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S Control Chart For Specified Values Of Cpu

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Abstract- Control Charts are the tools used in assessment of statistical control of the process. The capability indices are used to evaluate the capability of the process to meet specification. The basic process capability indices are Cp, Cpk, Cpm and Cpmk. The process capability indices are affected by the distribution of the process. In this paper, S control charts for any specified values of capability index Cpu is presented. Normally distributed data is used to study the proposed control chart.

Keywords- Capability index; Control chart; Normal Distribution;

1. INTRODUCTION

Control charts are the basic tools for measuring process performance. W. A. Shewhart initialized the concept of statistical process control. Assignable causes of variation are due to non-random causes and are rectifiable. A typical control chart consists of three horizontal lines. They are, a central line (CL), Upper control limit (UCL), Lower control limit (LCL). The S chart is used to monitor the variability of a process. Control chart is an on-line process monitoring technique widely used to detect the occurrence of assignable causes.

Chen et. al [1], constructed the control chart of unilateral specification index Cpl and Cpu to monitor the stability of process and process capability. Subramani et. al [2] proposed $\bar{X}$ and R control chart based on process capability indices Cp and Cpk. Liaquat Ahmad et. al [3] proposed a $\bar{X}$ control chart based on process capability index (Cp) using repetitive sampling and the performance of the proposed chart was evaluated by ARL_1 and the performance is efficient for the quick detection of false alarms. Muhammad Aslam et. al [4], presented a mixed control chart with attribute and variable data using the process capability index Cpk. O.A. Adeoti et. al [5], proposed a process capability index based control chart using Downton estimator with a specified Cp value. O.A. Adeoti et. al [6], presented a new process capability index control chart for monitoring the process mean using repetitive sampling. Souha Ben Amara et. al [7], investigates the effects of measurement system variability evaluated by measurement system discrimination ratio (DR) on the process capability indices Cp and Cpm, on the expected non-conforming units of product per million(ppm), on the expected mean value of the Taguchi loss function (E(Loss) and on the Shewhart charts properties.

2. BASIC PROCESS CAPABILITY INDICES

The capability index Cp is used to measure the capability of process dispersion in achieving the dispersion of the customer specification and it is given by,

$$C_P = \frac{USL - LSL}{6\sigma}$$

By replacing σ with the unbiased estimator $\hat{\sigma} = \frac{\bar{s}}{c_4}$

$$\hat{C}_P = \frac{USL - LSL}{6\frac{\bar{s}}{c_4}} \qquad (1)$$

Kane [8] discussed the index Cpk, which attempts to measure the variability and shift in process mean simultaneously.

$$C_{Pk} = \min(C_{Pu}, C_{Pl})$$

$$C_{Pu} = \frac{USL - \bar{\bar{X}}}{3\frac{\bar{s}}{c_4}} \qquad (2)$$

$$C_{Pl} = \frac{\bar{\bar{X}} - \text{LSL}}{3\frac{\bar{s}}{c_4}}$$

1.1. S Control Chart for Specified values of Cpu:

In this paper, S – control chart for any Specified value of Cp is discussed. According to Montgomery [9], the control limits for Shewhart $\bar{X}$ and S control chart by replacing σ with the unbiased estimator is given below.

Control Limits of Shewhart S chart:

$$U\,C\,L = \bar{s} + 3\left(\frac{\bar{s}}{C_4}\right)\sqrt{1 - {C_4}^2}$$
$$= B_4\bar{s} \qquad (5)$$

$$Centrl\ Line = \bar{s} \qquad (6)$$

$$LCL = \bar{s} - 3\left(\frac{\bar{s}}{C_4}\right)\sqrt{1 - {C_4}^2}$$
$$= B_3\bar{s} \qquad (7)$$

Where,

$$\bar{s} = \frac{\sum_{i=1}^{m} S_i}{m}$$

$$s_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(X_{ij} - \bar{X}_i\right)^2}$$

$$c_4 = \left(\frac{2}{n-1}\right)^{1/2}\left(\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n-1}{2}\right)}\right)$$

The standard deviation for any specified value of Cpu is:

$$\bar{s} = \left(\frac{c_4(USL-\mu_{obs})}{3 * \mathrm{Cpu}_{\mathrm{exp}}}\right) \qquad (8)$$

The control limits of the proposed modified S chart for any specified value of capability index Cpu is derived by substituting (8) in (5), (6) and (7)

2.2. Operating Procedure:

Step-1: Calculate the average of subgroup standard deviation and calculate the estimate of population standard deviation.

Step-2: Calculate Cpu(observed) using (2).

Step-3: Calculate the control limits of the proposed S chart by substituting (8) in (5), (6) and (7) using Cpu(expected).

Step-4: Eliminate the point's lies above and below the UCL and LCL respectively.

3. ILLUSTRATION

The control chart proposed is illustrated using a simulated normally distributed data with mean μ = 20 and standard deviation σ = 1. The following parameters, USL = 22 and LSL = 16 are considered for illustrating the proposed capability index based control chart. The parameters calculated from the simulated data are as follows:

$\bar{\bar{X}} = 20.05672$; $\bar{s} = 1.021505$; Cp=1.00, Cpu = 0.65 ; Cpl = 1.32 and Cpk = 0.65

From the above result it is observed that the process average is 20.05672 with standard deviation of 1.021505. The control limits of Shewhart $\bar{X}$ and S control chart are calculated from (5), (6) and (7). From figure 1, since all the points in Shewhart $\bar{X}$ and S control chart are within the control limits, it is inferred that the process is statistically control. But as the Cp value is 1.00, it is clear that the process spread and specification spread are equal. Also the values of the indices Cpu, Cpl and Cpk are 0.65, 1.32 and 0.65 respectively. Since the Cp value is not equal to Cpk it is clear that the process mean has shifted and also since the Cpk is 0.65, the process is not capable to meet specification. But from figure 2, the capability analysis output, it is clear that the process is not capable in meeting specifications. Hence even the process is statistically control (figure1), from the capability analysis it is evident that the process is not capable of meeting customer specification (figure 2). Thus the capability index based control chart can be used to reduce variation so as to converge the capability of the process towards meeting customer specification. In these type of control charts the Cpu is fixed in prior and named as Cpu(expected). In illustration the Cpu is 0.65, By fixing Cpu(expected) = 1.00, the control limits of the proposed control chart is computed by substituting (8) in (5), (6) and (7).

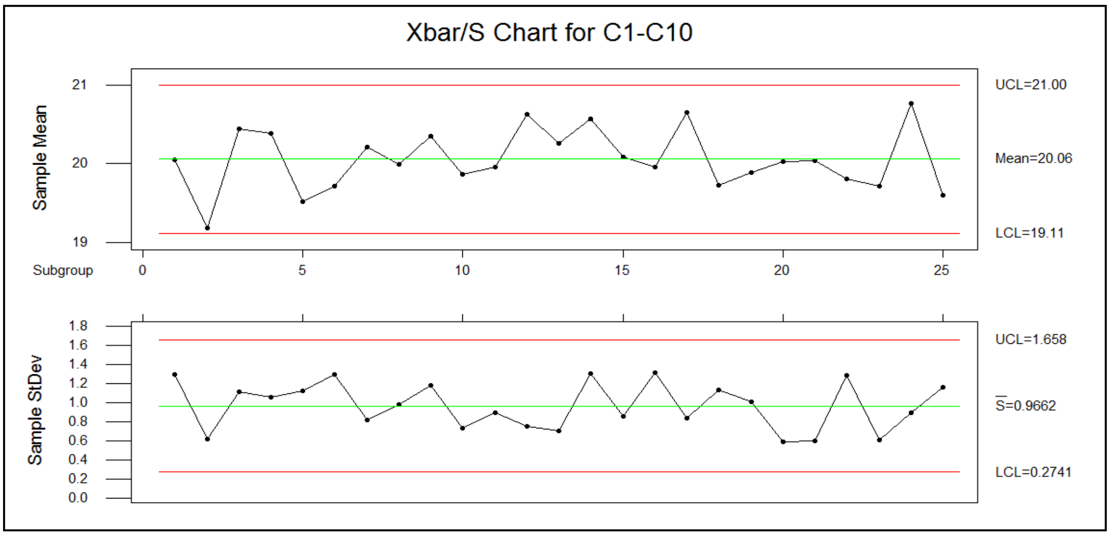

Figure 1 Shewhart $\bar{X}$ and S control chart

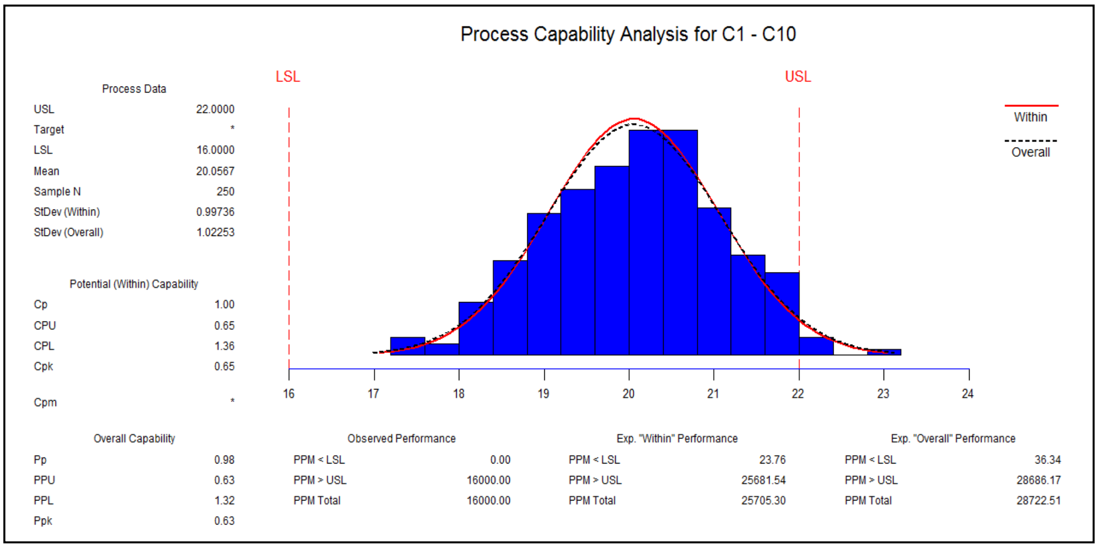

Figure 2 Process capability indices Values

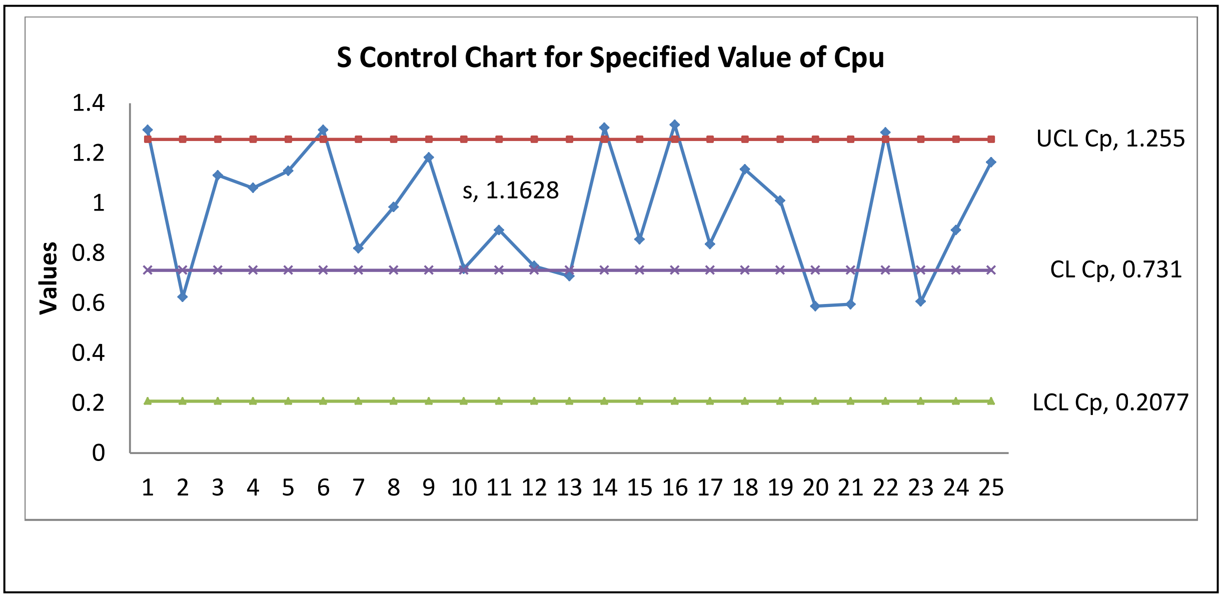

Figure 3 S Control Chart for Specified Value of Cpu

In figure 3, the S control chart for specified value of Cpu=1.00 is presented. It is clear that five points (1, 6,14, 16 and 22) lies outside the control limits. Since the upper control limit of Shewhart S control chart is 1.658 and the counterpart of the S control chart based on Cpu is 1.255, it is clear that the upper control limit of the proposed control chart is less than the control limits of Shewhart control chart. Also Since the lower control limit of Shewhart S control chart is 0.2741 and the counterpart of the S control chart based on Cpu is 0.2077, the lower control limit of the proposed control chart chart is greater than the control limits of Shewhart control chart. Thus the width control limits of the proposed control have reduced as compared with the limits of Shewhart S control chart. Thus this control chart can be used for reducing variation.

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