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Existence, Uniqueness And Stability Results Of Random Impulsive Semilinear Integro-Differential Systems

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Abstract: In this article, we study the existence, uniqueness and stability of random impulsive semilinear integro-differential systems. The results are obtained by using the contraction mapping principle. finally an example is given to illustrate the applications of the abstract results.

Key words : Semilinear integro-differential equation; random impulses; stability; contraction principle.
Subject classification : 35R09 ; 35R60; 35B35; 34D99.

1. INTRODUCTION

Impulsive differential equation have become more important in recent years in some mathematical models of processes and phenomena studied in physics, optimal control, chemotherapy, biotechnology, population dynamics and ecology. There have been much research activity concerning the theory of impulsive differential equations see [2-6]. The impulses may exists at deterministic or random points. There are lot of papers which investigate the properties of deterministic impulses see [9] and the references therein.

Thus the random impulsive equations gives more realistic than deterministic impulsive equations.There are few publications in this field, Wu and Duan brought forword random impulsive ordinary differential equations and investigated boundedness of soloutions to these models by Liapunov's direct function in [11] . Wu et al, studied some qualitative properties of random impulses in [7,8,10]. In [12-14] the author studied the existence results for the random impulsive neutral functional differential equations and differential inclutions with delays. In [13] , the authors generalized the distribution of random impulses with the Erlang distribution.

The stabilities like continuous dependence, Hyers-Ulam stability, Hyers- Ulam-Rassias stability, exponential stability and asymptotic stability have attracted the attention of many mathematicians see [15-18]. Motivated by the above mentioned works, the main purpose of this paper is to study of random impulsive semilinear integrodifferential systems. We relaxed the Lipchitz condition on the impulsive term and under our assumption it is enough to be bounded.

This article is organized as follows: In section 2, we recall some notations, definitions, concepts of random impulsive semilinear integrodifferential systems, In section 3, the assumptions, existence and uniqueness of solutions of random impulsive semilinear integrodifferential systems, In section 4, we study the stability of random impulsive semilinear integrodifferential systems, In section 5, we provide an example to illustrate the applications of the obtained result.

2. PRELIMINARIES

Let ⦀ • ⦀ denote the Euclidean norm in $\mathcal{R}^n$. If B is a vector or a matrix, its transpose is denoted by B^T; if b is a matrix, its Frobenius norm is represented by ⦀ B ⦀ = $\{trace(B^T B)\}^{\frac{1}{2}}$. Let $\mathcal{R}^n$ be the n-dimensional Euclidean space and Ω a nonempty set. Assume that δ_k is a random variable defined from Ω to $D_k \overset{\text{def}}{=} (0\ , d_k)$ for all k=1,2,…where $0< d_k < +\infty$. Furthermore, assume that δ_i and δ_j are independent with each other as $i \neq j$ for i,j = 1,2,…. Let δ,T ∈ $\mathscr{R}$ be two constants satisfying δ < T. For the sake of simplicity, we denote $\mathcal{R}_\delta = [\delta, T]$.

We consider semilinear integro- differential systems with random impulses of the form

x' (t) = Bx(t) + f (t, x(t)) + $\int_0^t g(s, x_s)$, $t \neq \eta_k$, $t \geq \delta$, (2.1

$x(\eta_k) = b_k(\delta_k)\, x(\eta_k^-)$, k = 1,2,… , (2.2)

$x_{t_0} = x_0$ (2.3)

Where B is a matrix of dimension $n \times n$: the functions f,g : $\mathcal{R}^n \times \mathcal{R}^n \rightarrow \mathcal{R}^n$; $b_k : D_k \rightarrow \mathcal{R}^{n \times n}$ is a matrix valued function for each k = 1,2,…; $\eta_0 = t_0$ and $\eta_k = \eta_{k-1} + \delta_k$ for k = 1,2,…, here $t_0 \in \mathcal{R}_\delta$ is arbitrary real number. Obviously, $t_0 = \eta_0 < \eta_1 < \ldots < \eta_k < \ldots$; $x(\eta_k^-) = \lim_{t\uparrow\eta_k} x(t)$ according to their paths with the norm ⦀ x ⦀ = $sup_{\delta \leq s \leq t}$ ‖ x(s) ‖ for each t satisfying $\delta \leq s \leq T$.

Let us denote $\{A_t , t \geq 0\}$ by the simple counting process generated by $\{\eta_n\}$, that is , $\{A_t \geq n\} = \{\eta_n \leq t\}$, and denote $\mathcal{F}_t$ the $\sigma - algebra$ generated by $\{A_t , t \geq 0\}$. Then $(\Omega , P, \{\mathcal{F}_t\})$ is a probability space. Let $\mathfrak{B}$ be the Banach space with the norm defined for any $\psi \in \mathfrak{B}$, $\| \psi\|^2 = (sup_{t\in[\delta,T]}\, E\| \psi(t)\|^2)$, where $\psi(t)$, for any given t ∈ $[\delta, T]$.

Definition 2.1: For a given T ∈ $(\delta, +\infty)$, a stochastic process {x(t), $\delta \leq t \leq T$} is called a solution to equations (2.1)-(2.3) in $(\Omega , P, \{\mathcal{F}_t\})$, if

(i) x(t) is $\mathcal{F}_t$ - adapted;

(ii) x(t) = $\sum_{k=0}^{+\infty}$ [$\prod_{i=1}^{k} b_i(\delta_i)\, \Phi(t-t_0)x_0$ + $\sum_{i=1}^{k}\prod_{j=1}^{k} b_j(\delta_j)\int_{\eta_{i-1}}^{\eta_i}\Phi(t-s)$[f (s, x(s)) + $\int_0^s g(\tau,x_\tau)$]ds + $\int_{\eta_k}^{t}\Phi(t-s)$[f (s, x(s)) + $\int_0^s g(\tau,x_\tau)$]ds] $I_{[\eta_k,\eta_{k+1})}$(t) , t ∈ $[\delta,T]$, (2.4)

where $\prod_{j=m}^{n}(.) = 1$ as m > n, $\prod_{j=1}^{k} b_j(\delta_i) = b_k(\delta_k)\, b_{k-1}(\delta_{k-1})\ldots b_i(\delta_i)$, $I_B(.)$ is the index function, ie., $I_B(t) = \begin{cases} 1, & if\ t\ \in B \\ 0, & if\ t\ \notin B \end{cases}$.

3. EXISTENCE AND UNIQUENESS

In this section we give the existence and uniqueness of the system (2.1) – (2.3). We start with thefollowing assumptions,

(I) The function f satisfies the Lipchitz condition. ie; for $\alpha,\gamma \in \mathcal{R}^n$ and $\delta \le t \le T$ there exists a constant L > 0 such that E$\| f(t,\alpha) - f(t,\gamma)\|^2 \le LE\ \|\alpha-\gamma\|^2$, E$\| f(t,0)\|^2 \le \frac{k}{2}$, $where\ k > 0\ is\ a\ constant.$

(II) The condition $\max_{i,k}\{\prod_{j=i}^{k}\|b_j(\delta_j)\|\}$ is uniformly bounded if, there is a constant C > 0 such that $\max_{i,k}\{\prod_{j=i}^{k}\|b_j(\delta_j)\|\} \le C\ for\ all\ \ \delta_j \in D_j$,j = 1,2,…

Theorem 3.1 : Let the hypotheses (I) ,(II) hold. If the following inequality
$\Delta = M^2 \max\{1, C^2\} L(T-\delta)^2\,[1+(T-\delta)] < 1$, is satisfied , then the (2.1)-(2.3) has a unique solution in $\mathfrak{B}$.

Proof : Let T be an arbitrary number $\delta \le T < +\infty$. First we define the nonlinear operator ⊕ : $\mathfrak{B} \to \mathfrak{B}$ as follows

$(\oplus x)$(t) = $\sum_{k=0}^{+\infty}$ [$\prod_{i=1}^{k} b_i(\delta_i)\,\Phi(t-t_0)x_0$ + $\sum_{i=1}^{k}\prod_{j=1}^{k} b_j(\delta_j)\int_{\eta_{i-1}}^{\eta_i}\Phi(t-s)$[f (s, x(s)) + $\int_0^s g(\tau,x_\tau)$]ds + $\int_{\eta_k}^{t}\Phi(t-s)$[f (s, x(s)) + $\int_0^s g(\tau,x_\tau)$]ds] $I_{[\eta_k,\eta_{k+1})}$(t) , t ∈ $[\delta,T]$,

It is easy to prove the continuity of ⊕ . Now , we have to show that ⊕ $maps$ $\mathfrak{B}$ into itself.

$\|(\oplus x)(t)\|^2 \le [\sum_{k=0}^{+\infty} [\ \|\prod_{i=1}^{k} b_i(\delta_i)\|\|\Phi(t-t_0)\|\|x_0\| + \sum_{i=1}^{k}\|\prod_{j=1}^{k} b_j(\delta_j)\|\int_{\eta_{i-1}}^{\eta_i}\|\Phi(t-s)\,[f(s,x(s)) + \int_0^s g(\tau,x_\tau)]\|ds + \int_{\eta_k}^{t}\|\Phi(t-s)\,[f(s,x(s)) + \int_0^s g(\tau,x_\tau)]\|ds\ ]\, I_{[\eta_k,\eta_{k+1})}(t)\ ]^2$

$\le 2M^2 \max_k\{\prod_{i=1}^{k}\|b_i(\delta_i)\|^2\}\|x_0\|^2 + 2M^2\,[\max_{i,k}\{1,\prod_{j=1}^{k}\|b_j(\delta_j)\|\}]^2\,[\int_{t_0}^{t}\|[f(s,x(s)) + \int_0^s g(\tau,x_\tau)]\|ds\, I_{[\eta_k,\eta_{k+1})}(t)]^2$

$\le 2M^2C^2\|x_0\|^2 + 2M^2\max\{1, C^2\}\,[\int_{t_0}^{t}\|[f(s,x(s)) + \int_0^s g(\tau,x_\tau)]\|ds\,]^2$

$\le 2M^2C^2\|x_0\|^2 + 2M^2\max\{1, C^2\}(t-t_0)\int_{t_0}^{t}\|[f(s,x(s)) + \int_0^s g(\tau,x_\tau)]\|^2 ds$

$E\|(\oplus x)(t)\|^2 \le 2M^2C^2\|x_0\|^2 + 2M^2\max\{1, C^2\}(T-\delta)\int_{t_0}^{t}E\|[f(s,x(s)) + \int_0^s g(\tau,x_\tau)]\|^2 ds$

$\le 2M^2C^2\|x_0\|^2 + 4M^2\max\{1, C^2\}(T-\delta)^2(\frac{k}{2}+\frac{k}{2}) + 4M^2\max\{1, C^2\}(T-\delta)\, L\{\int_{t_0}^{t}E\|x(s)\|^2 ds + \int_0^s E\|x_\tau\|^2\}ds.$

Thus ,

$sup_{t\in[\delta,T]}\, E\|(\oplus x)(t)\|^2 \le 2M^2C^2\|x_0\|^2 + 4M^2\max\{1, C^2\}(T-\delta)^2 k + 4M^2\max\{1, C^2\}(T-\delta)^2 L\, sup_{t\in[\delta,T]}E\|x(s)\|^2 + 4M^2\max\{1, C^2\}(T-\delta)^3\, sup_{t\in[\delta,T]}E\|x_\tau\|^2$ for all t ∈ $[\delta,T]$.

Therefore ⊕ maps $\mathfrak{B}$ into itself.

Now , we have to show ⊕ is a contraction mapping

$\|(\oplus x)(t) - (\oplus y)(t)\|^2 \le \sum_{k=0}^{+\infty}[\ \sum_{i=1}^{k}\prod_{j=1}^{k}\|b_j(\delta_j)\|\int_{\eta_{i-1}}^{\eta_i}\|\Phi(t-s)\|\|\,[f(s,x(s)) + \int_0^s g(\tau,x_\tau)] - [f(s,y(s)) + \int_0^s g(\tau,y_\tau)]\|ds + \int_{\eta_k}^{t}\|\Phi(t-s)\|\|\,[f(s,x(s)) + \int_0^s g(\tau,x_\tau)] - [f(s,y(s)) + \int_0^s g(\tau,y_\tau)]\|ds\ ]\, I_{[\eta_k,\eta_{k+1})}(t)\ ]^2$

$\le M^2\,[\max_{i,k}\{1,\prod_{j=1}^{k}\|b_j(\delta_j)\|\}]^2\,(\int_{t_0}^{t}\|[f(s,x(s)) + \int_0^s g(\tau,x_\tau)] - [f(s,y(s)) + \int_0^s g(\tau,y_\tau)]\|ds\ \, I_{[\eta_k,\eta_{k+1})}(t))^2$

$\le M^2\max\{1, C^2\}(t-t_0)\int_{t_0}^{t}\|[f(s,x(s)) + \int_0^s g(\tau,x_\tau)] - [f(s,y(s)) + \int_0^s g(\tau,y_\tau)]\|^2 ds$

$E\|(\oplus x)(t) - (\oplus y)(t)\|^2 \le M^2\max\{1, C^2\}(t-t_0)\int_{t_0}^{t}E\|[f(s,x(s)) + \int_0^s g(\tau,x_\tau)] - [f(s,y(s)) + \int_0^s g(\tau,y_\tau)]\|^2 ds$

$\le M^2\max\{1, C^2\}(T-\delta)\, L\{\int_{t_0}^{t}E\|x(s)-y(s)\|^2 ds + \int_0^s E\|x_\tau - y_\tau\|^2\}ds$

Taking the supremum over t, we get,

$\|(\oplus x)(t) - (\oplus y)(t)\|^2 \le M^2\max\{1, C^2\}[(T-\delta)^2 L\|x(s)-y(s)\|^2 + (T-\delta)^3 L\|x_\tau - y_\tau\|^2]$.

Thus ,

$\|(\oplus x)(t) - (\oplus y)(t)\|^2 \le \Delta\{\|x(s)-y(s)\|^2 + \|x_\tau - y_\tau\|^2\}$,

Since $0 < \Delta < 1$. This shows that the operator ⊕ satiafies the Contraction mapping principle and therefore, ⊕ has a unique fixed point which is the solution of the system (2.1)- (2.3).

4. STABILITY

In this section, we study the stability of the system (2.1) –(2.3) through the continuous dependence of solutions on intial condition.

Definition 4.1 : A solution x(s) of the system (2.1) – (2.2) with intial value θ which satisfies (2.3) is said to be stable in the mean square if for all $\epsilon > 0$ *there exists* $\rho > 0$ such that

E‖x(s) − y(s)‖ $^2 \le \varepsilon$ whenever E‖$\theta - \hat{\theta}$‖ $^2 <$ ρ, for all s ∈ $[\delta, T]$, (4.1)

Where y(s) is another solution of the system (2.1) – (2.2) with intial value $\hat{\theta}$ defined in (2.3).

Theorem 4.1: Let x(t) and $\bar{x}$ (t) be solutions of the system (2.1) –(2.3) with intial values x_0 and $\overline{x_0} \in \mathcal{R}^n$ respectively. If the assumptions of theorem 3.1 are satisfied, then the solution of the system (2.1) – (2.3) is stable in the mean square.

Proof: By the assumptions, x and $\bar{x}$ are the two solutions of the system (2.1) –(2.3) for t ∈ $[\delta, T]$. Then,

$$[\,x(t) - \bar{x}(t)] = \sum_{k=0}^{+\infty} \Big[\prod_{i=1}^{k} b_i(\delta_i)\, \Phi(t-t_0)[x_0 - \overline{x_0}] + \sum_{i=1}^{k}\prod_{j=1}^{k} b_j(\delta_j) \int_{\eta_{i-1}}^{\eta_i} \Phi(t-s)\{ [f(s, x(s)) + \int_0^s g(\tau, x_\tau)] - [f(s, \bar{x}(s)) + \int_0^s g(\tau, \bar{x}_\tau)] \}ds + \int_{\eta_k}^{t} \Phi(t-s)\{ [f(s, x(s)) + \int_0^s g(\tau, x_\tau)] - [f(s, \bar{x}(s)) + \int_0^s g(\tau, \bar{x}_\tau)] \}ds \Big] I_{[\eta_k,\eta_{k+1})}(t)$$

By using hypotheses (I) , (II), we get

$$E\| x(t) - \bar{x}(t) \|^2 \le 2 \sum_{k=0}^{+\infty} \Big[\prod_{i=1}^{k} \| b_i(\delta_i) \|^2 \|\Phi(t-t_0)\|^2 E\|x_0 - \overline{x_0}\|^2 I_{[\eta_k,\eta_{k+1})}(t) \Big] + 2E \Big[\sum_{k=0}^{+\infty}\Big[\sum_{i=1}^{k}\prod_{j=1}^{k} \| b_j(\delta_j)\| \int_{\eta_{i-1}}^{\eta_i} \|\Phi(t-s)\|\{ \|[f(s,x(s)) + \int_0^s g(\tau, x_\tau)] - [f(s,\bar{x}(s)) + \int_0^s g(\tau, \bar{x}_\tau)]\| \}ds + \int_{\eta_k}^{t} \|\Phi(t-s)\|\{ \|[f(s,x(s)) + \int_0^s g(\tau, x_\tau)] - [f(s,\bar{x}(s)) + \int_0^s g(\tau, \bar{x}_\tau)]\| \}ds \Big] I_{[\eta_k,\eta_{k+1})}(t) \Big]^2$$

$$\le 2M^2 \max_k\{ \prod_{i=1}^{k} \| b_i(\delta_i) \|^2\} E\|x_0 - \overline{x_0}\|^2 + 2M^2 [\max_{i,k}\{ 1, \prod_{j=1}^{k} \| b_j(\delta_j) \|\}]^2 \times E\Big[\int_{t_0}^{t} \| \big[f(s,x(s)) + \int_0^s g(\tau, x_\tau)\big] - [f(s,\bar{x}(s)) + \int_0^s g(\tau, \bar{x}_\tau)]\|ds\, I_{[\eta_k,\eta_{k+1})}(t)\Big]^2$$

$$\le 2M^2C^2 E\|x_0 - \overline{x_0}\|^2 + 2M^2 \max\{1, C^2\}(t - t_0) \Big[\int_{t_0}^{t} E\| \big[f(s,x(s)) + \int_0^s g(\tau, x_\tau)\big] - [f(s,\bar{x}(s)) + \int_0^s g(\tau, \bar{x}_\tau)]\|ds\, I_{[\eta_k,\eta_{k+1})}(t)\Big]^2$$

$$sup_{t\in[\delta,T]} E\| x(t) - \bar{x}(t) \|^2 \le 2M^2C^2 E\|x_0 - \overline{x_0}\|^2 + 2M^2 \max\{1, C^2\}(T - \delta)L\{\int_{t_0}^{t} sup_{t\in[\delta,T]} E\|x(s) - \bar{x}(s)\|^2 ds + \int_0^s E\| x_\tau - \bar{x}_\tau \|^2\}ds$$

By applying Grownwall's inequality, we have

$$sup_{t\in[\delta,T]} E\| x(t) - \bar{x}(t) \|^2 \le 2M^2C^2 E\|x_0 - \overline{x_0}\|^2 \exp(2M^2 \max\{1, C^2\}(T - \delta)^2[1 + (T - \delta)]\, L)$$
$$\le \lambda\, E\|x_0 - \overline{x_0}\|^2.$$

Where $\lambda = 2M^2C^2 \exp(2M^2 \max\{1, C^2\}(T - \delta)^2[1 + (T - \delta)]\, L)$

Now given $\epsilon > 0$, *choose* $\rho = \frac{\varepsilon}{\lambda}$ such that E‖$x_0 - \overline{x_0}$‖ $^2 < \rho$. Then

$sup_{t\in[\delta,T]}$ E‖ x(t) − $\bar{x}$ (t) ‖$^2 \le \epsilon$.

Thus, it is apparent that the difference between the solution x(t) and $\bar{x}$ (t) in the interval $[\delta, T]$ is small provided the change in the intial point (t_0, x_0) as well as in the function f$(t, x(t))$ do not exceed prescribed amounts. This completes the proof.

5. APPLICATION

Let $\widetilde{\Omega} \subset \mathcal{R}^n$ be a bounded domain with smooth boundary $\partial\widetilde{\Omega}$.

$$\begin{cases} u_s(x,s) = u_{xx}(x,s) + \int_{-r}^{t} \mu(\theta) u(s+\theta, x)d\theta, s \ne \xi_k, s \ge \tau, \\ u(x,\xi_k) = q(k)\tau_k u(x,\xi_k^-) \quad a.s.\, x \in \widetilde{\Omega}, \\ u(x,s) = \varphi(x,s) \quad a.s.\, x \in \widetilde{\Omega}, -r \le s \\ u(x,s) = 0 \quad a.s.\, x \in \partial\widetilde{\Omega} \end{cases} \quad (5.1)$$

Let X = L^2 $(\widetilde{\Omega})$, and τ_k be a random variable defined on $D_k \equiv (0, d_k)$ for k = 1,2,…, where 0 < $d_k < +\infty$ and $\mu: [-r, 0] \to \mathcal{R}$ is a positive function. Furthermore, assume that τ_k follow Erlang distribution, where k = 1,2,…; and τ_i *and* τ_j *are* independent with each other as i ≠ j for i,j = 1,2,…; q is a function of k; $\xi_0 = s_0$; $\xi_k = \xi_{k-1} + \tau_k$ for k = 1,2,… and $s_0 = \mathcal{R}^+$ is an arbitrarily given real number .

Define B is an operator on X by Bu = $\frac{\partial^2 u}{\partial x^2}$ with the domain

D(B) = {u ∈ X| u and $\frac{\partial u}{\partial x}$ are absolutely continuous , $\frac{\partial^2 u}{\partial x^2} \in X$, $u = 0$ *on* $\partial\widetilde{\Omega}$ } .

It is well known that B generates a strongly continuous semigroup S(s) which is compact, analytic and self adjoint. Moreover, the operator B can be expressed as B (u)= $\sum_{n=1}^{\infty} n^2 \langle u, u_n \rangle u_n$, u ∈ D(B),

Where u_n (ω) = $(\frac{2}{\pi})^{1/2}$ sin(nω) , n = 1, 2,…, is the orthonormal set of eigenvectors of B and for every $u \in X$,

S(s)u = $\sum_{n=1}^{\infty} \exp(-n^2 s) < u$, which satisfies ‖ S(s) ‖≤ $\exp(-\pi^2(s - s_0))$, s > s_0. Hence S(s) is a contraction semigroup.

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