv)Since $A \subseteq B$, $C(B) \subseteq C(A)$.

By using Theorem 3.19 (iv), $NP^{+}int(C(B)) \subseteq NP^{+}int(C(A))$.

Taking complement on both sides, $C(NP^{+}int(C(B))) \supseteq C(NP^{+}int(C(A)))$.

By Theorem 4.11 (ii), $NP^{+}cl(A) \subseteq NP^{+}cl(B)$. This proves (iv).

Theorem 4.13. Let (X, τ^{+}) be a SENTS. Then for a simple extension neutrosophic subset A and B of a SENTS X, we have

(i) $NP^{+}cl(A \cup B) = NP^{+}cl(A) \cup NP^{+}cl(B)$ and

(ii) $NP^+ cl(A \cap B) \subseteq NP^+ cl(A) \cap NP^+cl (B)$.

Proof : Since $NP^+cl (A \cup B) = NP^+cl (C(C (A \cup B)))$,
By using Theorem 4.11 (i), $NP^+cl (A \cup B) = C(NP^+int (C(A \cup B)))$
$NP^+cl (A \cup B) = C (NP^+int (C (A) \cup C (B)))$.
Again using Theorem 3.20 (i), $NP^+cl (A \cup B) = C(NP^+int (C (A)) \cap NP^+int (C(B)))$
$NP^+cl (A \cup B) = C(NP^+int(C(A))) \cup C(NP^+int(C (B)))$.
By using Theorem 4.11 (i), $NP^+cl (A \cup B) = NP^+cl(C(C (A))) \cup NP^+cl (C (C (B)))$
$NP^+cl (A \cup B) = NP^+cl (A) \cup NP^+cl (B)$. Thus proved (i).
(ii)Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$,
By using Theorem 4.12 (iv), $NS\ Cl (A \cap B) \subseteq NP^+cl (A)$ and $NP^+cl (A \cap B) \subseteq NP^+cl (B)$.
This implies that $NP^+cl (A \cap B) \subseteq NP^+cl (A) \cap NP^+cl (B)$. This proves(ii).

5. CONCLUSION

With the induction of the above definitions in simple extension neutrosophic topological spaces. We can extend its scope by generalizing this new concept. This would open new avenues of research in the existing neutrosophic topological setting.

REFERENCES

[1] K. Atanassov, Intuitionistic fuzzy sets, in V.Sgurev, ed., VII ITKRS session, sofia (June 1983 central Sci. and Tech. Library, Bulg, Academy of sciences (1984)).
[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems 20(1986) 81-86.
[3] K.Atanassov, Review and new result on intuitionistic fuzzy sets, preprint Im-MFAIS-1-88, Sofia, 1988.
[4] C.L. Chang, Fuzzy Topological Spaces, J. Math. Anal Appl.,24 (1968), 182-1 90.
[5] Dogan Coker, An introduction to Intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88, 81-89 (1997).
[6] P.Ishwarya and K.Bageerathi, On Neutrosophic semi-open sets in Neutrosophic topological spaces, International Journal of Mathematics Trends and Technology, Vol 37(3), 214-223(2016).
[7] B. Kanchana and F. Nirmala Irudayam, " A new class of contra continuous Functions via b-open sets in SETS", International Journal of Science and Research, Vol.5, Issue.8, Aug 2016, 1240-1245.
[8] N. Levine , "Simple extension of topology", Amer.math.Monthly, 71 (1964),22-105.
[9] T. Madhumathi and F.Nirmala Irudayam, "On Ω_{gb}^+-closed sets in simple extension topologicalspaces,"ElixirAppl.Math.105(2017) ,46305-46307.
[10] F. Nirmala Irudayam and Sr.I Arokiarani, "A note on the weaker form of bI set and its generalization in SEITS", International Journal of Computer application, Issue 2, vol.4(Aug 2012),42-54.
[11] Noiri T., Arockiarani Sr.I., and Nirmala Irudayam F., "On Some Topologies Induced By BI^+ Open Sets in Simple Extension Ideal Topological Spaces", International Journal of Mathematical Archive-4(1), 2013, 30-135.
[12] Florentin Smarandache, neutrosophy and neutrosophic Logic, First International conference on neutrosophy, Neutrosophic Logic, set, probability and statistics university of New mexico, Gallup NM 87301, USA(2002).
[13] F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Research Press, Rehoboth, NM (1999).
[14] A.A. Salama, S.A. AL-Blowi, Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Math., 3 (2012), 31-35.
[15] A. A. Salama, S. A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, Comp. Sci. Engg., 2 (2012), 129-132.
[16] V. Venkateswara Rao and Y. Srinivasa Rao, Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology, International Journal of ChemTech Research, Vol.10 No.10, pp: 449-458, 2017.
[17] L.A. Zadeh, Fuzzy sets, Information and control 8(1965) 338-353.