

A New Application of Shehu Transform for Handling Volterra Integral Equations of First Kind

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Abstract: Many problems of thermodynamics, nuclear reactor theory, chemotherapy and electrical systems have been described in the form of Volterra integral equations. In this paper, we have given a new application of Shehu transform for handling Volterra integral equations of first kind. In application section of this paper, some numerical applications are given to explain the importance of Shehu transform for handling Volterra integral equations of first kind. The results show that Shehu transform is a very useful integral transform for handling Volterra integral equations of first kind.

Keywords: Volterra integral equation of first kind, Shehu transform, Convolution theorem, Inverse Shehu transform.

1. INTRODUCTION

In the advance time, integral transforms methods (Laplace transform [1-2], Fourier transform [1], Kamal transform [3-9, 36], Mahgoub transform [10-16], Mohand transform [17-20, 37-40], Aboodh transform [21-26, 41-44], Elzaki transform [27-29, 45-46], Sumudu transform [30, 47-48] and Shehu transform [49-50]) are convenient mathematical methods for solving advance problems of engineering and sciences which are mathematically expressed in terms of differential equations, system of differential equations, partial differential equations, integral equations, system of integral equations, partial integro-differential equations and integro differential equations. Recently the comparative study of Mohand and other integral transforms was discussed by Aggarwal et al. [31-35]. Shehu transform of the function $F(t), t \geq 0$ is given by [49]:

$$S\{F(t)\} = \int_0^\infty F(t)e^{-\frac{vt}{u}} dt = H(v, u), v > 0, u > 0,$$

where operator S is called the Shehu transform operator. The Shehu transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. These mention two conditions are the only sufficient conditions for the existence of Shehu transform of the function $F(t)$.

The Volterra integral equation of first kind is given by [7, 14, 24, 27]

$$f(x) = \int_0^x k(x, t)g(t)dt \dots \dots \dots (1)$$

where the unknown function $g(x)$, that will be determined, occurs only inside the integral sign. The kernel $k(x, t)$ of integral equation (1) and the function $f(x)$ are known real-valued functions.

The aim of this work is to find out exact solutions of Volterra integral equation of first kind using the new and advance integral transform “Shehu transform” without large computational work.

2. LINEARITY PROPERTY OF SHEHU TRANSFORMS [49-50]

If $S\{F(t)\} = H_1(v, u)$ and $S\{G(t)\} = H_2(v, u)$ then $S\{aF(t) + bG(t)\} = aS\{F(t)\} + bS\{G(t)\}$
 $\Rightarrow S\{aF(t) + bG(t)\} = aH_1(v, u) + bH_2(v, u)$, where a, b are arbitrary constants.

3. SHEHU TRANSFORM OF SOME ELEMENTARY FUNCTIONS [49-50]

S.N.	$F(t)$	$S\{F(t)\} = H(v, u)$
1.	1	$\frac{u}{v}$
2.	t	$\left(\frac{u}{v}\right)^2$
3.	t^2	$2! \left(\frac{u}{v}\right)^3$
4.	$t^n, n \in N$	$n! \left(\frac{u}{v}\right)^{n+1}$
5.	$t^n, n > -1$	$\Gamma(n + 1) \left(\frac{u}{v}\right)^{n+1}$
6.	e^{at}	$\frac{u}{v - au}$
7.	$\sin at$	$\frac{au^2}{(v^2 + a^2u^2)}$
8.	$\cos at$	$\frac{uv}{(v^2 + a^2u^2)}$

9.	<i>sinh</i>	$\frac{au^2}{(v^2 - a^2u^2)}$
10.	<i>cosh</i>	$\frac{uv}{(v^2 - a^2u^2)}$
11.	$J_0(at)$	$\frac{u}{\sqrt{(v^2 + a^2u^2)}}$

4. SHEHU TRANSFORM OF THE DERIVATIVES OF THE FUNCTION $F(t)$ [49-50]:

If $S\{F(t)\} = H(v, u)$ then

- a) $S\{F'(t)\} = \frac{v}{u}H(v, u) - F(0)$
- b) $S\{F''(t)\} = \frac{v^2}{u^2}H(v, u) - \frac{v}{u}F(0) - F'(0)$
- c) $S\{F^{(n)}(t)\} = \frac{v^n}{u^n}H(v, u) - \sum_{k=0}^{n-1} \left(\frac{v}{u}\right)^{n-(k+1)} F^{(k)}(0).$

5. SHEHU TRANSFORM OF BESSEL FUNCTION OF ORDER ONE

Bessel function of order one is denoted by $J_1(t)$ and it is given by the following relation

$$\frac{d}{dt}J_0(t) = -J_1(t) \dots \dots \dots (2)$$

where $J_0(t)$ is the Bessel function of order zero.

Operating Shehu transform on both sides of (2), we get

$$S\left\{\frac{d}{dt}J_0(t)\right\} = -S\{J_1(t)\} \dots \dots \dots (3)$$

Applying the property, Shehu transform of derivative of function, in (3), we have

$$\begin{aligned} \Rightarrow \frac{v}{u}S\{J_0(t)\} - J_0(0) &= -S\{J_1(t)\} \\ \Rightarrow S\{J_1(t)\} &= 1 - \frac{v}{u}S\{J_0(t)\} \\ \Rightarrow S\{J_1(t)\} &= 1 - \frac{v}{u} \left[\frac{u}{\sqrt{(v^2 + u^2)}} \right] \\ \Rightarrow S\{J_1(t)\} &= 1 - \frac{v}{\sqrt{(v^2 + u^2)}} \end{aligned}$$

6. CONVOLUTION PROPERTY OF SHEHU TRANSFORMS

If Shehu transform of functions $F_1(t)$ and $F_2(t)$ are $H_1(v, u)$ and $H_2(v, u)$ respectively then Shehu transform of their convolution $F_1(t) * F_2(t)$ is given by

$$\begin{aligned} S\{F_1(t) * F_2(t)\} &= S\{F_1(t)\}S\{F_2(t)\} \\ \Rightarrow S\{F_1(t) * F_2(t)\} &= H_1(v, u)H_2(v, u), \end{aligned}$$

where $F_1(t) * F_2(t)$ is defined by

$$\begin{aligned} F_1(t) * F_2(t) &= \int_0^t F_1(t-x) F_2(x) dx \\ &= \int_0^t F_1(x) F_2(t-x) dx. \end{aligned}$$

Proof: By the definition of Shehu transform, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty e^{-\frac{vt}{u}} [F_1(t) * F_2(t)] dt$$

$$\Rightarrow S\{F_1(t) * F_2(t)\}$$

$$= \int_0^\infty e^{-\frac{vt}{u}} \left[\int_0^t F_1(t-x) F_2(x) dx \right] dt$$

By changing the order of integration, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x) \left[\int_x^\infty e^{-\frac{vt}{u}} F_1(t-x) dt \right] dx$$

Put $t-x = p$ so that $dt = dp$ in above equation, we have

$$S\{F_1(t) * F_2(t)\} = \int_0^\infty F_2(x) \left[\int_0^\infty e^{-\frac{v(p+x)}{u}} F_1(p) dp \right] dx$$

$$\Rightarrow S\{F_1(t) * F_2(t)\}$$

$$= \int_0^\infty F_2(x) e^{-\frac{xv}{u}} \left[\int_0^\infty e^{-\frac{pv}{u}} F_1(p) dp \right] dx$$

$$\Rightarrow S\{F_1(t) * F_2(t)\} = H_1(v, u)H_2(v, u).$$

7. INVERSE SHEHU TRANSFORM

If $S\{F(t)\} = H(v, u)$ then $F(t)$ is called the inverse Shehu transform of $H(v, u)$ and mathematically, it is defined as $F(t) = S^{-1}\{H(v, u)\}$, where the operator S^{-1} is called the inverse Shehu transform operator.

8. LINEARITY PROPERTY OF INVERSE SHEHU TRANSFORMS [50]

If $S^{-1}\{H_1(v, u)\} = F(t)$ and $S^{-1}\{H_2(v, u)\} = G(t)$ then

$$\begin{aligned} S^{-1}\{a H_1(v, u) + b H_2(v, u)\} \\ = aS^{-1}\{H_1(v, u)\} + bS^{-1}\{H_2(v, u)\} \\ \Rightarrow S^{-1}\{a H_1(v, u) + b H_2(v, u)\} = aF(t) + bG(t), \end{aligned}$$

where a, b are arbitrary constants.

9. INVERSE SHEHU TRANSFORM OF SOME ELEMENTARY FUNCTIONS [50]

S.N.	$H(v, u)$	$F(t) = S^{-1}\{H(v, u)\}$
1.	$\frac{u}{v}$	1
2.	$\left(\frac{u}{v}\right)^2$	t
3.	$\left(\frac{u}{v}\right)^3$	$\frac{t^2}{2!}$
4.	$\left(\frac{u}{v}\right)^{n+1}, n \in N$	$\frac{t^n}{n!}$
5.	$\left(\frac{u}{v}\right)^{n+1}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{u}{v-au}$	e^{at}
7.	$\frac{u^2}{(v^2 + a^2u^2)}$	$\frac{\sin at}{a}$
8.	$\frac{uv}{(v^2 + a^2u^2)}$	$\cos at$

9.	$\frac{u^2}{(v^2 - a^2u^2)}$	$\frac{\sinh at}{a}$
10.	$\frac{uv}{(v^2 - a^2u^2)}$	$\cosh at$
11.	$\frac{u}{\sqrt{(v^2 + a^2u^2)}}$	$J_0(at)$

10. SHEHU TRANSFORMS FOR HANDLING VOLTERRA INTEGRAL EQUATIONS OF FIRST KIND

In this paper, we will consider that the kernel $k(x, t)$ of integral equation (1) is a convolution type kernel that can be expressed by the difference $(x - t)$ so (1) can be expressed as

$$f(x) = \int_0^x k(x - t)g(t)dt \dots \dots \dots (4)$$

Applying the Shehu transform to both sides of (4), we have

$$S\{f(x)\} = S\left\{\int_0^x k(x - t)g(t)dt\right\} \dots \dots \dots (5)$$

Using convolution theorem of Shehu transform, we have

$$S\{f(x)\} = S\{k(x)\}S\{g(x)\} \\ \Rightarrow S\{g(x)\} = \frac{S\{f(x)\}}{S\{k(x)\}} \dots \dots \dots (6)$$

Operating inverse Shehu transform on both sides of (6), we have

$$g(x) = S^{-1}\left\{\frac{S\{f(x)\}}{S\{k(x)\}}\right\} \dots \dots \dots (7)$$

which is the required solution of (4).

11. APPLICATIONS

In this section, some numerical applications are given in order to demonstrate the importance of Shehu transform for handling Volterra integral equations of first kind.

Application: 1 Consider Volterra integral equation of first kind

$$x = \int_0^x e^{(x-t)} g(t)dt \dots \dots \dots (8)$$

Applying the Shehu transform to both sides of (8), we have

$$S\{x\} = S\left\{\int_0^x e^{(x-t)} g(t)dt\right\} \dots \dots \dots (9)$$

Using convolution theorem of Shehu transform on (9), we have

$$\left(\frac{u}{v}\right)^2 = S\{e^x\}S\{g(x)\} \\ \Rightarrow \left(\frac{u}{v}\right)^2 = \left[\frac{u}{v-u}\right]S\{g(x)\} \\ \Rightarrow S\{g(x)\} = \frac{u}{v} - \left(\frac{u}{v}\right)^2 \dots \dots \dots (10)$$

Operating inverse Shehu transform on both sides of (10), we have

$$g(x) = S^{-1}\left\{\frac{u}{v} - \left(\frac{u}{v}\right)^2\right\} = S^{-1}\left\{\frac{u}{v}\right\} - S^{-1}\left\{\left(\frac{u}{v}\right)^2\right\} \\ \Rightarrow g(x) = 1 - x \dots \dots \dots (11)$$

which is the required exact solution of (8).

Application: 2 Consider Volterra integral equation of first kind

$$\sin x = \int_0^x e^{(x-t)} g(t) dt \dots \dots \dots (12)$$

Applying the Shehu transform to both sides of (12), we have

$$S\{\sin x\} = S\left\{\int_0^x e^{(x-t)} g(t) dt\right\} \dots \dots \dots (13)$$

Using convolution theorem of Shehu transform on (13), we have

$$\frac{u^2}{(v^2 + u^2)} = S\{e^x\}S\{g(x)\} \\ \Rightarrow \frac{u^2}{(v^2 + u^2)} = \left[\frac{u}{v-u}\right]S\{g(x)\} \\ \Rightarrow S\{g(x)\} = \frac{u(v-u)}{(v^2 + u^2)} \\ = \frac{uv}{(v^2 + u^2)} - \frac{u^2}{(v^2 + u^2)} \dots (14)$$

Operating inverse Shehu transform on both sides of (14), we have

$$g(x) = S^{-1}\left\{\frac{uv}{(v^2 + u^2)}\right\} - S^{-1}\left\{\frac{u^2}{(v^2 + u^2)}\right\} \\ \Rightarrow g(x) = \cos x - \sin x \dots \dots \dots (15)$$

which is the required exact solution of (12).

Application: 3 Consider Volterra integral equation of first kind

$$\sin x = \int_0^x J_0(x-t)g(t) dt \dots \dots \dots (16)$$

Applying the Shehu transform to both sides of (16), we have

$$S\{\sin x\} = S\left\{\int_0^x J_0(x-t)g(t) dt\right\} \dots \dots \dots (17)$$

Using convolution theorem of Shehu transform on (17), we have

$$\frac{u^2}{(v^2 + u^2)} = S\{J_0(x)\}S\{g(x)\} \\ \Rightarrow \frac{u^2}{(v^2 + u^2)} = \left[\frac{u}{\sqrt{(v^2 + u^2)}}\right]S\{g(x)\} \\ \Rightarrow S\{g(x)\} = \frac{u}{\sqrt{(v^2 + u^2)}} \dots \dots \dots (18)$$

Operating inverse Shehu transform on both sides of (18), we have

$$g(x) = S^{-1}\left\{\frac{u}{\sqrt{(v^2 + u^2)}}\right\} = J_0(x) \dots \dots \dots (19)$$

which is the required exact solution of (16).

Application: 4 Consider Volterra integral equation of first kind

$$x^2 = \frac{1}{2} \int_0^x (x-t)g(t) dt \dots \dots \dots (20)$$

Applying the Shehu transform to both sides of (20), we have

$$S\{x^2\} = \frac{1}{2} S \left\{ \int_0^x (x-t)g(t) dt \right\} \dots \dots \dots (21)$$

Using convolution theorem of Shehu transform on(21), we have

$$\begin{aligned} 2! \left(\frac{u}{v}\right)^3 &= \frac{1}{2} S\{x\}S\{g(x)\} \\ \Rightarrow 2! \left(\frac{u}{v}\right)^3 &= \frac{1}{2} \left(\frac{u}{v}\right)^2 S\{g(x)\} \\ \Rightarrow S\{g(x)\} &= 4 \left(\frac{u}{v}\right) \dots \dots \dots (22) \end{aligned}$$

Operating inverse Shehu transform on both sides of(22), we have

$$g(x) = 4S^{-1} \left\{ \left(\frac{u}{v}\right) \right\} = 4 \dots \dots \dots (23)$$

which is the required exact solution of (20).

Application: 5 Consider Volterra integral equation of first kind

$$x = \int_0^x e^{-(x-t)} g(t) dt \dots \dots \dots (24)$$

Applying the Shehu transform to both sides of (24), we have

$$S\{x\} = S \left\{ \int_0^x e^{-(x-t)} g(t) dt \right\} \dots \dots \dots (25)$$

Using convolution theorem of Shehu transform on (25), we have

$$\begin{aligned} \left(\frac{u}{v}\right)^2 &= S\{e^{-x}\}S\{g(x)\} \\ \Rightarrow \left(\frac{u}{v}\right)^2 &= \left[\frac{u}{v+u}\right] S\{g(x)\} \\ \Rightarrow S\{g(x)\} &= \frac{u}{v} + \left(\frac{u}{v}\right)^2 \dots \dots \dots (26) \end{aligned}$$

Operating inverse Shehu transform on both sides of (26), we have

$$\begin{aligned} g(x) &= S^{-1} \left\{ \frac{u}{v} + \left(\frac{u}{v}\right)^2 \right\} = S^{-1} \left\{ \frac{u}{v} \right\} + S^{-1} \left\{ \left(\frac{u}{v}\right)^2 \right\} \\ \Rightarrow g(x) &= 1 + x \dots \dots \dots (27) \end{aligned}$$

which is the required exact solution of (24).

Application: 6 Consider Volterra integral equation of first kind

$$\sin x = \int_0^x g(x-t) g(t) dt \dots \dots \dots (28)$$

Applying the Shehu transform to both sides of (28), we have

$$S\{\sin x\} = S \left\{ \int_0^x g(x-t) g(t) dt \right\} \dots \dots \dots (29)$$

Using convolution theorem of Shehu transform on (29), we have

$$\frac{u^2}{(v^2 + u^2)} = S\{g(x)\}S\{g(x)\}$$

$$\begin{aligned} \Rightarrow [S\{g(x)\}]^2 &= \frac{u^2}{(v^2 + u^2)} \\ \Rightarrow S\{g(x)\} &= \pm \frac{u}{\sqrt{(v^2 + u^2)}} \dots \dots \dots (30) \end{aligned}$$

Operating inverse Shehu transform on both sides of (30), we have

$$\begin{aligned} g(x) &= \pm S^{-1} \left\{ \frac{u}{\sqrt{(v^2 + u^2)}} \right\} \\ \Rightarrow g(x) &= \pm J_0(x) \dots \dots \dots (31) \end{aligned}$$

which is the required exact solution of (28).

Application: 7 Consider Volterra integral equation of first kind

$$x = \int_0^x g(t) dt \dots \dots \dots (32)$$

Applying the Shehu transform to both sides of (32), we have

$$S\{x\} = S \left\{ \int_0^x g(t) dt \right\} \dots \dots \dots (33)$$

Using convolution theorem of Shehu transform on (33), we have

$$\begin{aligned} \left(\frac{u}{v}\right)^2 &= S\{1\}S\{g(x)\} \\ \Rightarrow \left(\frac{u}{v}\right)^2 &= \left(\frac{u}{v}\right) S\{g(x)\} \\ \Rightarrow S\{g(x)\} &= \left(\frac{u}{v}\right) \dots \dots \dots (34) \end{aligned}$$

Operating inverse Shehu transform on both sides of (34), we have

$$g(x) = S^{-1} \left\{ \left(\frac{u}{v}\right) \right\} = 1 \dots \dots \dots (35)$$

which is the required exact solution of (32).

Application: 8 Consider Volterra integral equation of first kind

$$1 - J_0(x) = \int_0^x g(t) dt \dots \dots \dots (36)$$

Applying the Shehu transform to both sides of (36), we have

$$S\{1\} - S\{J_0(x)\} = S \left\{ \int_0^x g(t) dt \right\} \dots \dots \dots (37)$$

Using convolution theorem of Shehu transform on (37), we have

$$\begin{aligned} \left(\frac{u}{v}\right) - \frac{u}{\sqrt{(v^2 + u^2)}} &= S\{1\}S\{g(x)\} \\ \Rightarrow \left(\frac{u}{v}\right) - \frac{u}{\sqrt{(v^2 + u^2)}} &= \left(\frac{u}{v}\right) S\{g(x)\} \\ \Rightarrow S\{g(x)\} &= 1 - \frac{u}{\sqrt{(v^2 + u^2)}} \dots \dots \dots (38) \end{aligned}$$

Operating inverse Shehu transform on both sides of (38), we have

$$g(x) = S^{-1} \left\{ 1 - \frac{u}{\sqrt{(v^2 + u^2)}} \right\} = J_1(x) \dots \dots \dots (39)$$

which is the required exact solution of (36).

Application: 9 Consider Volterra integral equation of first kind

$$J_0(x) - \cos x = \int_0^x J_0(x-t)g(t) dt \dots \dots \dots (40)$$

Applying the Shehu transform to both sides of (40), we have

$$S\{J_0(x)\} - S\{\cos x\} = S\left\{\int_0^x J_0(x-t)g(t) dt\right\} \dots (41)$$

Using convolution theorem of Shehu transform on(41), we have

$$\begin{aligned} \frac{u}{\sqrt{(v^2+u^2)}} - \frac{uv}{(v^2+u^2)} &= S\{J_0(x)\}S\{g(x)\} \\ \Rightarrow \frac{u}{\sqrt{(v^2+u^2)}} - \frac{uv}{(v^2+u^2)} &= \frac{u}{\sqrt{(v^2+u^2)}}S\{g(x)\} \\ \Rightarrow S\{g(x)\} &= 1 - \frac{v}{\sqrt{(v^2+u^2)}} \dots \dots \dots (42) \end{aligned}$$

Operating inverse Shehu transform on both sides of (42), we have

$$g(x) = S^{-1}\left\{1 - \frac{v}{\sqrt{(v^2+u^2)}}\right\} = J_1(x) \dots \dots \dots (43)$$

which is the required exact solution of (40).

Application: 10 Consider Volterra integral equation of first kind

$$\cos x - J_0(x) = -\int_0^x J_1(x-t)g(t) dt \dots \dots \dots (44)$$

Applying the Shehu transform to both sides of (44), we have

$$S\{\cos x\} - S\{J_0(x)\} = -S\int_0^x J_1(x-t)g(t) dt \dots (45)$$

Using convolution theorem of Shehu transform on (45), we have

$$\begin{aligned} \frac{uv}{(v^2+u^2)} - \frac{u}{\sqrt{(v^2+u^2)}} &= -S\{J_1(x)\}S\{g(x)\} \\ \Rightarrow \frac{uv}{(v^2+u^2)} - \frac{u}{\sqrt{(v^2+u^2)}} &= -\left[1 - \frac{v}{\sqrt{(v^2+u^2)}}\right]S\{g(x)\} \\ \Rightarrow S\{g(x)\} &= \frac{u}{\sqrt{(v^2+u^2)}} \dots \dots \dots (46) \end{aligned}$$

Operating inverse Shehu transform on both sides of (46), we have

$$g(x) = S^{-1}\left\{\frac{u}{\sqrt{(v^2+u^2)}}\right\} = J_0(x) \dots \dots \dots (47)$$

which is the required exact solution of (44).

12. CONCLUSION

In this paper, we have successfully discussed the new application of Shehu transform for handling Volterra integral equations of first kind. The given numerical applications show that the exact solution of Volterra integral equations of first kind obtained in very less computational work and spending a very little time using

Shehu transform. In future, Shehu transform can be applied for solving other Volterra integral equations and their systems.

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