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Characterizations of Bi-Maximal Spaces in Soft Topological Spaces

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Abstract

This paper investigates the ideas of soft αBbi-resolvable sets, soft αBbi-maximal spaces, soft αBsub-maximal spaces and soft αBbi-resolvable spaces. Some interesting properties and characterizations between them are established.

1 Introduction

Authors [4], [5], [3] and [2] said the concepts of soft sets, applications of soft sets and soft topological spaces. The notions of open sets, closed sets, closure, interior, neighbourhood of a point and seperation axioms are also introduced by soft set and their basic properties are investigated by Muhammad Shabir and Munazza Naz [5]. Zorulutna et al [8] introduced the concepts of soft continuity and soft compactness. They studied relationships between soft topology and fuzzy topology. S. Subashini and D. Vidhya [7] carried out the concept of soft αB-continuous function. This paper analyzes the concept of soft αBbi-maximal spaces. Also discussed the characterizations and interrelations are investigated.

2 Preliminaries

Definition 2.1. [4] A soft set is a pair (G, ν) over the universal set U, where G is a function $G : \nu \to P(U)$ where $\nu \subseteq E$, E is a parameter.

Definition 2.2. [5] A soft topology is a collection T on X having the following conditions

(i) Φ,$\widetilde{X}$ belongs to T,

(ii) $\cup_{j\in A}(J_i, \omega_i) \in T$,

(iii) $\cap_{j\in A}(J_i, \omega_i) \in T$ where A is finite.

(X, T, E) is a soft topological space over X. Every members of T are open sets and its complement are closed sets.

Definition 2.3. [5] If there exists a open set (H, δ) such that $x \in (H, \delta) \subseteq (F, \mu)$ over X then (F, μ) is soft neighborhood of x.

Definition 2.4. [7] The intersection of soft α-open(closed) and soft t-open(closed) is soft αB-open(closed).

Definition 2.5. [1] If every dense set is open then (X, T) is said to be a sub-maximal space.

3 Soft αBbi-maximal Spaces, Soft αBsub-maximal Spaces and Soft αBbi-resolvable Spaces

3.1 Soft αBbi-maximal Spaces

Definition 3.1. A soft set (C, γ) is a soft αB-neighborhood of x, if there exists a αB-open (D, ψ) such that $x \in (D, \psi) \subseteq (C, \gamma)$.

Definition 3.2. x is a soft αB-limit point of (F, μ), if every soft αB-neighborhood of x intersects (F, μ) in some point other than x itself.
The group of αB-limit points of (F, μ) is a soft αB-derived set of (F, μ) and is represented by $\alpha Bd(F, \mu)$.

Remark 3.1. The following conditions hold in (X, T, E)

(i) $\alpha Bd(\Phi) = \Phi$, $\alpha Bd(\widetilde{X}) = \widetilde{X}$.

(ii) $\alpha Bd(\bigcup\{A_i/i \in I\}) = \bigcup\{\alpha Bd(A_i)/i \in I\}$.

(iii) $\alpha Bd(\alpha Bd(G, \varepsilon)) = \alpha Bd(G, \varepsilon)$.

Definition 3.3. For any closed set (H, η), $\alpha Bd(F, \mu) \subseteq (H, \eta)$.

Definition 3.4. If $s\alpha Bcl(I, \theta) \sqcup s\alpha Bcl(I, \theta)' = \widetilde{X}$ for any soft set (I, θ) then (I, θ) is said to be soft αBbi-closure set. The complement of αBbi-closure is αBbi-interior.

Note 3.1. (i) $s\alpha Bcl(F, \mu) \sqcup s\alpha Bcl(F, \mu)'$ denoted by $s\alpha B\mathcal{BC}(F, \mu)$.

(ii) $s\alpha Bint(F, \mu) \sqcap s\alpha Bint(F, \mu)'$ denoted by $s\alpha B\mathcal{BI}(F, \mu)$.

Definition 3.5. Every soft αBbi-closure is soft αB-open then (X, T, E) is αBbi-maximal.

Remark 3.2. (i) $s\alpha B\mathcal{BC}(F,\mu)' = s\alpha B\mathcal{BC}(F,\mu)$.

(ii) $s\alpha B\mathcal{BI}(G,\nu)' = s\alpha B\mathcal{BI}(G,\nu)$.

Proposition 3.1. The following conditions are equivalent in (X, T, E).

(i) A soft αBbi-maximal space.

(ii) s$\alpha B\mathcal{BC}(J,\omega)$-$(J,\omega)$ is soft αB-closed, for any soft set (J,ω).

(iii) For each soft set (J,ω), if s$\alpha B\mathcal{BI}(J,\omega) = \Phi$, then (J,ω) is soft αB-closed and $\alpha Bd(J,\omega) = \Phi$.

(iv) s$\alpha B\mathcal{BC}(J,\omega)$-$(J,\omega)$ is soft αB-closed and αBd$(J,\omega) = \Phi$, for each soft set (J,ω).

Proof. (ii)⇒(i) Suppose (J,ω) is a soft αBbi-closure set. Consider,

$$\begin{aligned} s\alpha B\mathcal{BC}(J,\omega) - (J,\omega) &= [s\alpha Bcl(J,\omega) \sqcup s\alpha Bcl(J,\omega)'] \sqcap (J,\omega)' \\ &= \widetilde{X} \sqcap (J,\omega)' \\ &= (J,\omega)'. \end{aligned}$$

Since $s\alpha B\mathcal{BC}(J,\omega) - (J,\omega)$ then (J,ω) is soft αB-closed, (J,ω) is soft αB-open. Hence (X, T, E) is soft αBbi-maximal.

(i)⇒(iii) Let $s\alpha B\mathcal{BI}(J,\omega) = \Phi$. Then,$\{s\alpha B\mathcal{BI}(J,\omega)\}' = s\alpha B\mathcal{BC}(J,\omega)' = \widetilde{X}$. Thus, $(J,\omega)'$ is soft αBbi-closure. Since (X, T, E) is αBbi-maximal, $(J,\omega)'$ is soft αB-open set. Then, (J,ω) is a soft αB-closed set and $\alpha Bd(J,\omega) = \Phi$.

(iii)⇒(i) Let (J,ω) be a soft αBbi-closure set. Then,

$$\begin{aligned} s\alpha B\mathcal{BC}(J,\omega) &= \widetilde{X} \\ \Rightarrow s\alpha B\mathcal{BI}(J,\omega)' &= \Phi. \end{aligned}$$

By (iii), $(J,\omega)'$ is soft αB-closed. Thus, (J,ω) is soft αB-open and (i) is soft αBbi-maximal.

(iii)⇒(ii) Let (J,ω) be a soft set. Since by (iii), $s\alpha B\mathcal{BI}(J,\omega) = \Phi$, then $s\alpha B\mathcal{BI}(s\alpha B\mathcal{BC}(J,\omega) - (J,\omega)) = \Phi$. Hence, $s\alpha B\mathcal{BC}(J,\omega) - (J,\omega)$ is a soft αB-closed set.

(ii)⇒(iii) Suppose, $s\alpha B\mathcal{BI}(J,\omega) = \Phi$. Then,

$$\begin{aligned} (J,\omega)' &= (J,\omega)' \sqcup s\alpha B\mathcal{BI}(J,\omega) \\ &= (J,\omega)' \sqcup [s\alpha B\mathcal{BC}(J,\omega)']' \\ &= [(J,\omega) \sqcap s\alpha B\mathcal{BC}(J,\omega)']' \\ &= [s\alpha B\mathcal{BC}(J,\omega)' - (J,\omega)']'. \end{aligned}$$

Thus $(J,\omega)'$ is soft αB-open set. Hence (J,ω) is a soft αB-closed set. Thus, $\alpha Bd(J,\omega) \sqsubseteq (J,\omega)$. It is enough to prove that $\alpha Bd(s\alpha B\mathcal{BC}(J,\omega)) = \Phi$. Suppose not. Let $x \in s\alpha B\mathcal{BC}(J,\omega)$.
Since, $s\alpha B\mathcal{BI}(s\alpha B\mathcal{BC}(J,\omega)) = \Phi$, $s\alpha B\mathcal{BI}(s\alpha B\mathcal{BC}(J,\omega) - \{x\}) = \Phi$. Thus, $(s\alpha B\mathcal{BC}(J,\omega) - \{x\})$ is soft αB-closed and $\{x\} \sqcup s\alpha B\mathcal{BC}(J,\omega)'$ is soft αB-open. But then there is a soft αB-open neighborhood $(M,\vartheta) = \{x\} \sqcup s\alpha B\mathcal{BC}(J,\omega)'$ of x such that $(M,\vartheta) \sqcap (s\alpha B\mathcal{BC}(J,\omega) - \{x\}) = \Phi$. Thus, $x \notin \alpha Bd(s\alpha B\mathcal{BC}(J,\omega))$. It follows that $\alpha Bd(s\alpha B\mathcal{BC}(J,\omega)) = \Phi$. Therefore, $s\alpha Bcl(J,\omega) \sqcup s\alpha Bcl(J,\omega)'$ is soft closed and $\alpha Bd(J,\omega) = \Phi$.

(iii)$\Rightarrow$(iv) Since,

$$\begin{aligned}
s\alpha B\mathcal{BI}[s\alpha B\mathcal{BC}(J,\omega) - (J,\omega)] &= s\alpha Bint[\{s\alpha Bcl(J,\omega) \sqcup s\alpha Bcl(J,\omega)'\} - (J,\omega)] \sqcap \\
&\quad s\alpha Bint[\{s\alpha Bcl(J,\omega) \sqcup s\alpha Bcl(J,\omega)'\} - (J,\omega)]' \\
&= s\alpha Bint[(J,\omega) \sqcap \{s\alpha Bcl(J,\omega) \sqcup s\alpha Bcl(J,\omega)'\}'] \sqcap \\
&\quad s\alpha Bint[(J,\omega) \sqcap \{s\alpha Bcl(J,\omega) \sqcup s\alpha Bcl(J,\omega)'\}']' \\
&= s\alpha Bint[(J,\omega) \sqcap \{s\alpha Bint(J,\omega) \sqcap s\alpha Bint(J,\omega)'\}] \sqcap \\
&\quad s\alpha Bint[(J,\omega) \sqcap \{s\alpha Bint(J,\omega) \sqcap s\alpha Bint(J,\omega)'\}]' \\
&= \Phi \sqcap \widetilde{X} \\
&= \Phi.
\end{aligned}$$

By (iii), $s\alpha B\mathcal{BC}(J,\omega) - (J,\omega)$ is soft αB-closed. Hence, $s\alpha B\mathcal{BC}(J,\omega) - (J,\omega)$ is soft αB-closed and $\alpha Bd(J,\omega) = \Phi$

(iv)$\Rightarrow$(i) Let (J,ω) be a soft αBbi-closure set. Then, $s\alpha B\mathcal{BC}(J,\omega) = \widetilde{X}$. Since by (iv), $s\alpha B\mathcal{BC}(J,\omega) - (J,\omega)$ is soft αB-closed, $(J,\omega)'$ is soft αB-closed. Thus (J,ω) is soft αB-open. Hence (X,T,E) is soft αBbi-maximal. □

3.2 Soft αBsub-maximal Spaces

Definition 3.6. For any soft set (K,χ) is a soft αB-dense set, if $s\alpha Bcl(K,\chi) = \widetilde{X}$.

Definition 3.7. A soft topological space is said to be αBsub-maximal space if every αB-dense is αB-open.

Proposition 3.2. Prove that the conditions below are equivalent.

(i) (X,T,E) is αBsub-maximal.

(ii) $s\alpha Bcl(F,\mu) - (F,\mu)$ is soft αB-closed where (F,μ) is any soft set .

(iii) If $s\alpha Bint(F,\mu) = \Phi$, then (F,μ) is soft αB-closed and $\alpha Bd(F,\mu) = \Phi$ for each soft set (F,μ).

(iv) $s\alpha Bcl(F,\mu) - (F,\mu)$ is soft αB-closed and $\alpha Bd(F,\mu) = \Phi$, for any soft set (F,μ).

Proof. This proof is similar to the Proposition 3.1. □

3.3 Soft αBbi-resolvable Spaces

Definition 3.8. If (P, ϱ) and $s\alpha Bint(P, \varrho)'$ are soft αBbi-closure sets then (P, ϱ) is called a αBbi-resolvable set.

Definition 3.9. If each soft αBbi-resolvable set is αB-open then (X, T, E) is αBbi-resolvable.

Proposition 3.3. The conditions (i),(ii) and (iii)are equivalent.

(i) (X, T, E) is soft αBbi-resolvable space.

(ii) For every soft set (R, ϵ), $s\alpha B\mathcal{BC}(R, \epsilon) - (R, \epsilon)$ is soft αB-closed.

(iii) Any soft set (S, ρ), if s$\alpha B\mathcal{BC}(S, \rho) = \Phi$ and s$\alpha B\mathcal{BI}(s\alpha Bint(S, \rho)') = \Phi$, then (S, ρ) is soft αB-closed and $\alpha Bd(S, \rho) = \Phi$.

(iv) $s\alpha B\mathcal{BC}(R, \epsilon) - (R, \epsilon)$ is soft αB-closed and $\alpha Bd(R, \epsilon) = \Phi$, for each soft set (R, ϵ).

Proof. (ii)$\Rightarrow$(i) Let (R, ϵ) be a soft αBbi-resolvable set. Then (R, ϵ) and $s\alpha Bint(R, \epsilon)'$ is a soft αBbi-closure set. Consider, $s\alpha B\mathcal{BC}(R, \epsilon) - (R, \epsilon) = [s\alpha Bcl(R, \epsilon) \sqcup s\alpha Bcl(R, \epsilon)'] \sqcap (R, \epsilon)' = \widetilde{X} \sqcap (R, \epsilon)' = (R, \epsilon)'$. Since $s\alpha B\mathcal{BC}(R, \epsilon) - (R, \epsilon)$ is soft αB-closed then (R, ϵ) is αB-open. Hence (X, T, E) is αBbi-resolvable.

(i)$\Rightarrow$(iii) Suppose, $s\alpha B\mathcal{BI}(R, \epsilon) = \Phi$ and $s\alpha B\mathcal{BI}(s\alpha Bint(R, \epsilon)') = \Phi$. Then, $\{s\alpha B\mathcal{BI}(R, \epsilon)\}' = s\alpha B\mathcal{BC}(R, \epsilon)' = \widetilde{X}$.

Thus $(R, \epsilon)'$ is soft αBbi-closure. Since (X, T, E) is αBbi-resolvable, $(R, \epsilon)'$ is αB-open. Then, (R, ϵ) is a soft αB-closed set and $\alpha Bd(R, \epsilon) = \Phi$.

(iii)$\Rightarrow$(iv)

$$\begin{aligned}
&s\alpha B\mathcal{BI}[s\alpha B\mathcal{BC}(S, \rho) - (S, \rho)] \\
&\quad = s\alpha Bint[\{s\alpha Bcl(R, \epsilon) \sqcup s\alpha Bcl(S, \rho)'\} - (S, \rho)] \sqcap \\
&\qquad s\alpha Bint[\{s\alpha Bcl(S, \rho) \sqcup s\alpha Bcl(S, \rho)'\}(S, \rho)]' \\
&\quad = s\alpha Bint[(S, \rho) \sqcap \{s\alpha Bcl(S, \rho) \sqcup s\alpha Bcl(S, \rho)'\}'] \sqcap \\
&\qquad s\alpha Bint[(S, \rho) \sqcap \{s\alpha Bcl(S, \rho) \sqcup s\alpha Bcl(S, \rho)'\}']' \\
&\quad = s\alpha Bint[(S, \rho) \sqcap \{s\alpha Bint(S, \rho) \sqcap s\alpha Bint(S, \rho)'\}] \sqcap \\
&\qquad s\alpha Bint[(S, \rho) \sqcap \{s\alpha Bint(S, \rho) \sqcap s\alpha Bint(S, \rho)'\}]' \\
&\quad = \Phi \sqcap \widetilde{X} \\
&\quad = \Phi.
\end{aligned}$$

Also,

$$\begin{aligned}
&s\alpha B\mathcal{BI}[s\alpha Bint(s\alpha B\mathcal{BC}(S, \rho) - (S, \rho))]' \\
&\quad = s\alpha Bint[s\alpha Bint(s\alpha B\mathcal{BC}(S, \rho) - (S, \rho))]' \sqcap \\
&\quad = \Phi.
\end{aligned}$$

By (iii), $s\alpha B\mathcal{BC}(S, \rho) - (S, \rho)$ is a soft αB-closed set and $\alpha Bd(S, \rho) \sqsubseteq (S, \rho)$. It is enough to prove that $\alpha Bd(s\alpha B\mathcal{BC}(S, \rho)) = \Phi$. Suppose not. Let $x \in s\alpha B\mathcal{BC}(S, \rho)$. Since, $s\alpha B\mathcal{BI}(s\alpha B\mathcal{BC}(S, \rho)) = \Phi$, $s\alpha B\mathcal{BI}(s\alpha B\mathcal{BC}(S, \rho)) - \{x\} = \Phi$. Thus, $(s\alpha B\mathcal{BC}(S, \rho) -$

$\{x\}$) is soft αB-closed and $\{x\} \sqcup s\alpha B\mathcal{BC}(S,\rho)'$ is soft αB-open. But then there is a soft αB-open neighborhood $U = \{x\} \sqcup s\alpha B\mathcal{BC}(S,\rho)'$ of x such that $U \sqcap (s\alpha B\mathcal{BC}(S,\rho) - \{x\}) = \Phi$. Thus, $x \notin \alpha Bd(s\alpha B\mathcal{BC}(S,\rho))$. It follows that $\alpha Bd(s\alpha B\mathcal{BC}(S,\rho)) = \Phi$. Therefore, $s\alpha Bcl(S,\rho) \sqcup s\alpha Bcl(S,\rho)'$ is soft closed and $\alpha Bd(S,\rho) = \Phi$.

(ii)⇒(iii) Suppose s$\alpha B\mathcal{BI}(R,\epsilon) = \Phi$. Then,

$$\begin{aligned}(R,\epsilon)' &= (R,\epsilon)' \sqcup s\alpha B\mathcal{BI}(R,\epsilon)\\ &= (R,\epsilon)' \sqcup [s\alpha B\mathcal{BC}(R,\epsilon)']'\\ &= [(R,\epsilon) \sqcap s\alpha B\mathcal{BC}(R,\epsilon)]'.\end{aligned}$$

By (ii), $[s\alpha B\mathcal{BC}(R,\epsilon)' - (R,\epsilon)']'$ is soft αB-open set. Hence, (R,ϵ) is soft αB-closed and $\alpha Bd(R,\epsilon) = \Phi$.

(iii)⇒(i) Let (R,ϵ) be a soft αBbi-resolvable set. Then, (R,ϵ) and $s\alpha Bint(R,\epsilon)'$ are soft αBbi-closure sets. Since, $s\alpha B\mathcal{BC}(R,\epsilon) = \widetilde{X}$ and $s\alpha B\mathcal{BC}(s\alpha Bint(R,\epsilon)') = \widetilde{X}$. Thus, $s\alpha B\mathcal{BI}(R,\epsilon)' = \Phi$. By (iii), $(R,\epsilon)'$ is a αB-closed set. Thus (R,ϵ) is a soft αB-open set. Hence, the space (X,T,E) is soft αBbi-resolvable.

Proof (iii)⇒(ii), (iv)⇒(i) and (iv)⇒(iii) is similar to (iii)⇒(iv), (ii)⇒(i) and (ii)⇒(iii) respectively. □

4 Interrelations

Remark 4.1. The interrelations among the spaces introduced are given clearly in the following diagram and the converses need not to be true as shown in the following Example 4.1, 4.2 and 4.3.

$$\begin{array}{c}\text{soft } \alpha B\text{bi-resolvable space}\\ \Uparrow\\ \text{soft } \alpha B\text{bi-maximal space}\\ \Downarrow\\ \text{soft } \alpha B\text{sub-maximal space}\end{array}$$

Example 4.1. $X = \{a,b,c\}$ and $E = \{e_1,e_2,e_3\}$ and $\varsigma = \{e_1,e_2\}$. Let F,G,H,I be mapping from E to $P(X)$ defined by, $(F,\varsigma) = \{(e_1,\{a\}),(e_2,\{b,c\})\}$, $(G,\varsigma) = \{(e_1,\{b\}),(e_2,\{a\})\}$, $(H,\varsigma) = \{(e_1,\{a,b\}),(e_2,\{\phi\})\}$, $(I,\varsigma) = \{(e_1,\{a,b\}),(e_2,\{\widetilde{X}\})\}$ are soft sets. Then $T = \{\Phi,\widetilde{X},(F,\varsigma),(G,\varsigma),(H,\varsigma),(I,\varsigma)\}$ is a soft topological space. Here (F,ς) is soft αBbi-closure but not a soft αB-dense.

Example 4.2. Let $X = \{a,b,c\}$ and $E = \{e_1,e_2,e_3\}$ and $\nu = \{e_1,e_2\}$. Let F,G be mapping from E to $P(X)$ defined by, $(F,\nu) = \{(e_1,\{a\}),(e_2,\{c\})\}$, $(G,\nu) = \{(e_1,\{b,c\}),(e_2,\{a,b\})\}$, are soft sets over X. $T = \{\Phi,\widetilde{X},(F,\nu),(G,\nu)\}$ is a soft topological space. Take $(H,\nu) = \{(e_1,\{c\}),(e_2,\{a\})\}$. Here (H,ν) is αBbi-closure but not αB-open. Therefore (X,T,E) is αBsub-maximal but not αBbi-maximal.

Example 4.3. Let $X = \{a, b, c\}$ and $E = \{e_1, e_2, e_3\}$ and $\kappa = \{e_1, e_2\}$. Let F, G be mapping from E to $P(X)$ defined by the soft sets, $(F, \kappa) = \{(e_1, \{a\}), (e_2, \{c\})\}$, $(G, \kappa) = \{(e_1, \{b, c\}), (e_2, \{a, b\})\}$. Thus T is $T = \{\Phi, \widetilde{X}, (F, \kappa), (G, \kappa)\}$. For any soft set $(H, \kappa) = \{(e_1, \{c\}), (e_2, \{a\})\}$ is αBbi-closure but not αB-open. (X, T, E) is αBbi-resolvable but not αBbi-maximal.

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