International Journal of Research in Advent Technology, Vol.7, No.4, April 2019
E-ISSN: 2321-9637
Available online at www.ijrat.org

System For Finding Location Domination Number Of A Graph By The Fusion Of Vertex

Rajasekar. G [1], Nagarajan. K [2]
Department of Mathematics[1,2], Jawahar Science College, Neyveli[1,2]
Email: grsmaths@gmail.com [1] , n2a4g1r1j1@gmail.com[2]

Abstract- Locating-Dominating set (LD -set) of a graph G, is a dominating set S with the property that every vertices v in $V(G)-S$ can be identified uniquely by the set of neighbours of v which are in S. In this paper a procedure for finding the locating-domination number of graph obtained by fusion of a single vertex from two graphs has been formulated. By this procedure the location domination number of $C_m \bowtie_f C_n$, Windmill graph, Dutch windmill graph and Friendship graph are been found.

Keywords-Dominating set; Locating set; Locating domination set; Fusion of vertex

1. INTRODUCTION

For any vertex v in $V(G)-S$, $S(v)$ is the set of vertices in S which are adjacent to v. Locating-dominating set was introduced by Slater [1, 2] which is defined as follows. A dominating set S is defined to be a locating-dominating set if $S(v) \neq S(w)$, for any $v, w \in V(G)-S$. A locating dominating set is denoted by LD-set. The minimum cardinalities of an LD-set in G is called the location-domination number of G and it is denoted by $RD(G)$. An LD-set with $RD(G)$ elements is called as a referencing-dominating set or an RD-set. The set of all RD-set of the given graph is denoted by $\mathscr{S}$.

G. Rajasekar et al [3,4,5] have found location domination number of sum of graphs, graph connected by bridge and graph obtained by fusion of vertex.

The windmill graph $K_n^{(m)}$ or $Wd(n,m)$ is the graph obtained by taking m copies of the complete graph K_n with a vertex in common. The Dutch windmill graph $D_n^{(m)}$ is the graph obtained by taking m copies of the cycle C_n with a vertex in common.

2. PRELIMINARIES

Let G be the graph obtained from G_1 and G_2 by vertex fusion of vertices $v_1 \in G_1$ and $v_2 \in G_2$ to form a single new vertex v. Let us denote this graph G by $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$.

Theorem 2.1. [4]**:** Let G_1, G_2 be any two graphs and G be a graph obtained by fusion of vertices $v_1 \in G_1$ and $v_2 \in G_2$ to form a single vertex v, then $RD(G_1)+RD(G_2)-2 \le RD(G) \le RD(G_1)+RD(G_2)$

Theorem 2.2. [4]**:** Let G_1, G_2 be any two graphs and $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$. For $i=1,2$, let G_i have an RD-set S_i such that $v_i \in S_i$ and $S_i(u_i) \neq \{v_i\}$ for all $u_i \in V(G_i)-S_i$ .then $RD(G) = RD(G_1)+RD(G_2)-2$ if and only if for $i=1,2$, $S_i - \{v_i\}$ is the RD-set of $G_i - \{v_i\}$ and $N_{G_i}(v_i) \bigcap S_i \neq \Phi$.

Theorem 2.3. [4]**:** Let G_1, G_2 be any two graphs and $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$. If for both $i=1$ and 2 v_i does not belongs to anyone of the RD-set of G_i then $RD(G) = RD(G_1)+RD(G_2)$.

Theorem 2.4. [4]**:** Let G_1, G_2 be any two graphs and $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$. Let either G_1 or G_2 has atleast one RD-set S_1 or S_2 respectively such that $v_1 \in S_1$ or $v_2 \in S_2$ (say $v_1 \in S_1$). If $S_1 - \{v_1\}$ is the RD-set of $G_1 - \{v_1\}$ for some RD-set S_1 then $RD(G)$ will be equal $RD(G_1)+RD(G_2)-1$ otherwise $RD(G)$ will be equal $RD(G_1)+RD(G_2)$.

Theorem 2.5. [4]**:** Let G_1, G_2 be any two graphs and $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$. For $i=1,2$, let G_i have an RD-set S_i such that $v_i \in S_i$. If for every RD-set S_i, there exist a vertex $u_i \in V(G_i)-S_i$ such that $S_i(u_i) = \{v_i\}$ for both $i=1$ and 2 then $RD(G) = RD(G_1)+RD(G_2)$.

Theorem 2.6. [4]**:** Let G_1, G_2 be any two graphs and $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$. For $i=1,2$, let G_i have an RD-set S_i such that $v_i \in S_i$. If G_2 has an RD-set S_2 such that $S_2(u_2) \neq \{v_2\}$ for all $u_2 \in V(G_2)-S_2$ and G_1 have no RD-set S_1 such that $S_1(u_1) \neq \{v_1\}$ for all $u_1 \in V(G_1)-S_1$ then

$$RD(G) = RD(G_1)+RD(G_2)-1.$$

Remark 2.1. [4]**:** Let G_1, G_2 be any two graphs and $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$. For $i=1,2$, let G_i have an

RD-set S_i such that $v_i \in S_i$ and $S_i(u_i) \neq \{v_i\}$ for all $u_i \in V(G_i) - S_i$. For either $i = 1$ or 2 if G_i does not have any RD-set S_i satisfying the both conditions

(i) $S_i - \{v_i\}$ is the RD-set of $G_i - \{v_i\}$

(ii) $N_{G_i}(v_i) \cap S_i \neq \Phi$

then $RD(G) = RD(G_1) + RD(G_2) - 1$.

3. LOCATION DOMINATION NUMBER OF A GRAPH BY THE FUSION OF VERTEX

Algorithm for finding location domination number of graph obtained by fusion of vertex is formulated with the aid of Theorem 2.2, 2.3, 2.4, 2.5, 2.6 and Remark 2.1. We see that location domination number of graph obtained by fusion of vertex depends on three feature of RD-set of the graph. The three properties are defined as follows:

***Property* 1:** G has an RD-set $S \in \mathscr{S}$ such that $v \in S$ and $S - \{v\}$ is the RD-set of $G - \{v\}$ where v is the vertex considered for fusion.

***Property* 2:** G has atleast one RD-set $S \in \mathscr{S}$ such that $S(u) \neq \{v\}$ for all $u \in V(G) - S$ and v is the considered for fusion.

***Property* 3:** G has an RD-set $S \in \mathscr{S}$ such that $v \in S$ and $N_G(v) \cap S \neq \Phi$ where v is the vertex considered for fusion.

Based on these three properties we formulate a procedure in form of Table 1. and Table 2. to find the location domination number of graphs obtained by fusion of vertex.

Remark 3.1. Let $G = G_1\{v_1\} \bowtie_f G_2\{v_2\}$ where v_1 and v_2 are fused to form a single vertex v. If one of the following condition is true then RD-set S of G doesn't contains the vertex v.

(i) Both G_1 and G_2 doesn't have any RD-set which contains the vertex v_1 and v_2 respectively

(ii) Both G_1 and G_2 have any RD-set satisfying Property 1, Property 2 and Property 3 simultaneously

Remark 3.2. If graph G doesn't satisfy Property 1 then there is no need to check Property 3.

Table 1. Procedure for finding the location domination number of graphs obtained by fusion of vertex- Table 1

$RD(G)$	$v_2 \notin S_2$ for all $S_2 \in \mathscr{S}_2$	$v_2 \in S_2$ for some $S_2 \in \mathscr{S}_2$
$v_1 \notin S_1$ for all $S_1 \in \mathscr{S}_1$	$RD(G_1) + RD(G_2)$	If G_2 satisfies Property 1 then $RD(G) = RD(G_1) + RD(G_2) - 1$ otherwise $RD(G) = RD(G_1) + RD(G_2)$
$v_1 \in S_1$ for some $S_1 \in \mathscr{S}_1$	If G_1 satisfies Property 1 then $RD(G) = RD(G_1) + RD(G_2) - 1$ otherwise $RD(G) = RD(G_1) + RD(G_2)$	Check Property 2 for the graph G_1 and G_2. And refer Table 2

Table 2. Procedure for finding the location domination number of graphs obtained by fusion of vertex- Table 2

$RD(G)$	G_2 satisfies Property 2	G_2 doesn't satisfy Property 2
G_1 satisfies Property 2	If both graphs G_1 and G_2 have RD-sets which satisfies Property 1, Property 2 and Property 3 simultaneously then $RD(G) = RD(G_1) + RD(G_2) - 2$ otherwise $RD(G) = RD(G_1) + RD(G_2) - 1$	$RD(G) = RD(G_1) + RD(G_2) - 1$
G_1 doesn't satisfy Property 2	$RD(G) = RD(G_1) + RD(G_2) - 1$	$RD(G) = RD(G_1) + RD(G_2)$

4. LOCATION DOMINATION NUMBER OF $C_m \bowtie_f C_n$, WINDMILL GRAPH, DUTCH WINDMILL GRAPH AND FRIENDSHIP GRAPH

This section deals with finding location domination number of $C_m \bowtie_f C_n$, Windmill graph, Dutch windmill graph and Friendship graph which are obtained by vertex fusion. So let us first study the properties of C_n and K_n related to vertex fusion.

Properties of C_n: Let the vertex label of C_n be $v_1, v_2, ..., v_{n-1}, v_n$. For cycle it is always possible to have a RD-set which contains the vertex considered for fusion.

- For $n = 4$, $\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}$ and $\{v_4, v_1\}$ are the possible RD-set. And Property 1 and Property 2 are not satisfied for any of the possible RD-set.
- For $n \equiv 0 \bmod 5$, i.e. $n = 5k$, $k \geq 1$. $\{v_2, v_4, v_7, v_9, ..., v_{5k-8}, v_{5k-6}, v_{5k-3}, v_{5k-1}\}$ is the only possible RD-set and it doesn't satisfies Property 1, Property 2 as well as Property 3.
- For $n \equiv 1 \bmod 5$, i.e. $n = 5k+1$, $k \geq 1$. $\{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}, v_{5k+1}\}$ is a RD-set such that $\{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}\}$ is the RD-set of $C_n - \{v_{5k+1}\}$ and $S(v_i) \neq \{v_{5k+1}\}$ for all $v_i \in V(C_n) - \{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}, v_{5k+1}\}$.

  Hence Property 1 and Property 2 is satisfied with respect to vertex v_{5k+1}. Whereas none of the vertices in the set $\{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}, v_{5k+1}\}$ are adjacent, so Property 3 is not satisfied.
- For $n \equiv 2 \bmod 5$, i.e. $n = 5k+2$, $k \geq 1$. $\{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}, v_{5k+2}\}$ is a RD-set such that $S(v_i) \neq \{v_1\}$ for all $v_i \in V(C_n) - \{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}, v_{5k+2}\}$.

  Hence Property 2 is satisfied with respect to vertex v_1. Whereas Property 1 and Property 3 is not fulfilled.
- For $n \equiv 3 \bmod 5$, i.e. $n = 5k+3$, $k \geq 0$. $\{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}, v_{5k+2}, v_{5k+3}\}$ is a RD-set such that v_{5k+3} satisfies Property 1, Property 2 and Property 3
- For $n \equiv 4 \bmod 5$, i.e. $n = 5k+4$, $k \geq 1$. $S = \{v_2, v_4, v_7, v_9, ..., v_{5k-3}, v_{5k-1}, v_{5k+2}, v_{5k+4}\}$ is a RD-set such that $S(v_i) \neq \{v_{5k+4}\}$ for all $v_i \in V(C_n) - S$. Hence Property 2 is satisfied with respect to vertex v_{5k+4}. But Property 1 and Property 3 is not fulfilled.

Properties of C_n are concatenated in the following Table 3

Properties of K_n: Let the vertex label of K_n be $v_1, v_2, ..., v_{n-1}, v_n$. For complete graph it is always possible to have a RD-set which contains the vertex considered for fusion.

For $n = 2$, Property 1, Property 2 and Property 3 fails.

Clearly for $n \geq 3$, $S = \{v_1, v_2, ..., v_{n-2}, v_{n-1}\}$ is an RD-set such that $S - \{v_1\}$ is the RD-set of $K_n - \{v_1\}$ $S(v_n) \neq \{v_1\}$, $S - \{v_1\}$ is the RD-set of $K_n - \{v_1\}$ and $N_G(v_1) \bigcup S = \{v_2, v_3, ..., v_{n-2}, v_{n-1}\} \neq \Phi$. Thus K_n satisfy Property 1, Property 2 and Property 3.

Theorem 4.1. Let $G = C_m \bowtie_f C_n$ then

$$RD(G) = \begin{cases} RD(C_m) + RD(C_n), & \text{if } m, n \equiv 0 \bmod 5 \text{ or} \\ & m, n = 4 \text{ or} \\ & m \equiv 0 \bmod 5 \text{ and } n = 4 \text{ or} \\ & m = 4 \text{ and } n \equiv 0 \bmod 5 \\ RD(C_m) + RD(C_n) - 2, & \text{if } m, n \equiv 3 \bmod 5 \\ RD(C_m) + RD(C_n) - 1, & \text{otherwise} \end{cases}$$

Proof. In cycle C_n, as vertex considered for fusion is in the RD-set, it is enough to check Table 2 alone.

As $n \equiv 3 \bmod 5$ satisfies Property 1, Property 2 and Property 3 while $n \equiv 0 \bmod 5$ and $n = 4$ doesn't satisfies Property 1, Property 2 and Property 3 we have that

Table 3. Properties of C_n related to vertex fusion

C_n	Vertex considered for fusion is in the RD-set	Property 1	Property 2	Property 3
$n \equiv 0 \bmod 5$ and $n = 4$	Yes	Not satisfied	Not satisfied	Not satisfied
$n \equiv 1 \bmod 5$	Yes	Satisfied	Satisfied	Not satisfied
$n \equiv 2 \bmod 5$ $n \equiv 4 \bmod 5$ except $n = 4$	Yes	Not satisfied	Satisfied	Not satisfied
$n \equiv 3 \bmod 5$	Yes	Satisfied	Satisfied	Satisfied

$$RD(G)=\begin{cases} RD(C_m)+RD(C_n), & \text{if } m,n\equiv 0 \bmod 5 \text{ or} \\ & m,n=4 \text{ or} \\ & m\equiv 0\bmod 5 \text{ and } n=4 \text{ or} \\ & m=4 \text{ and } n\equiv 0 \bmod 5 \\ RD(C_m)+RD(C_n)-2, & \text{if } m,n\equiv 3\bmod 5 \end{cases}$$

From Table 2, it is clear that for all other cases $RD(G)$ will be equal to $RD(C_m)+RD(C_n)-1$.

Corollary 4.1. Let F_n be the friendship graph then $RD(F_n)=n$.

Proof. $F_2 \cong C_3 \bowtie_f C_3$, hence by Theorem 4.1 $RD(C_3 \bowtie_f C_3)=RD(C_3)+RD(C_3)-2=2$ and by Remark 3.1 the fused vertex will not be in any RD-set.

F_n is obtained by fusion any vertex of C_3 to the vertex with maximum degree in F_{n-1} and by Table 1 we have

$$RD(F_n)=RD(F_{n-1})+RD(C_3)-1$$
$$RD(F_{n-1})=RD(F_{n-2})+RD(C_3)-1$$
$$\vdots$$
$$RD(F_4)=RD(F_3)+RD(C_3)-1$$
$$RD(F_3)=RD(F_2)+RD(C_3)-1=2+2-1=3$$

Thus $RD(F_n)=n-1+2-1=n$.

Theorem 4.2. For the Windmill graph $K_n^{(m)}$,

$$RD\left(K_n^{(m)}\right)=\begin{cases} m, & \text{if } n=2 \\ m(n-2), & \text{if } n\geq 3 \end{cases}.$$

Proof. $K_2^{(m)} \cong S_m$ and hence

$$RD\left(K_2^{(m)}\right)=RD(S_m)=m.$$

For $n\geq 3$, K_n satisfy all Property 1, Property 2 and Property 3, hence

$$RD\left(K_n^{(2)}\right)=RD(K_n)+RD(K_n)-2$$
$$=2(n-2)$$

and by Remark 3.1 the fused vertex will not be in any RD-set.

$K_n^{(m)}$ is obtained by fusion any vertex of K_n to the vertex with maximum degree in $K_n^{(m-1)}$ and by Table 1 we have

$$RD\left(K_n^{(m)}\right)=RD\left(K_n^{(m-1)}\right)+RD(K_n)-1$$
$$RD\left(K_n^{(m-1)}\right)=RD\left(K_n^{(m-2)}\right)+RD(K_n)-1$$
$$\vdots$$
$$RD\left(K_n^{(4)}\right)=RD\left(K_n^{(3)}\right)+RD(K_n)-1$$
$$RD\left(K_n^{(3)}\right)=RD\left(K_n^{(2)}\right)+RD(K_n)-1=2(n-2)+n-1-1$$
$$=3(n-2)$$

Hence $RD\left(K_n^{(m)}\right)=m(n-2)$.

Theorem 4.3. For the Dutch windmill graph $D_n^{(m)}$,

$$RD\left(D_n^{(m)}\right)=\begin{cases} m\left\lceil \frac{2n}{5}\right\rceil, & \text{if } n\equiv 0\bmod 5 \text{ or} \\ & n=4 \\ m\left\lceil \frac{2n}{5}\right\rceil-m, & \text{if } n\equiv 3\bmod 5 \\ m\left\lceil \frac{2n}{5}\right\rceil-(m-1), & \text{otherwise} \end{cases}$$

Proof. As $D_n^{(2)} \cong C_n \bowtie_f C_n$, by Theorem 4.1 we have that

$$RD\left(D_n^{(2)}\right)=\begin{cases} 2\left\lceil \frac{2n}{5}\right\rceil, & \text{if } n\equiv 0\bmod 5 \text{ or} \\ & n=4 \\ 2\left\lceil \frac{2n}{5}\right\rceil-2, & \text{if } n\equiv 3\bmod 5 \\ 2\left\lceil \frac{2n}{5}\right\rceil-1, & \text{otherwise} \end{cases}$$

$D_n^{(m)}$ is obtained by fusion of any vertex of C_n to the vertex of $D_n^{(m-1)}$ with maximum degree.

Case 1: For $n\equiv 0\bmod 5$ and $n=4$, $D_n^{(2)}$ and C_n do not satisfies Property 2. Moreover $D_n^{(m-1)}$ also do not satisfies Property 2 for $n\equiv 0\bmod 5$ and $n=4$. Hence we have

$$RD\left(D_n^{(3)}\right)=RD\left(D_n^{(2)}\right)+RD(C_n)=3\left\lceil \frac{2n}{5}\right\rceil$$
$$\vdots$$
$$RD\left(D_n^{(m-1)}\right)=RD\left(D_n^{(m-2)}\right)+RD(C_n)=(m-1)\left\lceil \frac{2n}{5}\right\rceil$$
$$RD\left(D_n^{(m)}\right)=RD\left(D_n^{(m-1)}\right)+RD(C_n)=m\left\lceil \frac{2n}{5}\right\rceil$$

Case 2: For $n\equiv 3\bmod 5$, any RD-set of $D_n^{(2)}$ does not contains the vertex considered for fusion. As C_n satisfies Property 1 for $n\equiv 3\bmod 5$ and by Table 1 we have

$$RD\left(D_n^{(3)}\right) = RD\left(D_n^{(2)}\right) + RD\left(C_n\right) - 1$$
$$= 3\left\lceil \frac{2n}{5} \right\rceil - 3$$
$$\vdots$$
$$RD\left(D_n^{(m-1)}\right) = RD\left(D_n^{(m-2)}\right) + RD\left(C_n\right) - 1$$
$$= (m-1)\left\lceil \frac{2n}{5} \right\rceil - (m-1)$$
$$RD\left(D_n^{(m)}\right) = RD\left(D_n^{(m-1)}\right) + RD\left(C_n\right) - 1 = m\left\lceil \frac{2n}{5} \right\rceil - m$$

Case 3: For all other possibilities we can similarly prove that $RD\left(D_n^{(m)}\right) = m\left\lceil \frac{2n}{5} \right\rceil - (m-1)$.

ACKNOWLEDGMENTS
This research is supported by UGC scheme RGNF. Award letter F1-17.1/2014-15/RGNF-2014-15-SC-TAM-80373/(SAIII/Website).

REFERENCES
[1] Slater, P. J. (1988): Dominating and reference sets in a graph, J. Math. Phys. Sci., **22**, pp. 445-455.
[2] Slater, P. J. (1987): Dominating and location in acyclic in graphs, Networks, **17**, pp. 55-64.
[3] Rajasekar, G.; Nagarajan, K. (2018): Algorithm for finding Location Domination Number of a Graph connected by a Bridge, Int. J. of Pure and Applied Math., **118** (6), pp. 313-321.
[4] Rajasekar, G.; Nagarajan, K. (2017): Location Domination Number of Graph obtained by the Fusion of Single Vertex, Global J. of Pure and Applied Math., **13** (9), pp. 4425–4436.
[5] Rajasekar, G.; Nagarajan, K. (2018): Location Domination Number of Sum of Graphs, Int. J. of Engineering & Technology, **7** (4.10), pp. 852-856.