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QUADRATIC NATURE OF ONE POINT COMPACTIFICATION OF UNIFORM SPACES

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Abstract. This paper deals with compactness and convergence of Hausdorff spaces and its subspaces in the real valued domain especially the functions are in the half range series. Uniqueness of one-point compactification function and its quadratic nature is also discussed in this paper. The same is extended into the real uniformed spaces. Convergence analysis of quadratic function is also arrived in a new manner.

Keywords: Unique compact Subspaces, One point compactification, Quadratic nature, Convergence analysis.
AMS Subject Classification: 54A05, 54A20, 54C05, 54C10.

1. Introduction

Uniformed spaces in topology was gradually revealed its applications in various fields especially convergence Analysis[3,6]. Despite of independent character of uniform space it is closely connected with the theory of topological spaces[2,7]. So the problem of defining and doing research in uniform spaces are of most important classes of topological spaces [1,5]. In a Hausdorff space, disjoint compact subspace can be separated by open sets. A best one point compactification problem is a problem of achieving the minimum distance between two sets through a function defined on one of the sets to the other[4,8]. With this logic, we separated the odd and even function and considered an even function in an uniformed spaces [9].

2. Preliminaries

Some basic concepts related to even functions are discussed in the following theorem.

Theorem 2.1. *Let $\theta \geq 0, 0 < p, q < 2$. If $g : (X, \|.,.\|) \to (X, \|.,.\|)$ is an even function such that*

$$\|T_f(x,y), z\| \leq \theta(\|x,z\|^p + \|y,z\|^q) \tag{1}$$

for each $x, y, z \in X$. Then there exists a unique compact subspaces in the real line $Q : X \to X$ satisfying (2.1) and

$$\|Q(x) - g(x), z\| \leq \frac{\theta \|x,z\|^q}{4 - 2^q} \tag{2}$$

for each $x, z \in X$.

Proof: Put $x = 0$ in (1), we have

$$\|g(2y) - 4g(y), z\| \leq \theta \|y,z\|^q \tag{3}$$

for each $y, z \in X$. Replacing y by x and dividing by 4 in (3), we get

$$\left\| \frac{g(2x)}{4} - g(x), z \right\| \leq \frac{\theta}{4} \|x,z\|^q \tag{4}$$

for each $x, z \in X$. By using induction on n, we get

$$\begin{aligned} \left\| \frac{g(2^n x)}{4^n} - g(x), z \right\| &\leq \frac{\theta}{4} \|x,z\|^q \sum_{j=0}^{n-1} 2^{(q-2)j} \\ &\leq \frac{\theta}{4} \|x,z\|^q \left(\frac{1 - 2^{(q-2)n}}{1 - 2^{q-2}} \right) \end{aligned} \tag{5}$$

for each $z \in X$. Therefore $\left\{ \frac{g(2^n x)}{4^n} \right\}$ is a Cauchy sequence in local compactness, for each $x \in X$. Since X is a Hausdorff space, the sequence $\left\{ \frac{g(2^n x)}{4^n} \right\}$ converges in X, for each $x \in X$. Define $Q : X \to X$ as

$$Q(x) = \lim_{n \to \infty} \frac{g(2^n x)}{4^n}$$

for each $x \in X$. By (5), we have

$$\begin{aligned}\lim_{n\to\infty}\left\|\frac{g(2^n x)}{4^n}-g(x),z\right\| \leq& \frac{\theta}{4}\left\|x,z\right\|^q\left(\frac{1}{1-2^{q-2}}\right)\\ \left\|Q(x)-g(x),z\right\| \leq& \frac{\theta}{4}\left\|x,z\right\|^q\left(\frac{4}{4-2^q}\right)\\ \leq& \frac{\theta\left\|x,z\right\|^q}{4-2^q}\end{aligned}$$

for each $x, z \in X$. Now we show that Q satisfies (**??**).

$$\begin{aligned}\left\|T_Q(x,y),z\right\| =& \lim_{n\to\infty}\frac{1}{4^n}\left\|T_f(2^n x,2^n y),z\right\|\\ \leq& \lim_{n\to\infty}\theta\left[2^{(p-2)n}\left\|x,z\right\|^p+2^{(q-2)n}\left\|y,z\right\|^q\right]\\ =&0\end{aligned}$$

for each $x, y, z \in X$. Therefore, $\|T_Q(x,y),z\| = 0$ for each $z \in X$. So $T_Q(x,y) = 0$. Since f is an even function we have Q is an even function and Q is a one-point compactification function.

Next we show that Q is unique. Let $Q' : X \to X$ be another one-point compactification function which satisfies (2.1) and (2). Since Q and Q' are quadratic.
$Q(2^n x) = 4^n Q(x), Q'(2^n x) = 4^n Q'(x)$ for each $x \in X$. It follows that

$$\begin{aligned}\left\|Q'(x)-Q(x),z\right\| =& \frac{1}{4^n}\left\|Q'(2^n x)-Q(2^n x),z\right\|\\ \leq& \frac{1}{4^n}\left[\left\|Q'(2^n x)-f(2^n x),z\right\|+\left\|f(2^n x)-Q(2^n x),z\right\|\right]\\ \leq& \frac{1}{4^n}\frac{2\theta\left\|2^n x,z\right\|^q}{2-2^q}\\ =& \frac{2\theta\left\|x,z\right\|^q 2^{(q-2)n}}{2-2^q}\\ \to& 0 \ as \ n\to\infty\end{aligned}$$

for each $x \in X$.

Hence $Q'(x) = Q(x)$ for each $x \in X$.

Corollary 2.2. *Let $\theta \geq 0, r > 0, 0 < p, q < 2$, $(X, \|.\|)$ be a real uniformed space. If $g : (X, \|.\|) \to (X, \|., .\|)$ is an even function satisfying the inequality*

$$\|T_f(x,y), z\| \leq \theta(\|x\|^p + \|y\|^q) \|z\|^r \tag{6}$$

for each $x, y, z \in X$. Then there exists a unique one-point compactification function $Q : X \to X$ satisfying (6) and

$$\|Q(x) - g(x), z\| \leq \frac{\theta \|x\|^q \|z\|^r}{4 - 2^q} \tag{7}$$

for each $x, z \in X$.

Proof: The proof follows from Theorem 2.1 by taking

$$\theta \left(\|x, z\|^p + \|y, z\|^q\right) = \theta \left(\|x\|^p + \|y\|^q\right) \|z\|^r \tag{8}$$

for all $x, y, z \in X$ which is the desired result.

3. Convergence in uniform continuity

Theorem 3.1. *Let $\theta \geq 0$ with $p, q > 2$. If $g : X \to X$ is an even function such that*

$$\|T_f(x,y), z\| \leq \theta(\|x, z\|^p + \|y, z\|^q) \tag{9}$$

for each $x, y, z \in X$. Then there exists a unique quadratic function $Q : X \to X$ satisfying (9) and

$$\|g(x) - Q(x), z\| \leq \frac{\theta \|x, z\|^q}{2^q - 4} \tag{10}$$

for each $x, z \in X$.

Proof: Put $x = 0$ in (9), we get

$$\|g(2y) - 4g(y), z\| \leq \theta \|y, z\|^q \tag{11}$$

for each $y, z \in X$. Replacing y by x in (11), we get

$$\|g(2x) - 4g(x), z\| \leq \theta \|x, z\|^q \tag{12}$$

for each $x, z \in X$. Replacing x by $\frac{x}{2}$ in (12), we get

$$\left\|f(x) - 4f\left(\frac{x}{2}\right), z\right\| \leq \theta \left\|\frac{x}{2}, z\right\|^q \tag{13}$$

for each $x, z \in X$. By using induction on n, we get

$$\begin{aligned}\left\|g(x) - 4^n g\left(\frac{x}{2^n}\right), z\right\| &\leq \frac{\theta}{2^q}\|x, z\|^q \sum_{j=0}^{n-1} 2^{(2-q)j} \\ &\leq \theta 2^{-q}\|x, z\|^q \left(\frac{1 - 2^{(2-q)n}}{1 - 2^{2-q}}\right) \qquad (14)\end{aligned}$$

for each $x, z \in X$.

Therefore $\left\{4^n g\left(\frac{x}{2^n}\right)\right\}$ is a Cauchy sequence in X, for each $x \in X$. Since X is a Hausdorff space, the sequence $\left\{4^n g\left(\frac{x}{2^n}\right)\right\}$ converges in X, for each $x \in X$. Define $Q : X \to X$ as

$$Q(x) = \lim_{n\to\infty} 4^n g\left(\frac{x}{2^n}\right)$$

for each $x \in X$. By (14), we have

$$\begin{aligned}\lim_{n\to\infty}\left\|f(x) - 4^n f\left(\frac{x}{2^n}\right), z\right\| &\leq \theta 2^{-q}\|x, z\|^q \left(\frac{1}{1 - 2^{2-q}}\right) \\ \|g(x) - Q(x), z\| &\leq \theta 2^{-q}\|x, z\|^q \left(\frac{2^q}{2^q - 4}\right) \\ &\leq \frac{\theta\|x, z\|^q}{2^q - 4}\end{aligned}$$

for each $x, z \in X$.

The rest of the proof is similar to the proof of Theorem 2.1.

Corollary 3.2. *Let $\theta \geq 0$, $r > 0$ with $p, q > 2$, $(X, \|.\|)$ be a real uniformed space. If $f : (X, \|.\|) \to (X, \|., .\|)$ is an even function such that*

$$\|T_f(x, y), z\| \leq \theta(\|x\|^p + \|y\|^q)\|z\|^r \qquad (15)$$

for each $x, y, z \in X$. Then there exists a unique quadratic function $Q : X \to X$ and satisfying (15) and

$$\|g(x) - Q(x), z\| \leq \frac{\theta\|x\|^q\|z\|^r}{2^q - 4} \qquad (16)$$

for each $x, z \in X$.

Proof: The proof follows from Theorem 3.1 by taking

$$\theta\left(\|x, z\|^p + \|y, z\|^q\right) = \theta\left(\|x\|^p + \|y\|^q\right)\|z\|^r \qquad (17)$$

for all $x, y, z \in X$ which leads to the expected result.

Theorem 3.3. *Let $\theta \geq 0, 0 < p, q < 3$. If $f : (X, \|.,.\|) \to (X, \|.,.\|)$ is an odd function such that*

$$\|T_f(x,y), z\| \leq \theta(\|x,z\|^p + \|y,z\|^q) \tag{18}$$

for each $x, y, z \in X$. Then there exists a mertic function $C : X \to X$ satisfying (18) and

$$\|C(x) - g(x), z\| \leq \frac{\theta \|x,z\|^q}{8 - 2^q} \tag{19}$$

for each $x, z \in X$.

Proof: Put $x = 0$ in (18), we get

$$\|g(2y) - 8g(y), z\| \leq \theta \|y,z\|^q \tag{20}$$

for each $y, z \in X$. Replacing y by x in (20), we get

$$\|g(2x) - 8g(x), z\| \leq \theta \|x,z\|^q \tag{21}$$

for each $x, z \in X$. Dividing by 8 in (21), we get

$$\left\|\frac{g(2x)}{8} - g(x), z\right\| \leq \frac{\theta \|x,z\|^q}{8} \tag{22}$$

for each $x, z \in X$. Therefore by using induction on n, we get

$$\begin{aligned} \left\|\frac{g(2^n x)}{8^n} - g(x), z\right\| &\leq \frac{\theta}{8}\|x,z\|^q \sum_{j=0}^{n-1} 2^{(q-3)j} \\ &\leq \frac{\theta}{8}\|x,z\|^q \left(\frac{1 - 2^{(q-3)n}}{1 - 2^{q-3}}\right) \end{aligned} \tag{23}$$

for each $x, z \in X$. Therefore $\left\{\frac{g(2^n x)}{8^n}\right\}$ is a Cauchy sequence in X, for each $x \in X$. Since X is a Hausdorff space, the sequence $\left\{\frac{g(2^n x)}{8^n}\right\}$ converges in X, for each $x \in X$. Define $A : X \to X$ as

$$C(x) = \lim_{n\to\infty} \frac{f(2^n x)}{8^n}$$

for each $x \in X$. By (23), we have

$$\begin{aligned}\lim_{n\to\infty}\left\|\frac{g(2^n x)}{8^n}-g(x),z\right\| &\leq \frac{\theta}{8}\|x,z\|^q\left(\frac{1}{1-2^{q-3}}\right)\\ \|C(x)-g(x),z\| &\leq \frac{\theta}{8}\|x,z\|^q\left(\frac{8}{8-2^q}\right)\\ &\leq \frac{\theta\|x,z\|^q}{8-2^q}\end{aligned}$$

for each $x, z \in X$.

The rest of the proof is similar to the proof of Theorem 2.1.

Corollary 3.4. *Let $\theta \geq 0, r > 0, 0 < p, q < 3$, $(X, \|.\|)$ be a real uniformed space. If $f : (X, \|.\|) \to (X, \|., .\|)$ is an odd function satisfying the inequality*

$$\|T_f(x,y),z\| \leq \theta(\|x\|^p + \|y\|^q)\|z\|^r \tag{24}$$

for each $x, y, z \in X$. Then there exists a unique inclusion function in topology $C : X \to X$ satisfying (24) and

$$\|C(x)-g(x),z\| \leq \frac{\theta\|x\|^q\|z\|^r}{8-2^q} \tag{25}$$

for each $x, z \in X$.

Proof: The proof follows from Theorem 3.3 by taking

$$\theta\left(\|x,z\|^p+\|y,z\|^q\right) = \theta\left(\|x\|^p+\|y\|^q\right)\|z\|^r \tag{26}$$

for all $x, y, z \in X$. Then we get the desired result.

Theorem 3.5. *Let $\theta \geq 0$, $p, q > 3$. If $g : X \to X$ be an odd function such that*

$$\|T_f(x,y),z\| \leq \theta(\|x,z\|^p + \|y,z\|^q) \tag{27}$$

for each $x, y, z \in X$. Then there exists a unique cubic function $C : X \to X$ satisfying (27) and

$$\|g(x)-C(x),z\| \leq \frac{\theta\|x,z\|^q}{2^q-8} \tag{28}$$

for each $x, z \in X$.

Proof: Put $x = 0$ in (27) , we get

$$\|g(2y) - 8g(y), z\| \leq \theta \|y, z\|^q \tag{29}$$

for each $y, z \in X$. Replacing y by x in (29), we get

$$\|g(2x) - 8g(x), z\| \leq \theta \|x, z\|^q \tag{30}$$

for each $x, z \in X$. Replacing x by $\frac{x}{2}$ in (30), we get

$$\left\|g(x) - 8g\left(\frac{x}{2}\right), z\right\| \leq \theta 2^{-q} \|x, z\|^q \tag{31}$$

for each $x, z \in X$. By using induction on n, we get

$$\begin{aligned} \left\|g(x) - 8^n f\left(\frac{x}{2^n}\right), z\right\| &\leq \theta 2^{-q} \|x, z\|^q \sum_{j=0}^{n-1} 2^{(3-q)j} \\ &\leq \theta 2^{-q} \|x, z\|^q \left(\frac{1 - 2^{(3-q)n}}{1 - 2^{3-q}}\right) \end{aligned} \tag{32}$$

for each $x, z \in X$.

Therefore $\left\{8^n f\left(\frac{x}{2^n}\right)\right\}$ is a Cauchy sequence in X, for each $x \in X$. Since X is a Hausdorff space, the sequence $\left\{8^n f\left(\frac{x}{2^n}\right)\right\}$ converges in X, for each $x \in X$. Define $C : X \to X$ as

$$C(x) = \lim_{n\to\infty} 8^n g(2^{-n}x)$$

for each $x \in X$. By 32, we have

$$\begin{aligned} \lim_{n\to\infty} \left\|g(x) - 8^n f\left(\frac{x}{2^n}\right), z\right\| &\leq \theta 2^{-q} \|x, z\|^q \left(\frac{1}{1 - 2^{3-q}}\right) \\ \|g(x) - C(x), z\| &\leq \frac{\theta}{2^q} \|x, z\|^q \left(\frac{2^q}{2^q - 8}\right) \\ &\leq \frac{\theta \|x, z\|^q}{2^q - 8} \end{aligned}$$

for each $x, z \in X$.

The rest of the proof is similar to the proof of Theorem 2.1.

Corollary 3.6. *Let $\theta \geq 0, r > 0$ with $p, q > 3$, $(X, \|.\|)$ be a real uniformed space. If $f : (X, \|.\|) \to (X, \|., .\|)$ is an odd function such that*

$$\|T_f(x, y), z\| \leq \theta(\|x\|^p + \|y\|^q) \|z\|^r \tag{33}$$

for each $x, y, z \in X$. Then there exists a unique cubic function $C : X \to X$ satisfying (33) and

$$\|g(x) - C(x), z\| \leq \frac{\theta \|x\|^q \|z\|^r}{2^q - 8} \tag{34}$$

for each $x, z \in X$.

Proof: The proof follows from Theorem 3.5 by taking

$$\theta \left(\|x, z\|^p + \|y, z\|^q\right) = \theta \left(\|x\|^p + \|y\|^q\right) \|z\|^r \tag{35}$$

for all $x, y, z \in X$. Then we get the desired result.

Theorem 3.7. *Let $\theta \geq 0, 0 < p < 3,\ f : X \to X$ be a function satisfying the inequality*

$$\|T_f(x, y), z\| \leq \theta(\|x, z\|^p + \|y, z\|^p) \tag{36}$$

for each $x, y, z \in X$. Then there exists a unique quadratic function $Q : X \to X$ and a unique cubic function $C : X \to X$ satisfying (36) and

$$\|f(x) - Q(x) - C(x), z\| \leq \theta \|x, z\|^p \left[\frac{1}{4 - 2^p} + \frac{1}{8 - 2^p}\right] \tag{37}$$

for each $x, z \in X$.

Proof: Let $x = y = 0$ in (36), we have $\|f(0), z\| = 0$ for each $z \in X$, so we have $f(0) = 0$. Define $f_o : X \to X,\ f_e : X \to X$ as

$$f_o(x) = \tfrac{1}{2}\left[f(x) - f(-x)\right]$$
$$f_e(x) = \tfrac{1}{2}\left[f(x) + f(-x)\right].$$

Then g_o is an odd function and g_e is an even function. Since $f(0) = 0$. We have $g_o(0) = g_e(0) = 0$. Also

$$\|D_{g_o}(x, y), z\| \leq \theta \left(\|x, z\|^p + \|y, z\|^p\right)$$
$$\|D_{g_e}(x, y), z\| \leq \theta \left(\|x, z\|^p + \|y, z\|^p\right)$$

for each $x, y, z \in X$.

Therefore by Theorem 2.1, there exists a unique quadratic function $Q : X \to X$ satisfying (36) and the inequality.

$$\|g_e(x) - Q(x), z\| \leq \frac{\theta \|x, z\|^p}{4 - 2^p} \tag{38}$$

for each $x, z \in X$.

Also by Theorem 3.3, there exists a unique cubic function $C : X \to X$ satisfying (38) and the inequality.

$$\|g_o(x) - C(x), z\| \leq \frac{\theta \|x, z\|^p}{8 - 2^p} \tag{39}$$

for each $x, z \in X$.

Now by (38) and (39), we have

$$\begin{aligned} \|g(x) - Q(x) - C(x), z\| &\leq \|g_e(x) - Q(x), z\| + \|g_o(x) - C(x), z\| \\ &\leq \left[\frac{\theta \|x, z\|^p}{4 - 2^p} + \frac{\theta \|x, z\|^p}{8 - 2^p}\right] \\ &\leq \theta \|x, z\|^p \left[\frac{1}{4 - 2^p} + \frac{1}{8 - 2^p}\right] \end{aligned}$$

for each $x, z \in X$.

Corollary 3.8. *Let $\theta \geq 0$, $r > 0$, $0 < p < 3$, $(X, \|.\|)$ be a real uniformed space. Suppose $f : (X, \|.\|) \to (X, \|., .\|)$ be a function satisfying the inequality*

$$\|T_f(x, y), z\| \leq \theta(\|x\|^p + \|y\|^p) \|z\|^r \tag{40}$$

for each $x, y, z \in X$. Then there exists a unique quadratic function $Q : X \to X$ and a unique cubic function $C : X \to X$ satisfying (40) and

$$\|f(x) - Q(x) - C(x), z\| \leq \theta \|x\|^p \|z\|^r \left[\frac{1}{4 - 2^p} + \frac{1}{8 - 2^p}\right] \tag{41}$$

for each $x, z \in X$.

Proof: The proof follows from Theorem 3.7 by taking

$$\theta \left(\|x, z\|^p + \|y, z\|^p\right) = \theta \left(\|x\|^p + \|y\|^p\right) \|z\|^r \tag{42}$$

for all $x, y, z \in X$. Then we get the desired result.

Theorem 3.9. *Let $\theta \geq 0, p > 2$, $f : X \to X$ be a function satisfying the inequality*

$$\|T_f(x, y), z\| \leq \theta(\|x, z\|^p + \|y, z\|^p) \tag{43}$$

for each $x, y, z \in X$. Then there exists a unique quadratic function $Q : X \to X$ and a unique cubic function $C : X \to X$ satisfying (43) and

$$\|f(x) - Q(x) - C(x), z\| \leq \theta \|x, z\|^p \left[\frac{1}{2^p - 4} + \frac{1}{2^p - 8}\right] \tag{44}$$

for each $x, z \in X$.

Proof: Let $x = y = 0$ in (43), we have $\|g(0), z\| = 0$ for each $z \in X$, so we have $f(0) = 0$. Define $f_o : X \to X,\ f_e : X \to X$ as

$$f_o(x) = \tfrac{1}{2}\left[f(x) - f(-x)\right]$$
$$f_e(x) = \tfrac{1}{2}\left[f(x) + f(-x)\right].$$

Then f_o is an odd function and f_e is an even function. Since $g(0) = 0$. We have $g_o(0) = g_e(0) = 0$. Also

$$\|T_{f_o}(x, y), z\| \leq \theta\left(\|x, z\|^p + \|y, z\|^p\right)$$
$$\|T_{f_e}(x, y), z\| \leq \theta\left(\|x, z\|^p + \|y, z\|^p\right)$$

for each $x, y, z \in X$.

Therefore by Theorem 3.1, there exists a unique quadratic function $Q : X \to X$ satisfying (43) and the inequality.

$$\|g_e(x) - Q(x), z\| \leq \frac{\theta \|x, z\|^p}{2^p - 4} \tag{45}$$

for each $x, z \in X$.

Also by Theorem 3.5, there exists a unique cubic function $C : X \to X$ satisfying (45) and the inequality.

$$\|g_o(x) - C(x), z\| \leq \frac{\theta \|x, z\|^p}{2^p - 8} \tag{46}$$

for each $x, z \in X$. Now by (45) and (46), we have

$$\begin{aligned}\|g(x) - Q(x) - C(x), z\| &\leq \|f_e(x) - Q(x), z\| + \|f_o(x) - C(x), z\| \\ &\leq \theta \|x, z\|^p \left[\frac{1}{2^p - 4} + \frac{1}{2^p - 8}\right]\end{aligned}$$

for each $x, z \in X$.

Corollary 3.10. *Let $\theta \geq 0, r > 0,\ p > 2,\ (X, \|.\|)$ be a real uniformed space. Suppose $f : (X, \|.\|) \to (X, \|.,.\|)$ be a function satisfying the inequality*

$$\|T_f(x, y), z\| \leq \theta(\|x\|^p + \|y\|^p)\, \|z\|^r \tag{47}$$

for each $x, y, z \in X$. Then there exists a unique quadratic function $Q : X \to X$ and a unique cubic function $C : X \to X$ satisfying (47) and

$$\|g(x) - Q(x) - C(x), z\| \leq \theta \|x\|^p \|z\|^r \left[\frac{1}{2^p - 4} + \frac{1}{2^p - 8}\right] \tag{48}$$

for each $x, z \in X$.

Proof: The proof follows from Theorem 3.9 by taking

$$\theta \left(\|x, z\|^p + \|y, z\|^p\right) = \theta \left(\|x\|^p + \|y\|^p\right) \|z\|^r \tag{49}$$

for all $x, y \in X$ which leads to an expected result.

4. CONCLUSION

The compactness and convergence of Hausdorff spaces and its subspaces in the real valued domain are identified in the half range domain. Uniqueness of one-point compactification function and its quadratic nature is arrived in this paper and the same is verified in 2^3. Convergence analysis of quadratic function along with the cubic nature is optimized.

REFERENCES

[1] Abbas M, Khan A R, and Nazir T, Coupled common fixed point results in two generalized metric spaces, Applied Mathematics and Computation, vol. 217, no. 13, pp. 63286336, 2011.

[2] Alexandro P.S., Introduction to set theory and general topology, Nauka, Moscow, 1977.

[3] Alexandro P.S., Pasynkov B.A., Introduction to dimension theory, Nauka, Moscow 1973.

[4] Borubaev A.A., On one generalization of metric, normed and unitary spaces, Dokl. RAN, 455(2), pp.127-129, 2014.

[5] Engelking R, General Topology (Heldermann, Berlin, 1989).

[6] Kanetov, B.E., On some classes of uniform spaces and uniformly continuous mappings, Bishkek, pp.1-160 , 2013.

[7] OlaleruJ O and Olaoluwa H O, Commonfixedpoints of four mappings satisfying weakly contractive-like condition in cone metric spaces, AppliedMathematical Sciences, vol. 7, no. 5760, pp. 28972908, 2013.

[8] Sadiq Basha S, Best proximity points: optimal solutions, Journal of OptimizationTheory and Applications, vol. 151, no. 1, pp. 210216, 2011.

[9] Yu.V. Sadovnichy, On uniformity on probability measures spaces, in: V.V. Fedorchuk, ed., General Topology. Mappings, Products and Dimension of Spaces , 119-131, 1994.