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An Integrated Inventory Model for Imperfect Production Process with Backorder Price Discount under Strict Carbon Cap Policy

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Abstract- Carbon emissions originated from various activities of the organization create a significant threat to the environment. With the increasing environmental awareness in public and the implementation of environmental regulations, there is an increased pressure on organizations to become environmentally conscious. Currently, they are looking for solutions to reduce carbon emissions associated with their operations. Different carbon policies were adopted by Government to control those emissions. In this paper, strict carbon cap policy is considered for an integrated single-vendor single-buyer imperfect production inventory model. The vendor makes an investment to increase the process quality and price discounts are offered by the buyer to the customers. Major sources of emissions from inventory holding, production setup and transportation have been incorporated. The main aim is to determine the optimal order quantity, safety stock factor and the number of shipments. The objective is to minimize the total expected cost of the supply chain and satisfying the carbon emission constraint. An algorithm is used to determine the optimal solutions of the model. Finally, a numerical example is given to illustrate the model flourished.
Keywords: Imperfect production, Emissions, Strict carbon cap policy, Single vendor- buyer, Backorder price discount

1. INTRODUCTION

Environmental problems are global. Many problems that arise as a consequence of population growth relate to the population activities, their use of resources and their aspirations. Human activities including their travel, construction and food production are the major source of greenhouse gases. Various environmental factors of sustainability include natural resources, energy, pollution and waste products. In the supply chain area, all activities related to inventory storage, production and transportation have an impact on carbon emissions. To make environmentally appropriate decisions, organisations and persons need to be fully involved in the planning and operations of the supply chain activities. It is the great challenge for organizations to control and minimize the carbon emissions of the entire supply chain. As the increment of the number of transportation increases, transportation cost increases which results in increasing percentage of carbon emissions. Minimizing stock holdings, recycling and refurbishing of products, designing the products such that to repair and reuse, reducing shipment distance etc are some of the measures to reduce resource usage and pollution.

Emissions of carbondioxide and other greenhouse gases will lead to major changes in the earth's climate system. As a result of these emissions, Global warming occurs. Greenhouse gas reduction, especially CO_2 emission reduction is the only way for human survival in facing global warming. In order to reduce the carbon emissions, Government and regulatory bodies have started implementing various carbon policies and commenced different carbon trading schemes. The most common carbon policies are Carbon tax, Strict carbon cap and carbon cap and trade. In the case of strict carbon cap policy, a certain emission limit is fixed by the regulatory bodies for the organizations known as cap, and the penalty for exceeding the cap is infinitely large. Hence, organizations are forced to handle their emission within the given limit. As reported by environmentalists, it is the most effective policy to curb carbon emission.

The rest of the paper is structured as follows: Section 2 presents the literature review. Section 3 provides fundamental assumptions and notations. Section 4 describes the mathematical model. Section 5 illustrates a numerical example.Section 6 concludes the paper. A list of references is also provided.

2. LITERATURE REVIEW

Goyal (1976) was the first researchers to analyze an integrated inventory model for single-vendor single-buyer system. Banerjee (1986) enhanced the model of Goyal (1976) and presented a joint economic lot-size model under lot-for-lot basis.

Pan and Yang (2002) proposed a model by considering lead time as a decision variable. Ben-Daya and Hariga (2004) assumed stochastic demand for single-vendor single-buyer integrated inventory model by relaxing the assumption of deterministic demand thereby allowing shortages. It is quite nonsensical to consider the products to be of perfect quality. The end products maynot withstand the quality standards set by the manufacturer due to faulty production process, mishandling during transportation etc. Porteus (1986) first introduced the logarithmic investment function to improve the process quality. Rosenblatt and Lee (1986) generalized the model of Porteus (1986) to EPQ. After that, Many researches focussed in the area of imperfect production and process quality improvement. Ben-Daya and Hariga (2000) proposed an economic lot scheduling problem with imperfect production process. Huang (2004) progressed an integrated vendor-buyer inventory model for imperfect quality items and presumed that the defective items follows a given distribution.

Dey and Giri (2014) considered vendor investment for process quality improvement in an integrated inventory model by assuming percentage of defective items produced to be a control parameter. Lin (2009) studied an integrated supply chain model with backorder price discount and investment to reduce the ordering cost. Jaggi and Arneja (2010) explored a periodic review inventory model with backorder price discounts where shortages are partially backlogged. Integration of environmental issues with the inventory model has been flourished further since inventory plays a vital role in influencing the environment. Bonney and Jaber (2011) discussed various environmental issues arising from the inventory and demonstrate an EOQ inventory model. Ghosh et al. (2016) developed a two echelon supply chain model with different carbon policies namely: Carbon cap and trade, carbon tax and Strict carbon cap policy. Ghosh et al. (2017) considered stochastic demand in a supply chain inventory model under strict carbon cap policy. Ivan Darma Wangsa (2017) discussed about direct and indirect emissions from transport and industries.Further, he considered penalty and incentive policies for these emissions. Mukherjee et al (2019) proposed an imperfect production inventory model with backorder price discount and investment to improve the quality of products.

3. NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used to establish the mathematical model.

Notations

D	Demand rate for non defective items
P	Production rate for the vendor ($P = 1/p$)
A	Ordering cost per order for the buyer
F	Transportation cost per delivery
B	Setup cost for the vendor
L	Lead-time
Q	Order quantity
h_v	Holding cost per item per unit time for the vendor
h_{b1}	Holding cost for defective items per unit time for the buyer
h_{b2}	Holding cost for non- defective items per unit time for the buyer
n	The number of shipments per production run from the vendor to the buyer
r	Reorder point
s	Screening cost per unit item for the buyer
x	Screening rate of the buyer

w	Warranty cost per unit defective item for the vendor
k	Safety stock factor
y	Percentage of defective items produced
y_0	Percentage of defective items produced prior to investment
η	Fractional annual opportunity cost
δ	Percentage decrease in defective items per dollar increase in investment
β	Fraction of the shortage that will be backordered at the buyer's end $(0 \leq \beta < 1)$
β_0	Upper bound of the backorder ratio $(0 \leq \beta \leq \beta_0 \leq 1)$
π_x	Unit backorder price discount
π_0	Marginal profit per unit
X	Lead time demand
b	Fixed carbon emission per production setup
f	Carbon emission per unit time due to transportation
α_{b1}	Carbon emission per unit item due to holding of defective items at the buyer
α_{b2}	Carbon emission per unit item due to holding of non-defective items at the buyer
α_v	Carbon emission per unit item due to inventory at the vendor
$\hat{C}$	Cap(maximum limit) on carbon emission per unit time

Assumptions

1. A single type of item is considered by single vendor and single buyer.
2. The buyer orders a quantity of nQ items to the vendor. The vendor produces Q items in n equal sized shipments and delivers to the buyer.
3. The buyer follows the (Q, r) continuous review policy with constant lead – time and partial backlogging.
4. The buyer provides price discount to the customers to make them wait for the orders arrive with next lot.
5. Lead time demand X is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$.
6. The reorder point $r =$ expected demand during lead time + safety stock(SS), i.e.,$r = DL + k\sigma\sqrt{L}$, where k is the safety stock factor.
7. The length of vendor's production cycle is $nQ(1-y)/D$, and the length of buyer's ordering cycle is $Q(1-y)/D$.
8. Fixed screening rate which is greater than the demand rate, i.e., $x > D$.
9. The vendor incurs warranty cost for each defective item produced.
10. The logarithmic investment function is given by $I(y) = \frac{1}{\delta} ln\left(\frac{y_0}{y}\right)$, where δ is the percentage decrease in y for a dollar increase in investment.
11. Carbon emission is considered from production setup, inventory holding at the buyer and vendor and transportation.

4. MATHEMATICAL MODEL

This paper extends the work of Mukherjee et al. (2019). In this paper, strict carbon cap policy is adopted for an integrated single-vendor single-buyer imperfect production inventory model.

Buyer's Perspective

The total annual expected cost of the buyer comprised of ordering cost, inventory holding cost, shortage cost, shipment cost and screening cost.

The expected shortage at the end of the cycle is given by

$$E(X-r)^{+}=\int_{r}^{\infty}(x-r)\,f_x(X)\,dx=\sigma\sqrt{L}\,\psi(k) \tag{1}$$

Where $\psi(k)=\phi(k)-k[1-\Phi(k)]>0$, ϕ,Φ denote the standard normal probability density function and cumulative distribution function respectively.

Hence, the expected stockout cost per unit time is

$$\frac{D}{Q(1-y)}\left[\left(\beta_0\pi_x^2/\pi_0\right)+\pi_0-\beta_0\pi_x\right]E(X-r)^{+} \tag{2}$$

Where π_x is the backorder price discount offered by the buyer for each unit of the item.

Since the items delivered from the vendor involves defective items due to imperfect production, the buyer conducts screening of items and separate defectives and non- defectives. Thus holding cost is calculated for both criteria.

The average inventory for non-defective items is

$$\frac{nQ(1-y)}{D}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+\frac{DQy}{2x(1-y)}+\left(1-\beta_0\pi_x/\pi_0\right)E(X-r)^{+}\right] \tag{3}$$

The average inventory level for defective items is

$$nQ^2y\left[\frac{(1-y)}{D}-\frac{1}{2x}\right] \tag{4}$$

The expected total cost of the buyer is given by

$$\begin{aligned}ETCB(Q,k,n)&=\frac{D(A+nF)}{nQ(1-y)}+h_{b1}\left(Qy-\frac{DQy}{2x(1-y)}\right)+\frac{sD}{1-y}\\&+h_{b2}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+\left(1-\beta_0\pi_x/\pi_0\right)\sigma\sqrt{L}\,\psi(k)+\frac{DQy}{2x(1-y)}\right]\\&+\frac{D}{Q(1-y)}\left[\pi_0-\beta_0\pi_x+\beta_0\pi_x^2/\pi_0\right]\sigma\sqrt{L}\,\psi(k)\end{aligned} \tag{5}$$

Vendor's Perspective

The total annual expected cost of vendor is obtained by the sum of setup cost, holding cost, warranty cost for defective items with investment and is given by

$$ETCV(Q,n)=\frac{BD}{nQ(1-y)}+h_v\frac{Q}{2}\left[(n-1)-(n-2)\frac{Dp}{1-y}\right]+\frac{wDy}{1-y}+\frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right) \tag{6}$$

Integrated Approach

The total expected cost of the supply chain is obtained by adding Eqs (5) and (6) given as

$$\begin{aligned}ETC(Q,k,n)=&\frac{D(A+B+nF)}{nQ(1-y)}+h_{b1}\left(Qy-\frac{DQy}{2x(1-y)}\right)+\frac{D(s+wy)}{1-y}\\&+h_{b2}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+(1-\beta_0\pi_x/\pi_0)\sigma\sqrt{L}\,\psi(k)+\frac{DQy}{2x(1-y)}\right]\\&+\frac{\eta}{\delta}\ln\left(\frac{y_0}{y}\right)+\frac{D}{Q(1-y)}\left[\pi_0-\beta_0\pi_x+\beta_0\pi_x^2/\pi_0\right]\sigma\sqrt{L}\,\psi(k)\\&+h_v\frac{Q}{2}\left[(n-1)-(n-2)\frac{Dp}{1-y}\right]\end{aligned} \tag{7}$$

Now, the major sources of carbon emissions are considered from holding inventory at the buyer and vendor , manufacturing set up and from transportation. Thus, the total expected carbon emission per unit time from these sources can be given as

$$\begin{aligned}TE(Q,k,n)=&\frac{bD}{nQ(1-y)}+\frac{fD}{Q(1-y)}+\alpha_{b1}\left(Qy-\frac{DQy}{2x(1-y)}\right)\\&+\alpha_{b2}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+(1-\beta_0\pi_x/\pi_0)\sigma\sqrt{L}\,\psi(k)+\frac{DQy}{2x(1-y)}\right]\\&+\alpha_v\frac{Q}{2}\left[(n-1)-(n-2)\frac{Dp}{1-y}\right]\end{aligned} \tag{8}$$

As we are considering strict carbon cap policy, the total carbon emission per unit time should not exceed the specified limit $\hat{C}$. Then, the carbon emission constraint can be written as

$$\begin{aligned}&\frac{bD}{nQ(1-y)}+\frac{fD}{Q(1-y)}+\alpha_{b1}\left(Qy-\frac{DQy}{2x(1-y)}\right)+\alpha_v\frac{Q}{2}\left[(n-1)-(n-2)\frac{Dp}{1-y}\right]\\&\quad+\alpha_{b2}\left[k\sigma\sqrt{L}+\frac{Q(1-y)}{2}+(1-\beta_0\pi_x/\pi_0)\sigma\sqrt{L}\,\psi(k)+\frac{DQy}{2x(1-y)}\right]\le\hat{C}\end{aligned} \tag{9}$$

Thus the problem is to find the optimal order quantity Q, safety stock factor k and the number of shipments n, that minimize the total expected cost (7) and satisties the carbon constraint (9).

Ignoring the carbon constraint initially, the optimal value of Q and k is obtained by taking the first partial derivative of Eq (7) with respect to Q and k and equating to zero, we get

$$Q_0 = \sqrt{\frac{\frac{D\left[A+B+nF\right]}{n} + D\left[\pi_0 - \beta_0\pi_x + \beta_0\pi_x^2/\pi_0\right]\sigma\sqrt{L}\,\psi(k)}{h_{b1}\left[y(1-y) - \frac{Dy}{2x}\right] + h_{b2}\left[\frac{(1-y)^2}{2} + \frac{Dy}{2x}\right] + \frac{h_v}{2}\left[(n-1)(1-y) - (n-2)Dp\right]}} \tag{10}$$

and

$$F(k) = 1 - \frac{h_{b2}}{h_{b2}\left(1 - \beta_0\pi_x/\pi_0\right) + \frac{D}{Q(1-y)}\left[\pi_0 - \beta_0\pi_x + \beta_0\pi_x^2/\pi_0\right]} \tag{11}$$

We need to determine the the optimal value of Q and k for fixed n that satisfies the carbon constraint. The method proposed by chen et al. (2013) is adopted for this purpose. The carbon constraint can be written as

$$\begin{aligned} &\frac{bD}{nQ(1-y)} + \frac{fD}{Q(1-y)} + \alpha_{b1}\left(Qy - \frac{DQy}{2x(1-y)}\right) + \alpha_v \frac{Q}{2}\left[(n-1) - (n-2)\frac{Dp}{1-y}\right] \\ &\quad + \alpha_{b2}\left[k\sigma\sqrt{L} + \frac{Q(1-y)}{2} + \left(1 - \beta_0\pi_x/\pi_0\right)\sigma\sqrt{L}\,\psi(k) + \frac{DQy}{2x(1-y)}\right] - \hat{C} \le 0 \end{aligned} \tag{12}$$

The roots of the quadratic equation of the above inequality (12) is given by

$$Q_1 = \frac{\hat{C} - \alpha_{b2}\left[k\sigma\sqrt{L} + \left(1-\beta_0\pi_x/\pi_0\right)\sigma\sqrt{L}\,\psi(k)\right] - \left[\begin{array}{l}\left(\alpha_{b2}\left[k\sigma\sqrt{L} + \left(1-\beta_0\pi_x/\pi_0\right)\sigma\sqrt{L}\,\psi(k)\right] - \hat{C}\right)^2 \\ -2\left(\begin{array}{l}\alpha_{b1}\left(2y - \frac{Dy}{x(1-y)}\right) + \alpha_{b2}\left((1-y) + \frac{Dy}{x(1-y)}\right) \\ +\alpha_v\left((n-1) - (n-2)\frac{Dp}{1-y}\right)\end{array}\right) \\ \left(\frac{bD}{n(1-y)} + \frac{fD}{1-y}\right)\end{array}\right]^{1/2}}{\alpha_{b1}\left(2y - \frac{Dy}{x(1-y)}\right) + \alpha_{b2}\left((1-y) + \frac{Dy}{x(1-y)}\right) + \alpha_v\left((n-1) - (n-2)\frac{Dp}{1-y}\right)} \tag{13}$$

and

$$Q_2=\frac{\hat{C}-\alpha_{b2}\left[k\sigma\sqrt{L}+(1-\beta_0\pi_x/\pi_0)\sigma\sqrt{L}\,\psi(k)\right]+\left[\left(\alpha_{b2}\left[k\sigma\sqrt{L}+(1-\beta_0\pi_x/\pi_0)\sigma\sqrt{L}\,\psi(k)\right]-\hat{C}\right)^2-2\left(\begin{array}{l}\alpha_{b1}\left(2y-\frac{Dy}{x(1-y)}\right)+\alpha_{b2}\left((1-y)+\frac{Dy}{x(1-y)}\right)\\+\alpha_v\left((n-1)-(n-2)\frac{Dp}{1-y}\right)\end{array}\right)\left(\frac{bD}{n(1-y)}+\frac{fD}{1-y}\right)\right]^{1/2}}{\alpha_{b1}\left(2y-\frac{Dy}{x(1-y)}\right)+\alpha_{b2}\left((1-y)+\frac{Dy}{x(1-y)}\right)+\alpha_v\left((n-1)-(n-2)\frac{Dp}{1-y}\right)} \tag{14}$$

Where Q_1 and Q_2 are the lower and upper bounds, respectively for the feasible range of Q and they are positive.

Now for fixed n, the optimal Q will be obtained at $\hat{Q}$ as given below in (15) which will satisfy the carbon constraint

$$\hat{Q}=\begin{cases}Q_0,\ if\ Q_1\le Q_0\le Q_2\\ Q_1,\ if\ Q_0\le Q_1\\ Q_2,\ if\ Q_0\ge Q_2\end{cases} \tag{15}$$

Finding the suitable value of $\hat{Q}$ from Eq (15) and putting in Eq. (11) as $Q=\hat{Q}$, the value of k is obtained for fixed n. We assume deterministic demand for initialization since it will be difficult to derive the value of one variable without knowing the other one. Set $\sigma\sqrt{L}=0$ in Eqs. (10), (13) and (14) to get the initial value of Q_0, Q_1 and Q_2 given as

$$Q_0=\sqrt{\frac{\frac{D[A+B+nF]}{n}}{h_{b1}\left[y(1-y)-\frac{Dy}{2x}\right]+h_{b2}\left[\frac{(1-y)^2}{2}+\frac{Dy}{2x}\right]+\frac{h_v}{2}\left[(n-1)(1-y)-(n-2)Dp\right]}} \tag{16}$$

$$Q_1=\frac{\hat{C}-\left[\left(\hat{C}\right)^2-2\left(\begin{array}{l}\alpha_{b1}\left(2y-\frac{Dy}{x(1-y)}\right)+\alpha_{b2}\left((1-y)+\frac{Dy}{x(1-y)}\right)\\+\alpha_v\left((n-1)-(n-2)\frac{Dp}{1-y}\right)\end{array}\right)\left(\frac{bD}{n(1-y)}+\frac{fD}{1-y}\right)\right]^{1/2}}{\alpha_{b1}\left(2y-\frac{Dy}{x(1-y)}\right)+\alpha_{b2}\left((1-y)+\frac{Dy}{x(1-y)}\right)+\alpha_v\left((n-1)-(n-2)\frac{Dp}{1-y}\right)} \tag{17}$$

and

$$Q_2 = \frac{\hat{C} + \left[\left(\hat{C}\right)^2 - 2 \left(\begin{array}{l} \alpha_{b1}\left(2y - \frac{Dy}{x(1-y)}\right) + \alpha_{b2}\left((1-y) + \frac{Dy}{x(1-y)}\right) \\ + \alpha_v\left((n-1) - (n-2)\frac{Dp}{1-y}\right) \end{array} \right) \left(\frac{bD}{n(1-y)} + \frac{fD}{1-y} \right) \right]^{1/2}}{\alpha_{b1}\left(2y - \frac{Dy}{x(1-y)}\right) + \alpha_{b2}\left((1-y) + \frac{Dy}{x(1-y)}\right) + \alpha_v\left((n-1) - (n-2)\frac{Dp}{1-y}\right)} \quad (18)$$

An algorithm is used to determine the optimal Q, k and n.

Algorithm:

Step 1: Set $n = 1$

Step 2: Compute initial Q_0,Q_1 and Q_2 from Eqs. (16), (17) and (18) respectively.

Step 3: Select a proper value of $\hat{Q}$ satisfying the condition given in Eq (15).

Step 4: Compute k using $\hat{Q}$ in Eq (11)

Step 5: Find Q_0,Q_1 and Q_2 from Eqs. (10), (13) and (14) respectively

Step 6: Choose the appropriate value of $\hat{Q}$ using the condition in Eq (15)

Step 7: Repeat steps 4, 5 and 6 till no change occurs in the value of $\hat{Q}$ and k.

Step 8: Set $\hat{Q}_{(n)} = \hat{Q}$ and $r_{(n)} = r$. Thus, $\left(\hat{Q}_{(n)}, r_{(n)}\right)$ is the optimal solution for fixed n and compute $ETC\left(\hat{Q}_{(n)}, r_{(n)}, n\right)$ using Eq (7).

Step 9: Set $n = n + 1$ and repeat Steps 2 to 8 to get new $ETC\left(\hat{Q}_{(n)}, r_{(n)}, n\right)$.

Step 10: If $ETC\left(\hat{Q}_{(n)}, r_{(n)}, n\right) \le ETC\left(\hat{Q}_{(n-1)}, r_{(n-1)}, n-1\right)$ Go to Step 9; otherwise go to step 11.

Step 11: Set $ETC(Q^*, r^*, n^*) = ETC\left(\hat{Q}_{(n-1)}, r_{(n-1)}, n-1\right)$, then (Q^*, r^*, n^*) is the optimal solution. Total emission is obtained from Eq.(8)

5. NUMERICAL EXAMPLE

$D = 1000$ units per year, $P = 3200$ units per year, A=$50 per order, $B = \$400$ per setup, $F = \$35$ per shipment, $L = 10$, $h_v = \$4$ per unit, $h_{b1} = \$6$ per unit, $h_{b2} = \$10$ per unit, $s = \$100$ per unit, $x = 2152$ units per unit time, $w = \$24$ per unit, $y_0 = 0.22$, $y = 0.036$, $\eta = 0.2$, $\delta = 0.0002$, $\sigma = 5$, $\pi_x = 75$, $\pi_0 = 150$, $\beta_0 = 0.2$, $\alpha_{b1} = 0.80$ ton per unit per year, $\alpha_{b2} = 0.80$ ton per unit per year, $\alpha_v = 0.80$ ton per unit per year, $b = 20$ ton per setup, $f = 0.004$ ton per shipment, $\hat{C} = 500$ Ton.

The optimal solution is given as

$\boldsymbol{n^* = 6}$, $\boldsymbol{Q^* = 97.68}$, $\boldsymbol{k^* = 2.48}$, $\boldsymbol{ETC = 109270.893}$, $\boldsymbol{TE = 252.005}$

6. CONCLUSION

In this paper, an integrated single vendor single buyer imperfect production inventory model with backorder price discount under Strict carbon cap policy is studied. Carbon emissions are unavoidable in every production system. Unless these emissions are controlled, our global ecosystem will greatly be affected. The strict carbon cap policy is the most effective tool for highly emitting industries to reduce excessive emissions.

REFERENCES

[1] Banerjee, A. (1986). A joint economic-lot-size model for purchaser and vendor. Decis. Sci. 17, 292– 311.

[2] Ben-Daya, M., & Hariga, M. (2000). Economic lot scheduling problem with imperfect production processes. Journal of the Operational Research Society, 51(7), 875-881.

[3] Ben-Daya, M., Hariga, M. (2004). Integrated single vendor single buyer model with stochastic demand and variable lead-time. Int. J. Prod. Econ. 92, 75–80.

[4] Bonney, M., & Jaber, M. Y. (2011). Environmentally responsible inventory models: Non-classical models for a non-

classical era. International Journal of Production Economics, 133, 43–53.

[5] Chen, X., Benjaafar, S., & Elomri, A. (2013). The carbon-constrained EOQ, Oper. Res. Lett. 41 (2) ,172–179.

[6] Ghosh, A., Jha, J.K., & Sarmah, S.P. (2016). Optimizing a two-echelon serial supply chain with different carbon policies. International journal of sustainable engineering, 9(6): 363-377.

[7] Ghosh, A., Jha, J.K., & Sarmah, S.P. (2017). Optimal lot sizing under strict carbon cap policy considering stochastic demand. Applied Mathematical Modelling, 44, 688-704.

[8] Goyal, S.K. (1976). An integrated inventory model for a single supplier-single customer problem. Int. J. Prod. Res. 15(1), 107–111.

[9] Huang, C.K. (2004). An optimal policy for a single-vendor single-buyer integrated productioninventory problem with process unreliability consideration. Int. J. Prod. Econ. 91(1), 91–98.

[10] Jaggi, C.K., & Arneja, N. (2010). Periodic inventory model with unstable lead-time and set-up cost with backorder price discount. Int J. Appl. Decis. Sci. 3(1), 53–73 .

[11] Lin, Y.J. (2009). An integrated vendor-buyer inventory model with backorder price discount and effective investment to reduce ordering cost. Comput. Ind. Eng. 56(4), 1597–1606.

[12] Mukherjee A., Dey O., & Giri B.C. (2019) An Integrated Imperfect Production–Inventory Model with Optimal Vendor Investment and Backorder Price Discount. Information Technology and Applied Mathematics. Advances in Intelligent Systems and Computing, 699, 187-203.

[13] Pan, J.C.H., & Yang, J.S. (2002). A study of an integrated inventory with controllable lead time. Int. J.Prod. Res. 40(5), 1263–1273.

[14] Porteus, E.L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. Oper. Res. 34(1), 137–144.

[15] Rosenblatt, M.J., & Lee, H.L. (1986). Economic production cycles with imperfect production process. IIE Trans. 18(1), 48–55.

[16] Wangsa, I.D. (2017). Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions. International Journal of Industrial Engineering Computations,8, 453-480.