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Fibonacci Mean Anti-magic Labeling of Graphs

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Abstract:A Fibonacci Mean Anti-magic labeling of a graph G is an injective function g: V (G) → { f_2, f_3, … f_{n+1}}, where f_n is n^{th} Fibonacci number with the induced a function g^* : E(G) → N defined by g ∗(e = uv) =
$$\begin{cases} \frac{g(u) + g(v)}{2} & \text{if } g(u) + g(v) \text{ is even} \\ \frac{g(u) + g(v) + 1}{2} & \text{if } g(u) + g(v) \text{ is odd} \end{cases}$$
and all these edge labeling are distinct. A graph is called Fibonacci Mean Anti-magic if it admits a Fibonacci Mean Anti-magic labeling. In this paper we prove that some special graphs caterpillar, spider, SF(n,1), $B_{n.n}$, Triangular Snake, Quadrilateral Snake, Ladder and $P_{n(m)}$ graphs are Fibonacci Mean Anti-magic graphs.

Keywords: Fibonacci mean anti-magic graph, caterpillar, spider, SF(n,1), $B_{n.n}$, Triangular Snake, Quadrilateral Snake, Ladder and $P_{n(m)}$

AMS Subject Classification (2010): 05C78.

1. INTRODUCTION

In this paper we mean a *graph* G by finite, connected, undirected graph G = (V, E) without any loops and multiple edges with │V│ = p vertices and │E│= q edges. We follow the notation and terminology of [9].

The concept of graph labeling was introduced by Rosa [6] in 1967.

Definition 1.1

A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain conditions. The process of vertex labeling is label the vertices with integers. Under this vertex labeling, the edge weight of an edge e = uv is defined as W (e) = W (uv) = g (u) + g(v).

In 1994, N.Hartsfield and Ringel [1] introduced the concept of *anti-magic graph*.

Definition 1.2

Each vertex labeling f of a graph G = (p,q) from{0,1,2,…q} induces a edge labeling g_f where g ∗ (e) is sum the labels of end vertices of an edge e. Labeling f is called *anti-magic* if and only if all the edge labelings are pair wisely distinct.

Definition 1.3

By an *edge anti-magic vertex labeling* we mean a one-to-one mapping V(G) into {0,1,2,…q} such that the set of edge weights of all edges in G is {1,2,…q}. Different kinds of anti-magic graphs were studied by T.Nicholas, S.Somasundaram and V.Vilfred [5].

S.Somasundaram and R.Ponraj [7] and [8] introduced the concept of *mean labeling*.

Definition 1.4

A graph G with p vertices and q edges is a *mean graph* if there is an injective function f from the vertices of G to {1,2…..q} such that when each edge uv is labelled with $\frac{f(u) + f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u) + f(v) + 1}{2}$ if f(u) + f(v) is odd then the resulting edges are distinct.

Fibonacci graceful labeling was introduced by Kathiresan.M and Amutha.S [2] and different kind of graphs were studied by [3] and [4].

Definition 1.5

The *Fibonacci numbers* can be defined by the linear recurrence relation Fn = F_{n-1} + F_{n-2}; n ≥ 3. This generates the infinite sequence of integer's beginning1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…

Definition 1.6

A *Caterpillar* is a tree which has path P_n = a_1 ,a_2……a_n of order n and is obtained by attaching X_i (possibly zero) end vertices at the vertex a_i of P_n, i = 1,2….n by end edges. It is denoted as T = S(X_1, X_2….. X_n). The order of T is n + ∑Xi.

Example 1.7

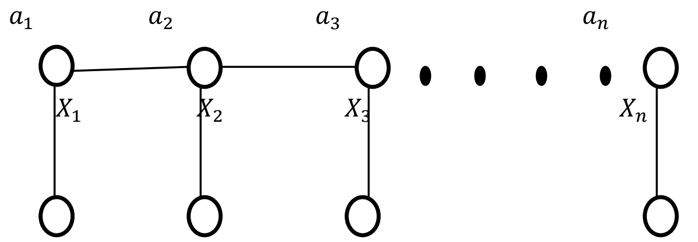

Fig (1.1) - T = S(X_1, X_2…..X_n)

Definition 1.8

A *Spider* SP ($P_{n,2}$) is a Caterpillar S(X_1,X_2,….X_n) where $X_n = 2$ and $X_i = 0$ for i = 1,2,…n-1

Example 1.9

The following graph is the example for the Spider SP ($P_{4,\,2}$)

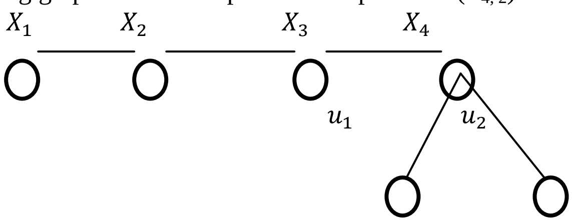

Fig (1.2) - SP ($P_{4,\,2}$)

Definition 1.10

A *Quadrilateral Snake Q_n* is obtained from a path {u_1 ,u_2…..u_n } by joining u_i and u_{i+1} to two vertices v_i and w_i , $1 \leq i \leq n - 1$ respectively and then joining v_i and w_i .

Definition 1.11

The Product P_2 X P_n is called a *Ladder* and it is denoted by L_n .

Definition 1.12

Triangular Snake T_n is obtained from a path u_1 , u_2 ,…….u_n by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n - 1$, that is every edge of a path is replaced by a triangle C_3.

Definition 1.13

An *SF (n,m)* is a graph consisting of a cycle C_n , $n \geq 3$, and n set of m independent vertices where each set joins each of the vertices of C_n.

Definition 1.14

Bi-Star is the graph obtained by joining the apex vertices of two copies of Star $K_{1,n}$.

Definition 1.15

The graph $P_{n_{(m)}}$ = G (V , E) such that V (G) = { v_{ij} , $1 \leq i \leq m$, and $1 \leq j \leq n$ }

E (G) = { $v_{ij}v_{i(j+1)}$ / $1 \leq i \leq m$ and $1 \leq j \leq n - 1$ } ∪{ $v_{i(n-1)}v_{i+1((n-1)}$/ $1 \leq i \leq m - 1$}

2. RESULTS ON FIBONACCI MEAN ANTI-MAGIC LABELING

In this section we investigate Fibonacci Mean Anti-magic labeling of some special graphs.

Definition 2.1

A Fibonacci Mean Anti-magic labeling of a graph G is an injective function g: V (G) → { f_2, f_3, … f_{n+1}}, where f_n is n^{th} Fibonacci number with the induced a function g^* : E(G) → N defined by $g*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & \text{if } g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & \text{if } g(u)+g(v) \text{ is odd} \end{cases}$ and all these edge labeling are distinct. A graph is called Fibonacci Mean Anti-magic if it admits a Fibonacci Mean Anti-magic labeling.

Theorem 2.2

The Caterpillar S(X_1 , X_2,……X_n) has Fibonacci Mean – Anti-magic labeling.

Proof:

The path vertices are denoted as v_1, v_2,…v_n and the end vertices are denoted as u_1,u_2,…u_n.

The assignment of vertex labels are g (v_i) = f_2, f_3, … f_{n+1}. The induced edge labels are g*: E (G) → N and all are distinct.

This completes the proof.

Theorem 2.3

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Every Spider SP ($P_{n,2}$) admits Fibonacci Mean – Anti-magic labeling.

Proof:

Define g: V (G) → { $f_2, f_3, \ldots f_{n+1}$}, where f_n is n^{th} Fibonacci number with the induced a function g^* : E(G) → N defined by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & \text{if } g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & \text{if } g(u)+g(v) \text{ is odd} \end{cases}$$

and all these edge labeling are distinct.

Example 2.4

The following graph is the example for the Spider SP ($P_{4,2}$)

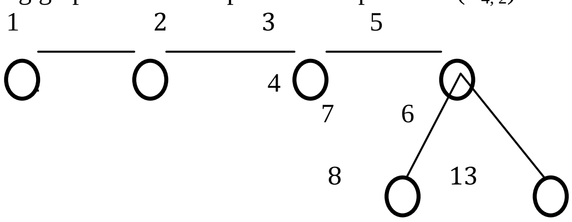

Fig (2.1) - SP ($P_{4,2}$)

Theorem 2.5

The graph G ⊙ K_1 is a Fibonacci Mean – Anti-magic graph where G = T_n for all integer n ≥ 2.

Proof:

Let {$u_1,u_2\ldots u_n$} be a path of length n. Let v_i, 1 ≤ i ≤ n - 1 be the new vertex joined to u_i and u_{i+1}. The resulting graph is called T_n and let x_i be the vertex which is joined to u_i , 1 ≤ i ≤ n, let y_i be the vertex which is joined to v_i, 1 ≤ i ≤ n - 1. The resulting graph is G_1 (i.e.) G ⊙ K_1 where G = T_n graph. Now the vertex set of V (G_1) = {u_1, $u_2\ldots u_n$, $v_1,v_2\ldots v_{n-1}$, x_1,x_2, …x_n, y_1y_2, …y_{n-1}} and the edge set E(G_1) = {u_iu_{i+1}, u_iv_i /1 ≤ i ≤ n-1} ∪{u_ix_i / 1 ≤ i ≤ n }∪ {v_iu_{i+1}, v_iy_i /1 ≤ i ≤ n - 1 }. Here │V (G_1) │= 4n - 2.

Let g: V (G) → { $f_2, f_3, \ldots f_{n+1}$}, where f_n is n^{th} Fibonacci number with the induced a function g^* : E(G) → N defined by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & \text{if } g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & \text{if } g(u)+g(v) \text{ is odd} \end{cases}$$

and all these edge labeling are distinct.

Theorem 2.6

The graph G ⊙ K_1 is a Fibonacci Mean – Anti-magic graph where G = Q_n for all integer n ≥ 2.

Proof:

Let {$u_1,u_2\ldots u_n$} be a path of length n. Let v_i w_i be two vertices joined to u_i and u_{i+1} respectively and then join v_i and w_i, 1 ≤ i ≤ n - 1. The resulting graph is called a quadrilateral snake Q_n. Let x_i be the vertex which is joined to u_i , 1 ≤ i ≤ n, let y_i be the new vertex which is joined to v_i, 1 ≤ i ≤ n - 1and let z_i be the new vertex which is joined to w_i, 1 ≤ i ≤ n - 1. The resulting graph is G_1 (i.e) G ⊙ K_1 where G = Q_n graph. Now the vertex set of V (G_1) = {u_1,u_2, …u_n, v_1,v_2, …v_{n-1}, w_1,w_2, …w_{n-1} x_1,x_2, …x_n, y_1y_2, …y_{n-1}, z_1,z_2, …z_{n-1}} and the edge set E(G_1) = {u_iu_{i+1} /1 ≤ i ≤ n-1}∪{u_ix_i /1 ≤ i ≤ n }∪{u_iv_i , v_iy_i , v_iw_i ,w_iz_i /1 ≤ i ≤ n-1 } ∪ {w_iu_{i+1} / 1 ≤ i ≤ n - 1 }. Here │V (G_1) │= 6n - 4.

Let g: V (G) → { $f_2, f_3, \ldots f_{n+1}$}, where f_n is n^{th} Fibonacci number with the induced a function g^* : E(G) → N defined by

$$g^*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & \text{if } g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & \text{if } g(u)+g(v) \text{ is odd} \end{cases}$$

and all these edge labeling are distinct.

Theorem 2.7

The graph G ⊙ K_1 is Fibonacci Mean anti-magic graph where G = L_n for all integer n ≥ 2.

Proof:

Let G be the ladder with vertices {u_1, $u_2\ldots v_1,v_2\ldots v_n$ }. Let u_i' be the new vertex joined to u_i 1 ≤ i ≤ n and v_i' be the new vertex joined to v_i, 1 ≤i ≤n in G. The resulting graph G_1 = G ⊙ K_1 where G=L_n graph. Now the vertex set V(G_1)={$u_1,u_2,\ldots v_1,v_2,\ldots v_n$, u_1',u_2',…u_n'v_1',v_2'…v_n'}.

The Edge set E(G_1) = { v_iv_{i+1}, u_iu_{i+1} / 1 ≤ i ≤ n – 1 } ∪ { u_iv_i, u_iu_i', v_iv_i'/ 1 ≤ i ≤ n}. Here │V (G_1) │ = 4n.

Let g: V (G) → { $f_2, f_3, \ldots f_{n+1}$}, where f_n is n^{th} Fibonacci number with the induced a function g^* : E(G) → N defined by

$$g*(e = uv) = \begin{cases} \frac{g(u)+g(v)}{2} & \text{if } g(u)+g(v) \text{ is even} \\ \frac{g(u)+g(v)+1}{2} & \text{if } g(u)+g(v) \text{ is odd} \end{cases}$$ and all these edge labeling are distinct.

Theorem 2.8

The graph SF (n, 1) admits a Fibonacci Mean Anti-magic labeling

Proof:

Let G denote the graph SF (n, 1).

Let v_1, v_2, ... v_n be the vertices of the cycle SF(n,1) and v_j' for j = 1,2,...n be the vertices joining the corresponding vertices v_j.

Here | V (G) | = 2n and | E (G) | = 2n.

Define g: V (G) → { f_2, f_3, … f_{n+1}}, where f_n is n^{th} Fibonacci number.

The distinct edge labels are determined by the condition

$$g^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

Hence Proved.

Theorem 2.9

The Bi- Star $B_{n,n}$ admits Fibonacci Mean Anti-magic Labeling.

Proof:

Consider the two copies of $K_{1,n}$. Let v_1, v_2, ... v_n and u_1, u_2, ... u_n be the corresponding vertices of each copy of $K_{1,n}$ with apex vertex v and u.

Let $e_i = v\,v_i$, $e_i' = u\,u_i$ and $e = uv$ of bistar graph.

Here | V ($B_{n,n}$) | = 2n + 2, | E ($B_{n,n}$) | = 2n + 1.

The vertices are assigned by Fibonacci numbers and the induced edge labels are distinct and the theorem follows.

Theorem 2.10

The graph $P_{n_{(m)}}$ is a Fibonacci Mean Anti-magic graph for all n, m ≥ 2.

Proof:

Let G =$P_{n_{(m)}}$.

Let V (G) = { v_{ij} , 1 ≤ i ≤ m, and 1 ≤ j ≤ n }and E (G) = {$v_{ij}v_{i(j+1)}$ / 1 ≤ i ≤ m and 1 ≤ j ≤ n – 1 } ∪{ $v_{i(n-1)}v_{i+1((n-1)}$ / 1 ≤ i ≤ m – 1 }.

Then | V (G) | = mn and | E(G) | = mn – 1.

The assignment of vertex labels are g (v_i) = f_2, f_3, … $f_{n+1.}$

All the edge labels are distinct.

Hence the theorem follows.

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