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Type –Reduction Method on Fuzzy Critical Path

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Abstract- In this paper, we propose a new method to find the fuzzy critical path in a acyclic project network using type reduction method. Type-2 discrete fuzzy numbers and its complement are used to identify the fuzzy critical path. An illustrative example is also included to demonstrate our proposed method.

Keywords: Fuzzy critical path, type-2 discrete fuzzy number, acyclic project network, centre of gravity.

1. INTRODUCTION [8],[10],[11]

Critical path method is a network-based method that provides assistance in planning and controlling complicated projects in real world applications. The objective of the critical path method(CPM) is to locate critical activities on the critical path so that resources may be concentrated on these activities in order to shorten the project length time. CPM enables the decision maker adopt a better strategy of optimizing the time and the available resources to ensure the speedy completion and the standard of the project.

This paper analyzes the critical path in a general project network with fuzzy activity times. Several type-reduction methods for type-2 fuzzy sets have been proposed. Coupland and john[4] proposed a type reduction method for type-2 fuzzy sets. We propound centre of gravity under type reduction method for fuzzy numbers to find a fuzzy critical path. The time span of each activity in a fuzzy project network is represented by type – 2 discrete fuzzy number.

The organization of the paper is as follows: In section 2 some basic concepts are discussed. section 3 gives some properties of total slack fuzzy time. Section 4 gives the network terminology. An algorithm to find the fuzzy critical path is discussed in section 5.The proposed algorithm is illustrated through a numerical example in section 6.

2. BASIC CONCEPTS

2.1. Type-2 discrete fuzzy number [1],[2]

Let X be a non-empty finite set, which is referred as the universal set. A type-2 fuzzy set $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u) : X \times I \rightarrow I$ *where* $x \in X,\ I \in [0,1]$ *and* $u \in J_x \subseteq I$ *that is* $\tilde{A} = \{((x,u); \mu_{\tilde{A}}(x,u) / x \in X, u \in J_x \subseteq I)\ \}$ where $0 \le \mu_{\tilde{A}}(x,u) \le 1$

$\tilde{A}$ can also be expressed as $\tilde{A} = \int_{x \in X} \int_{u \in J_x} \dfrac{\mu_{\tilde{A}}(x,u)}{(x,u)} = \int_{x \in U} \dfrac{P_x(u)/u}{x}$ $J_x \subseteq I$, where $\iint$ denotes union all admissible x and u. For, discrete $\int$ universe of discourse is replaced by $\sum$.

2.2.Complement of type-2 discrete fuzzy number

The complement of the type-2 fuzzy set houses a fuzzy membership function given in the following formula:

$$\overline{\tilde{A}} = \sum_{x \in X} \left[\sum_{u \in J_x} f_x(u) \Big/ (1-u) \right] \Big/ x$$

2.3 Centroid measure [9]

If $\tilde{A}$ is a type-2 fuzzy set in the discrete case, the centroid of $\tilde{A}$ can be defined as follows:

$$C_{\tilde{A}} = \int_{\theta_1 \in J_{x_1}} \cdot \int_{\theta_2 \in J_{x_2}} \cdot \ldots \int_{\theta_R \in J_{x_R}} \cdot [f_{x_1}(\theta_1).f_{x_2}(\theta_1)....f_{x_R}(\theta_R)] \Bigg/ \frac{\sum_{j=1}^{R} x_j \mu_A(x_j)}{\sum_{j=1}^{R} \mu_A(x_j)}$$

Where $\tilde{A} = \sum_{j=1}^{R} \left[\sum_{u \in J_{x_j}} f_{x_j}(u) \Big/ u \right] \Big/ x_j$

2.4 centre of gravity

The centre of gravity of a fuzzy set $\tilde{A}$ is represented as

$$f(\tilde{A}) = \frac{\sum_{i=1}^{R} x_i \mu_A(x_i)}{\sum_{i=1}^{R} \mu_A(x_i)} \quad \text{where } \tilde{A} = \sum_{i=1}^{R} \mu_A(x_i) / x_i$$

2.5. Notations:

t_{ij}= The activity between node i and j.

ESF_j =The earliest starting fuzzy time of node j.

LFF_i = The latest finishing fuzzy time of node i.

TSF_{ij} = The total slack fuzzy time of t_{ij}.

p_n= the n^{th} fuzzy path.

P = the set of all fuzzy paths in a project network

$F(p_n)$= The total slack fuzzy time of path p_n in a project network.

3. PROPERTIES [5],[6],[7]

Property :3.1 *(Forward pass calculation)*

To calculate the earliest starting fuzzy time in the project network, set the initial node to zero for starting (ie) $ESF_1 = (0.0,0.0,0.0,0.0)$

$$ESF_j = \max_i \{ESF_j + TSF_{ij}\},\ j \neq i,\ j \in N,$$

i=number of preceding nodes.(ESF_j =The earliest starting fuzzy time of node j).

Ranking value is utilized to identify the maximum value.

Earliest finishing fuzzy time = Earliest starting fuzzy time (+) Fuzzy activity time.

Property 3.2. *(Backward pass calculation)*

To calculate the latest finishing time in the project network set $LFF_n = ESFn$.

$$LFF_j = \min_j \{LFF_j(-)SET_{ij}\}, i \neq n, i \in N,$$

j = number of succeeding nodes. Ranking value is utilized to identify the minimum value.

Latest starting fuzzy time= Latest finishing Fuzzy time (-) Fuzzy activity time.

Property 3.3.

For the activity t_{ij}, i<j

Total fuzzy slack:

$$SFT_{ij} = LFF_j(-)(ESF_i(+)SFT_{ij}) \quad \text{(or)}$$

$$(LFF_j(-)SFT_{ij})(-)ESF_i,\ 1 \le i \le j \le n;$$

$$i, j \in N,$$

Property 3.4.

$$F(p_n) = \sum_{\substack{1 \le i \le j \le n \\ i,j \in p_k}} SFT_{ij}\ ,\ p_k \in P,$$

p_n denotes the number of possible paths in a network from source node to the destination node, k=1 to m.

4. NETWORK TERMINOLOGY

A directed acyclic project network consisting of six nodes and seven edges are considered. Each edge in this network is assigned by type-2 discrete fuzzy numbers. Then the set of all possible paths are denoted by P. The fuzzy critical path is identified from P.

5. Algorithm (for finding critical path) [3]

Step 1: Estimate the fuzzy activity time with respect to each activity.

Step 2: Let $ESF_1 = (0.0,0.0,0.0,0.0)$ and calculate ESF_j , j =2,3,…. n by using property 1.

Step 3: Let $LFF_n = ESF_n$ and calculate LFF_i ,i=n-1, n-2,….. 2,1. By using property 2.

Step 4: Calculate SFT_{ij} with respect to each activity in a project network by using property 3.

Step 5 : Calculate centre of gravity for each activity using definition 2.4.

Step 6 : If total float =0 (using centre of gravity), the corresponding path is a fuzzy critical path.

6. NUMERICAL EXAMPLE

The problem is to find the fuzzy critical path in a acyclic project network whose edges are assigned with type-2 discrete fuzzy numbers.

Solution :

The edge lengths are

$$\tilde{A} = (0.7/0.6 + 0.6/0.5)/3 + (0.6/0.9)/2$$
$$\tilde{B} = (0.5/0.7 + 0.7/0.6)/2 + (0.4/0.3)/3$$
$$\tilde{C} = (0.9/0.7)/3 + (0.7/0.8 + 0.6/0.7)/4$$
$$\tilde{D} = (0.8/0.4)/3$$
$$\tilde{E} = (0.7/0.8 + 0.6/0.8)/3 + (0.3/0.7)/4$$
$$\tilde{F} = (0.7/0.8 + 0.2/0.7)/2$$
$$\tilde{G} = (0.5/0.5)/3 + (0.3/0.4 + 0.6/0.5)/4$$

Complement of edge lengths are

$$\overline{\tilde{A}} = (0.7/0.4 + 0.6/0.5)/3 + (0.6/0.1)/2$$
$$\overline{\tilde{B}} = (0.5/0.3 + 0.7/0.4)/2 + (0.4/0.7)/3$$
$$\overline{\tilde{C}} = (0.9/0.3)/3 + (0.7/0.2 + 0.6/0.3)/4$$
$$\overline{\tilde{D}} = (0.8/0.6)/3$$
$$\overline{\tilde{E}} = (0.7/0.2 + 0.6/0.2)/3 + (0.3/0.3)/4$$
$$\overline{\tilde{F}} = (0.7/0.2 + 0.2/0.3)/2$$
$$\overline{\tilde{G}} = (0.5/0.5)/3 + (0.3/0.6 + 0.6/0.5)/4$$

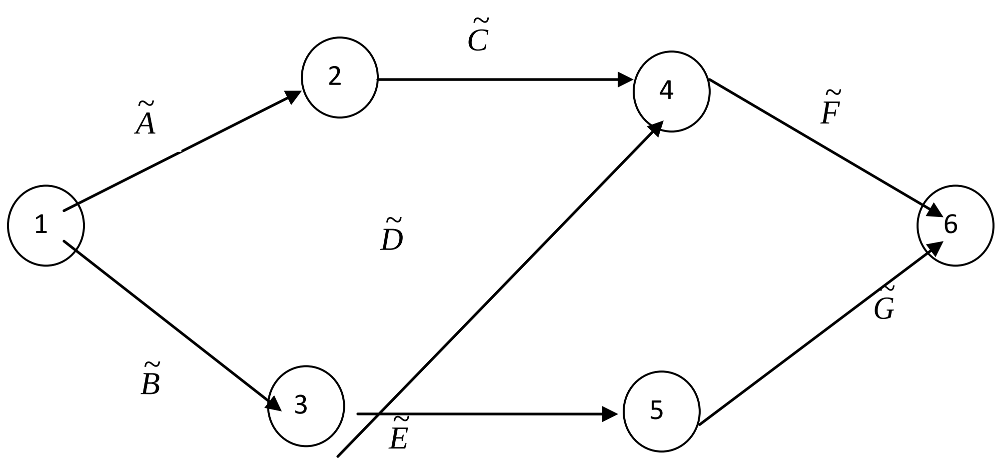

Fig .6.1 Acyclic project network

Activities, fuzzy durations and total slack time for each activity for type-2 discrete fuzzy number and its complement are given in the table 6.1 and table 6.2.

Table: 6.1

Activity (i-j)i<j	Fuzzy activity time	Total float
1-2	$(0.7/0.6 + 0.6/0.5)/3 + (0.6/0.9)/2$	1.4

1-3	$(0.5/0.7+0.7/0.6)/2+(0.4/0.3)/3$	0
2-4	$(0.9/0.7)/3+(0.7/0.8+0.6/0.7)/4$	1.4
3-4	$(0.8/0.4)/3$	2.0
3-5	$(0.7/0.8+0.6/0.8)/3+(0.3/0.7)/4$	0
4-6	$(0.7/0.8+0.2/0.7)/2$	1.4
5-6	$(0.5/0.5)/3+(0.3/0.4+0.6/0.5)/4$	0

Table: 6.2

Activity (i-j)i<j	Fuzzy activity time	Total float
1-2	$(0.7/0.4+0.6/0.5)/3+(0.6/0.1)/2$	1.55
1-3	$(0.5/0.3+0.7/0.4)/2+(0.4/0.7)/3$	0
2-4	$(0.9/0.3)/3+(0.7/0.2+0.6/0.3)/4$	1.55
3-4	$(0.8/0.6)/3$	2.1
3-5	$(0.7/0.2+0.6/0.2)/3+(0.3/0.3)/4$	0
4-6	$(0.7/0.2+0.2/0.3)/2$	1.55
5-6	$(0.5/0.5)/3+(0.3/0.6+0.6/0.5)/4$	0

The path 1-3-5-6 is identified as a fuzzy critical path by type reduction method from the possible paths P = {(1-2-4-6),(1-3-5-6),(1-3-4-6)}

5. CONCLUSION

In this paper, we have proposed type reduction process to identify fuzzy critical path in a acyclic project network using type-2 discrete fuzzy numbers and its complement. The same fuzzy critical path is obtained in both cases.

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