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Optimal N-Policy for Bernoulli feedback $M^X/G/1$ Machining System

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Abstract- This study applies N-policy to an $M^X/G/1$ repairable machining system with Bernoulli feedback and general setup time. The arrival rate of the failed units depends on the state of the system which may be in buildup, setup and busy states. Supplementary variable technique is employed to obtain the probability generating function for the system queue size distribution and the mean number of failed units in the system under steady state conditions. The Laplace-Stieltjes transform of the waiting time is obtained which is further used to evaluate its mean value. The total operational cost of the system is minimized in order to determine the optimal value of N.

Keywords: N-Policy, $M^X/G/1$ system, Batch arrival, Bernoulli feedback, Setup time, Supplementary variable technique, Queue size, waiting time.

1. INTRODUCTION

Queueing theory has emerged as one of the foremost areas of research because of its utility in simulating many real life systems.we proposed an optimal control N-policy for an $M/E_k/1$ system. We now shift our focus to a batch failure queueing model with feedback. Many congestion situations arising in data communication and telecommunication systems can be modeled as feedback queueing models where a unit receiving incomplete service may repeatedly seek service till its service is completed. In recent past, queueing systems with Bernoulli feedback have been extensively investigated by Disney et al. (1980), Simon (1984) and many others. An M/G/1 queue with Bernoulli feedback was studied by Rege (1993). Retrial queues with Bernoulli feedback have been analysed by Krishna Kumar et al. (2002) and recently by Atencia and Moreno (2004).

In many realistic situations, customers arrive in batches. Burke (1975) and Chaudhary and Templeton (1983) have contributed significantly to the study the bulk arrival queues. Lee and Lee (1991) examined an M/G/1 batch arrival queue with different server vacations. Batch arrival queues have been thoroughly investigated by Lee et al. (1995), Yue and Cao (1997), Lee et al. (1998), Bacot and Dshalalow (2001) and others. Chaudhary and Paul (2004) implemented an N-policy to a batch arrival queue with additional service channel.

The purpose of this paper is to investigate an optimal N-policy for an M/G/1 batch failure of units of multi-component machining system with Bernoulli feedback and state-dependent failure rates. At any time, the system can be in buildup, setup or busy states. We assume that the failed unit having received partial repair, rejoins at the head of the queue and hence assumes priority over the other failed units whose repair is yet to commence.

The rest of the paper is organized as follows: In section 2, the system under investigation is described along with requisite notations. Supplementary variable technique is employed in section 3 to establish the state equations and obtain the steady-state queue length. Section 4 covers the queue size distribution during an idle period. We obtain the Laplace-Stieltjes transform of the stationary waiting time of an arbitrary failed unit in the system in the next section 5. The total operating cost of the system is formulated as a function of N in section 6 to determine the optimal value of N. We conclude our investigation in section 7 with concluding remarks.

2. MODEL DESCRIPTION AND NOTATIONS

We study an $M^X/G/1$ machining system with Bernoulli feedback and set up times under N-policy. The underlying assumptions governing the machining system are:

(i) The life-time of units is considered to be exponential distributed. For repair of failed units, we assume i.i.d. general distribution.

(ii) The failed units arrive in batch of random size to a machining centre having single server who repairs these units according to FCFS rule.

(iii) The arrival rate of the failed units depends on the state of the system which may be in buildup, setup or busy states. Accordingly the arrival rate is $\lambda_1, \lambda_2, \lambda_3$, respectively.

(iv) The buildup period comprises of the time during which the server is idle until N failed units accumulate in the system.

(v) The server initiates the setup as soon as the number of failed units reaches N. The setup period is general distributed and is independent of other random variables involved.

(vi) At the end of the setup period, the server starts providing exhaustive repair of failed units till the system again becomes empty. This is called the busy period.

(vii) If repair of a failed unit is not completed successfully, it rejoins the queue for repair. However other failed units leave the system after getting repair.

The following notations are used to completely specify the queueing system:

X	:	Random variable denoting the batch size
X(z)	:	p.g.f. of the batch size X
S	:	Random variable for the repair of the failed units
s(x)	:	p.d.f. of S
$S^*(\theta)$	:	Laplace transform of s(x)
D	:	Random variable for setup time
d(x)	:	p.d.f. of D
$D^*(\theta)$	:	Laplace transform of d(x)
p,q	:	Probability that a failed unit is feedback to the head of the queue for retrial and probability that the failed unit departs from the system after completion of repair, respectively, (p+q = 1)
P_n	:	P_r (System is in buildup period and there are n failed units in the system under steady-state), n = 0, 1, 2, ..., N-1
$R_n(x)dx$	:	Pr (System is in setup period, there are n failed units in the system under steady state and elapsed setup time lies in (x, (x+ Δx)), n = N, N +1, N + 2, N+3, ...
$Q_n(x)dx$	:	Pr (Server is busy, there are n failed units in the system under steady state and elapsed repair time of the failed unit currently being repaired lies in (x,(x _ Δx)), n = 1, 2, 3, ...
$\beta(x)$	:	Conditional completion rates at time x for setup time
μ(x)	:	Conditional completion rates at time x for repair
P(z)	:	p.g.f. or P_n
R(x,z)	:	p.g.f. of $R_n(x)$
Q(x,z)	:	p.g.f. of $Q_n(x)$
$\pi(z)$	:	p.g.f. of system size at an arbitrary epoch under steady-state
$H^*(\theta)$	:	Laplace-Stieltjes transform of p.d.f. of total repair time H(t)

3. QUEUE SIZE DISTRIBUTION

The elapsed repair time and elapsed setup time are treated as supplementary variables. Assuming steady state, the stationary state equations are formulated as:

$$0 = -\lambda_1 P_n + \lambda_1 P_{n-1}, \qquad n = 1, 2, 3, \ldots\ldots N\text{-}1 \qquad (1)$$

$$\frac{dR_N(x)}{dx} = -[\lambda_2 + \beta(x)]R_N(x) \qquad (2)$$

$$\frac{dR_n(x)}{dx} = -[\lambda_2 + \beta(x)]R_n(x) + \lambda_2 \sum_{k=1}^{n-1} R_{n-k}(x).prob(X = k) \qquad n \geq N+1 \tag{3}$$

$$\frac{dQ_1(x)}{dx} = -[\lambda_3 + \mu(x)]Q_1(x) \tag{4}$$

$$\frac{dQ_n(x)}{dx} = -[\lambda_3 + \mu(x)]Q_n(x) + \lambda_3 \sum_{k=1}^{n-1} Q_{n-k}(x).prob(X = k) \qquad n = 2,3,4,\ldots.. \tag{5}$$

In fact, $\beta(x) = \dfrac{d(x)}{1 - D(x)}, \quad \mu(x) = \dfrac{s(x)}{1 - S(x)}$

The boundary conditions for the state equations are given by

$$R_N(0) = \lambda_1 P_{N-1}, \qquad R_n(0) = 0, \qquad n \geq N+1 \tag{6a}$$

$$Q_n(0) = q\int_0^\infty Q_{n+1}(x)\mu(x)dx + p\int_0^\infty Q_n(x)\mu(x)dx, \; n = 1,2,\ldots.N-1 \tag{6b}$$

$$Q_n(0) = q\int_0^\infty Q_{n+1}(x)\mu(x)dx + p\int_0^\infty Q_n(x)\mu(x)dx + \int_0^\infty R_n(x)\beta(x)dx,$$

$$n = N, N+1, N+2,.. \tag{6c}$$

The normalizing conditions is

$$\sum_{n=0}^{N} P_n + \sum_{n=N}^{\infty} \int_0^\infty R_n(x)\beta(x)dx + \sum_{n=1}^{\infty}\int_0^\infty Q_n(x)\mu(x)dx = 1 \tag{7}$$

Define probability generating functions as

$$P(z) = \sum_{n=0}^{N-1} P_n Z^n \tag{8a}$$

$$R(x,z) = \sum_{n=N}^{\infty} R_n(x) z^n \tag{8b}$$

$$Q(x,z) = \sum_{n=N}^{\infty} Q_n(x) z^n \tag{8c}$$

$$\pi(z) = P(z) + Q(z) + R(z) \tag{8d}$$

Now multiplying eq. (1) by z^n and summing, we have

$$P(z) = P_0 \frac{1 - z^N}{1 - z} \tag{9}$$

Multiplying eqs. (2) and (3) with appropriate powers of z^n and summing, we get

$$\frac{\partial R(x,z)}{\partial x} = -[\lambda_2(1 - X(z)) + \beta(x)]R(x,z) \tag{10}$$

In like manner, eqs. (4) and (5) yield :

$$\frac{\partial Q(x,z)}{\partial x} = -[\lambda_3(1 - X(z)) + \mu(x)]Q(x,z) \tag{11}$$

The boundary conditions (6a) – (6c) are used to obtain

$$\mathrm{R}(0,\mathrm{z}) = \lambda_1 \mathrm{P}_{\mathrm{N}-1}\mathrm{Z}^{\mathrm{N}} \tag{12}$$

and

$$Q(0,z) = \left[\frac{(q+pz)}{z}\right]\left[\int_0^\infty Q(x,z)\mu(x)dx - q\int_0^\infty Q_1(x)\mu(x)dx + \int_0^\infty R(x,z)\beta(x)dx\right] \tag{13}$$

Solving the partial differential equations (10) and (11), we get

$$R(x,z) = \lambda_1 P_{N-1} Z^N \exp\left[-\lambda_2(1 - X(z))x - \int_0^x \beta(r)dr\right] \tag{14}$$

and

$$Q(x,z) = Q(0,z)\exp\left[-\lambda_3(1 - X(z))x - \int_0^x \mu(r)dr\right] \tag{15}$$

Substituting values from (14) and (15) in (13) and simplifying, we have

$$Q(0,z) = \frac{P_0\left[z\lambda_1 - \lambda_1 z^{N+1} D^*(\lambda_2(1 - X(z)))\right]}{(pq+q)S^*(\lambda_3(1 - X(z))) - z} \tag{16}$$

where

$$D^*(\lambda_2(1 - X(z))) = \int_0^\infty e^{-\lambda_2(1-X(z))t} d(t)dt \tag{17a}$$

and

$$S^*(\lambda_2(1 - X(z))) = \int_0^\infty e^{-\lambda_3(1-X(z))t} s(t)dt \tag{17b}$$

Using (16) in (15), we find

$$Q(x,z) = P_0 z\lambda_1\left[\frac{\left[1 - z^N D^*(\lambda_2(1 - X(z)))\right]}{(pz+q)S^*(\lambda_3(1 - X(z))) - z}\right]\exp\left[-\lambda_3(1 - X(z))x - \int_0^x \mu(r)dr\right] \tag{18}$$

The partial probability generating functions are hence obtained as

$$R(z) = \int_0^\infty R(x,z)dx = \frac{\lambda_1}{\lambda_2}P_0 z^N\left[\frac{\left[1 - D^*(\lambda_2(1 - X(z)))\right]}{1 - X(z)}\right] \tag{19}$$

and

$$Q(z) = \int_0^\infty Q(x,z)dx$$

$$= \frac{\lambda_1}{\lambda_2} P_0 z \left[\frac{\left[1 - z^N D^*\left(\lambda_2\left(1 - X(z)\right)\right)\right]}{(pz+q)S^*\left(\lambda_3\left(1 - X(z)\right)\right)} \right] \left[\frac{1 - S^*\left(\lambda_3\left(1 - X(z)\right)\right)}{1 - X(z)} \right] \quad (20)$$

Therefore, the p.g.f. of queue size under steady-state is obtained as

$$n(z) = P(z) + Q(z) + R(z)$$

$$= \left\{ \frac{1 - z^N}{1 - z} + \frac{\lambda_1}{\lambda_2} \frac{z^N\left[1 - D^*\left(\lambda_2\left(1 - X(z)\right)\right)\right]}{1 - X(z)} \right.$$

$$\left. + \frac{\lambda_1}{\lambda_3} \frac{z\left[1 - z^N d^*\left(\lambda_2\left(1 - X(z)\right)\right)\right]\left[1 - S^*\left(\lambda_3\left(1 - X(z)\right)\right)\right]}{(1 - X)(z)\left[(pz+q)S^*\left(\lambda_3\left(1 - X(z)\right)\right) - z\right]} \right\} P_0 \quad (21)$$

To evaluate P_0, we use the normalizing condition $\pi(1) = 1$ to yield

$$P_0 = \frac{q - \lambda_3 E(X)E(S)}{\left(q - \lambda_3 E(X)E(S)\right)\left[N + \lambda_1 E(D)\right] + \lambda_1 E(S)\left[N + \lambda_2 E(X)E(D)\right]} \quad (22)$$

By differentiating equation (21) with respect to z and then evaluating at z = 1, we obtain the mean number of customers in the system under steady state as

$$L = \pi'(1) = \left[\frac{q - \lambda_3 E(X)E(S)}{\left(q - \lambda_3 E(X)E(S)\right)\left[N + \lambda_1 E(D)\right] + \lambda_1 E(S)\left[N + \lambda_2 E(X)E(D)\right]} \right].$$

$$\left\{ \frac{N(N-1)}{2} + \left(\frac{2N+1}{2}\right)\lambda_1 E(D) + \frac{\lambda_1 \lambda_2 E(X)E\left(D^2\right)}{2} - \frac{\lambda_1 E\left(X^2\right)E(D)}{2E(X)} \right.$$

$$+ \frac{\lambda_1}{2E(X)\left[q - \lambda_3 E(X)E(S)\right]} \left[E(X)E(S)\left\{ N(N+1) + 2(N+1)\lambda_2 E(X)E(D) \right. \right.$$

$$\left. + \lambda_2^2 \left[E(X)\right]^2 E\left(D^2\right) + \lambda_2 E\left(X^2\right)E(D) - \lambda_2 E(X)E(D) \right\} + \left[N + \lambda_2 E(X)E(D)\right].$$

$$\left. \left\{ \lambda_3 \left(E(X)\right)^2 E\left(S^2\right) + E\left(X^2\right)E(S) - E(X)E(S) \right\} \right] - \frac{3\lambda_1}{2\left[E(X)\right]^2\left[q - \lambda_3 E(X)E(S)\right]^2}.$$

$$\left[E(X)E(S)\left[N + \lambda_2 E(X)E(D)\right] \left\{ \left(q - \lambda_3 E(X)E(S)\right)\left(E\left(X^2\right) - E(X)\right) \right. \right.$$

$$\left. \left. \left. + E(X)\left(2P\lambda_3 E(X)E(S)\right) + \lambda_3^2 \left(E(X)\right)^2 E\left(S^2\right) + \lambda_3 E\left(X^2\right)E(S) - \lambda_3 E(X)E(S) \right) \right] \right] \right\}$$

……………………(23)

4. IDLE PERIOD

The idle period of the machining system under consideration can be expressed as the sum of the buildup period and setup period. Assuming that vacation begins at the end of each busy period, the model can be visualized as a vacation model. The basic purpose is to utilize the idle time for other operations. For exponential repair time, E(S) = 1/μ which substituted in (21) yields

$$\pi(z)=\left[\frac{\left(q-\lambda_3 \frac{E(X)}{\bullet}\right)}{\left(q-\lambda_3 \frac{E(X)}{\bullet}\right)[N+\lambda_1 E(D)]+\left(\frac{\lambda_1}{\mu}\right)[N+\lambda_2 E(X)E(D)]}\right].$$

$$\left\{\frac{1-z^N}{1-z}+\frac{\lambda_1}{\lambda_2}\frac{z^N[1-D^*(\lambda_2(1-X(z)))]}{1-X(z)}+\lambda_1\frac{[z-z^{N+1}D^*(\lambda_2(1-X(z)))]}{(pz+q)\mu-z[\lambda_3(1-X(z))+\mu]}\right\}$$

……………………(24)

Applying lim $\mu \to \infty$ in (24), we have

$$\lim_{\mu\to\infty}\pi(z)=\left\{\frac{1-z^N}{1-z}+\frac{\lambda_1}{\lambda_2}\frac{z^N[1-D^*(\lambda_2(1-X(z)))]}{1-X(z)}\right\}\left[\frac{1}{N+\lambda_1 E(D)}\right]=H(z)(say) \quad (25)$$

From equation (25), we note that H(z) represent the p.g.f. of additional queue size distribution due to typical vacation period composed of a buildup period and a random setup period.

Define B_0 = length of buildup period; and D_0 = length of setup period

Then $E(B_0)=\left(\frac{N}{\lambda}\right)$ and $E(D_0)=E(D)$

Since B_0 and D_0 together define a vacation cycle, we can write

$$\Pr(B_0)=\frac{E(B_0)}{E(B_0)+E(D_0)}=\frac{\left(\frac{N}{\lambda}\right)}{\left(\frac{N}{\lambda}\right)+E(D)}\equiv\Psi(say) \quad (26a)$$

and

$$\Pr(D_0)=1-\Pr(B_0)=\frac{E(D)}{\left(\frac{N}{\lambda}\right)+E(D)}=1-\Psi \quad (26b)$$

Rewriting equation (25), we get

$$\pi(z)=\left[\frac{\left(N/\lambda_1\right)}{\left(N/\lambda_1\right)E(D)}\right]\left[\frac{1-z^N}{N(1-z)}\right]+\left[\frac{E(D)}{\left(N/\lambda_1\right)E(D)}\right]\left[\frac{z^N[1-D^*(\lambda_2(1-X(z)))]}{\lambda_2 E(D)(1-X(z))}\right]$$

$$=\Psi A_N(z)+(1-\Psi)B_N(z) \quad (27)$$

where $A_N(z)=\left[\frac{1-z^N}{N(1-z)}\right]$; *and* $B_N(z)=\left[\frac{z^N[1-D^*(\lambda_2(1-X(z)))]}{\lambda_2 E(D)(1-X(z))}\right]$

Clearly, $A_N(z)$ and ($B_N(z)$) are the p.g.f.s of the conditional distribution of the number of failed units arriving during a buildup period and residual life of a setup period respectively, given that the system is idle.

Equation (27) clearly states that the additional queue size distribution due to vacation period of the batch arrival M/G/1 machining system with different arrival rates is the convex combination of queue sizes due to buildup period and setup period.

5. WAITING TIME

We consider all the situations that an arbitrary (tagged) failed unit can experience. We start by evaluating the probabilities that a failed unit arrives during the buildup, setup or busy period.

Denote I = Length of the buildup period

B = Length of the busy period

Then, $$E(I)=\left(\frac{N}{\lambda_1}\right) \tag{28}$$

The p.g.f. $\chi(z)$ of the number of failed units waiting for repair when the busy period begins is the sum of N failed units arrived during buildup and the number of arrivals during setup period.

Hence, $$\chi(z)=z^N D^*(\lambda_2(1-X(z))) \tag{29}$$

Defining $B^*(\theta)$ as Laplace-Stieltjes transform of the busy period in M/G/1 queue with Bernoulli feedback, the busy period B of the machining system under consideration would have the Laplace-Stieltjes transform $\chi(B^*(\theta))$.

With $(N+\lambda_2 E(X)E(D))$ failed units on the average, the mean busy period E(B) assumes the form

$$E(B)=-\frac{d}{d\theta}\chi(B^*(\theta))\Big|_{\theta\to 0}=\left[\frac{[N+\lambda_2 E(X)E(D)]E(S)}{q-\lambda_3 E(X)E(S)}\right] \tag{30}$$

Expected length of a cycle $E(C)=E(I)+E(D)+E(B)$.
Using the renewal reward theorem, we obtain

Pr(System is under buildup period) = $\frac{E(I)}{E(C)}$

$$=\frac{N(q-\lambda_3 E(X)E(S))}{(q-\lambda_3 E(X)E(S))[N+\lambda_1 E(D)]+\lambda_1 E(S)[N+\lambda_2 E(X)E(D)]} \tag{31a}$$

Pr(System is under setup period) = $\frac{E(D)}{E(C)}$

$$=\frac{\lambda_1 E(D)(q-\lambda_3 E(X)E(S))}{(q-\lambda_3 E(X)E(S))[N+\lambda_1 E(D)]+\lambda_1 E(S)[N+\lambda_2 E(X)E(D)]} \tag{31b}$$

Pr(System is under busy period) = $\frac{E(B)}{E(C)}$

$$=\frac{\lambda_1 E(S)(q-\lambda_2 E(X)E(S))}{(q-\lambda_3 E(X)E(S))[N+\lambda_1 E(D)]+\lambda_1 E(S)[N+\lambda_2 E(X)E(D)]} \tag{31c}$$

The Laplace-Stieltjes transform $W^*_{q(\theta)}$ of waiting time in the system can be analysed by conditioning at each period. Denote by t_j the time between the (j-1)th and jth arrivals during buildup period. Clearly, all t_j's are i.i.d. exponential random variables with parameters λ_1. Let H_i be the total repair time given to the ith failed unit arrived during buildup period. Then the waiting time in the queue of the jth failed unit arriving for repair in the buildup period is expressible as

$$(t_{j+1}+t_{j+2}+....+t_N)+D(H_1+H_2+....+H_{j-1})$$

Assigning equal probability 1/N to an arbitrary failed unit's position anywhere in the queue during buildup, we have

$$W_q''\ (\theta\ /\text{arrival during buildup}) = \frac{1}{N}\sum_{j=1}^{N}\left(\frac{\lambda_1}{\lambda_1+\theta}\right)^{N-j} D^*(\theta)\left(\frac{qS*-(\theta)}{1-pS*(\theta)}\right)^{j-1}$$

$$= \frac{1}{N}\left\{\frac{\left(\frac{\lambda_1}{\lambda_1+\theta}\right)^N - \left(\frac{qS*(\theta)}{1-pS^*(\theta)}\right)^N}{\left(\frac{\lambda_1}{\lambda_1+\theta}\right) - \left(\frac{qS*(\theta)}{1-pS*(\theta)}\right)}\right\} D*(\theta) \qquad (32)$$

The conditional mean waiting time is given by

$$\text{E(Wq/arrival during buildup period)} = -\frac{d}{d\theta}W_q^*(\theta / arrival\ during\ buildup)\Big|_{\theta\to 0}$$

$$= E(D) + \frac{N-1}{2}\left(\frac{1}{\lambda_1} + \frac{E(S)}{q}\right) + \frac{\lambda_1^2\left[2p(E(S)) + qE(S^2)\right] - 2q^2}{2q[1-\lambda_1(S)]\lambda_1} \qquad (33)$$

The waiting time of an arbitrary failed unit arriving during the setup period comprises of three parts –

(i) Time to serve N failed units arriving during buildup period
(ii) Remaining setup time from the epoch at which failed unit arrived.
(iii) Time to serve those failed units who arrived during setup period before it.

The conditional Laplace-Stieltjes transform is given by:

$$W_q^\theta\left[\theta\middle|\begin{array}{l}\text{arrival at setup period, D = x, tagged customer arrival at D y,}\\ \text{n arrivals during D y}\end{array}\right]$$

$$= S*(\theta)^{N+n}\exp(-\theta y)$$

Unconditioning {D=x}, {arbitrary failed unit arrived at D – y}, {n arrivals during D – y},

$$W_q^\theta[\theta|\text{arrival during setup period}]$$

$$= \left(\frac{qS*(\theta)}{1-pS*(\theta)}\right)^N \left[\frac{D*\left(\lambda_2 E(X)\left(\frac{1-S*(\theta)}{1-pS*(\theta)}\right)\right) - D*(\theta)}{E(D)\left[\theta - \lambda_2 E(X)\left(\frac{1-S*(\theta)}{1-pS*(\theta)}\right)\right]}\right] \qquad (34)$$

The conditional mean is then obtained as

$$E[W_q|arrival\ during\ setup\ period] = \frac{NE(S)}{q} + \frac{E(D^2)(q+\lambda_2 E(X)E(S))}{2E(D)q}$$

$$+ \frac{\lambda_2 E(X)}{2q}\left[\frac{2p(E(S))^2 + qE(S^2)}{q-\lambda_2 E(X)E(S)}\right] \qquad (35)$$

For the waiting of a tagged failed unit arriving during the busy period, we consider the busy period as a delayed cycle for which initial delay is the service time to serve total (N+X) failed units. Here X is the number of arrivals during setup period with p.g.f. $D*(\lambda_2(1-X(z)))$. Therefore,

$$W_q^*[\theta|arrival\ \ during\ \ busy\ \ period]$$

$$= \frac{q-\lambda_3 E(X)E(S)}{E(S)[N+\lambda_2 E(X)E(D)]}\left\{\frac{1-\left(\frac{qS*(\theta)}{1-pS*(\theta)}\right)^N D*\left(\lambda_2 E(X)\left(\frac{1-S*(\theta)}{1-pS*(\theta)}\right)\right)}{\theta - \lambda_3 E(X)\left(\frac{1-S*(\theta)}{1-pS*(\theta)}\right)}\right\} \qquad (36)$$

The conditional mean value is

$$E([W_q|\text{arrival at busy period}])=\left[\frac{2p(E(S))^2+qE(S^2)}{2E(S)[q-\lambda_3E(X)E(S)]}\right]$$
$$+E(S)\left\{\frac{N(N-1)+2N\lambda_2E(X)E(D)+\lambda_2^2[E(X)]^2E(D^2)}{2q[N+\lambda_2E(X)E(D)]}\right\} \tag{37}$$

Eqs. (31)-(37) are employed using the total probability arguments to obtain the Laplace-Stieltjes transform of an arbitrary failed unit's waiting time as :

$$W_q^*(\theta)=\left[\frac{q-\lambda_3E(X)E(S)}{(q-\lambda_3(X)E(S))[N+\lambda_1E(D)]+\lambda_1E(S)[N+\lambda_2E(X)E(D)]}\right]$$

$$\times\left\{D^*(\theta)\left[\frac{\left(\frac{\lambda_1}{\lambda_1+\theta}\right)^N-\left(\frac{qS^*(\theta)}{1-pS^*(\theta)}\right)^N}{\left(\frac{\lambda_1}{\lambda_1+\theta}\right)-\left(\frac{qS^*(\theta)}{1-pS^*(\theta)}\right)}\right]\right.$$

$$+\lambda_2\left(\frac{qS^*(\theta)}{1-pS^*(\theta)}\right)^N\left[\frac{D^*\left(\lambda_2E(X)\left(\frac{1-S^*(\theta)}{1-pS^*(\theta)}\right)\right)-D^*(\theta)}{\theta-\lambda_2E(X)\left(\frac{1-S^*(\theta)}{1-pS^*(\theta)}\right)}\right]$$

$$\left.+\lambda_3\left[\frac{1-\left(\frac{qS^*(\theta)}{1-pS^*(\theta)}\right)^N D^*\left(\lambda_2E(X)\left(\frac{1-S^*(\theta)}{1-pS^*(\theta)}\right)\right)}{\theta-\lambda_3E(X)\left(\frac{1-S^*(\theta)}{1-pS^*(\theta)}\right)}\right]\right\} \tag{38}$$

The corresponding mean value is obtained as:

$$E(W_q)=\left[E(D)+\frac{N-1}{2}\left(\frac{1}{\lambda_1}+\frac{E(S)}{q}\right)\right]$$
$$=\left[\frac{N(q-\lambda_3E(X)E(S))}{(q-\lambda_3E(X)E(S))[N+\lambda_1E(D)]+\lambda_1E(S)[N+\lambda_2E(X)E(D)]}\right]$$
$$\times\left[\frac{NE(S)}{q}+\frac{E(D^2)(q+\lambda_2E(X)E(S))}{2E(D)q}+\frac{\lambda_2E(X)}{2q}\left[\frac{2p(E(S))^2+qE(S^2)}{q-\lambda_2E(X)E(S)}\right]\right]$$
$$=\left[\frac{\lambda_1E(D)(q-\lambda_3E(X)E(S))}{(q-\lambda_3E(X)E(S))[N+\lambda_1E(D)]+\lambda_1E(S)[N+\lambda_2E(X)E(D)]}\right]$$
$$\times\left\{\left[\frac{2p(E(S))^2+qE(S^2)}{2E(S)[q-\lambda_3E(X)E(S)]}\right]+\frac{E(S)}{2q}\left[\frac{N(N-1)+2N\lambda_2E(X)E(D)+\lambda_2^2[E(X)]^2E(D^2)}{[N+\lambda_2E(X)E(D)]}\right]\right\}.$$
$$\times\left[\frac{\lambda_1E(S)(N-\lambda_2E(X)E(D))}{(q-\lambda_3E(X)E(S))[N+\lambda_1E(D)]+\lambda_1E(S)[N+\lambda_2E(X)E(D)]}\right] \tag{39}$$

6. OPTIMAL N-POLICY

We propose an optimal N-policy to minimize the mean cost per unit time. First we develop the mathematical expression for the mean cost per unit time.

There are two kinds of costs, setup cost and failed unit holding cost. In a cycle we incur setup cost C_s once. Let C_h be the holding cost of a failed unit per unit time in the system. Then, expected total cost per unit time is expressed as

$$E(TC) = \frac{(\text{Mean accumulative holding cost in a cycle}) + (\text{Setup cost})}{\text{Mean length of a cycle}} \tag{40}$$

The expected holding cost during buildup period is calculated as

$$C_h\left(\frac{1}{\lambda_1}+\frac{2}{\lambda_1}+......+\frac{N-1}{\lambda_1}\right)=\frac{N(N-1)}{2\lambda_1}C_h \tag{41}$$

Let Y_j be the arrival epoch of the j^{th} failed unit during setup period and $f_j(t)$ be the p.d.f. of the period unit j^{th} arrival assumed to be Erlangian (j,λ_2). Then we formulate the holding cost during setup period for $Y_j < D$ as follows.

$$\begin{aligned} C_h E\left[\sum_{j=1}^{\infty}(D-Y_j)\right] &= C_h E\left[\sum_{j=1}^{\infty}\int_0^D (D-t) f_j(t)dt\right] \\ &= C_h E\left[\int_0^D (D-1)\sum_{j=1}^{\infty} f_j(t)dt\right] \\ &= C_h E\left[\int_0^D (D-t)\lambda_2 E(X)dt\right] \\ &= \frac{C_h\lambda_2 E(X)E(D^2)}{2} \end{aligned} \tag{42}$$

Consequently, the holding cost during setup period is given by

$$=\left[\frac{\lambda_2 E(X)E(D^2)}{2}+NE(D)\right]C_h \tag{43}$$

The term $NE(D)C_h$ originates from the failed units who arrived during the buildup period.

For calculating the holding cost during busy period we start with N failed units (arrived during buildup period) plus those who arrived during setup period (denoted by Y). Then the accumulated holding cost h_n during the busy period when the server becomes busy with n failed units is.

$$h_n = \frac{C_h}{1-\rho}\left[\frac{n(n-1)E(S)}{2}+nE(S)+\frac{n\lambda[E(X)]^2 E(S^2)}{2(1-\rho)}\right] \quad \text{where } \rho=\lambda E(S) \tag{44}$$

Since the busy period starts with (N+Y) failed units in the model under investigation where Y has p.g.f. $D^x[\lambda_2(1-X(ZX))]$,

$$\begin{aligned} E(h_{N+Y}) &= \frac{qC_h}{q-\lambda_3 E(X)E(S)}\left\{\frac{E[(N+Y)(N+Y-1)]}{2}\frac{E(S)}{q}\right. \\ &\quad \left. +E(Y+N)\left[\frac{\lambda_3[E(X)]^2 E(S^2)}{2(q-\lambda_3 E(X)E(S))}+\frac{E(S)[1-\lambda_3 E(X)E(S)]}{(q-\lambda_3 E(X)E(S))}\right]\right\} \\ &= \frac{qC_h}{q-\lambda_3 E(X)E(S)}\left[\frac{\{N(N-1)+(2N-1)\lambda_2 E(X)E(D)+\lambda_2^2[E(X)]^2 E(D^2)\}E(S)}{2q}\right] \\ &\quad +[\lambda_2 E(X)E(D)+N]\left[\frac{\lambda_3[E(X)]^2 E(S^2)}{2(q-\lambda_3 E(X)E(S))}\right]+\frac{E(S)[1-\lambda_3 E(X)E(S)]}{(q-\lambda_3 E(X)E(S))} \end{aligned} \tag{45}$$

Therefore, the expected total cost per unit time assumes the form

$$E(TC) = \frac{\frac{N(N-1)C_h}{2\lambda_1} + \left(\frac{\lambda_2 E(X)E(D^2)}{2} + NE(D)\right)C_h + E(h_{N+Y}) + C_s}{E(C)} \quad (46)$$

Considering N as a continuous variable, we set $\frac{\partial\{E(TC)\}}{\partial N} = 0$ which provides optimal value of N as N^*. If N^* is not an integral value, it can be rounded off to the best possible integral value.

7. CONCLUDING REMARKS

Apart from interest from the point of view of theoretical structures, bulk arrival queues have been used for modeling in various situations. Ships arriving at a port in convoy, mail bags arriving at a central sorting station are some examples of arrivals in batches of fixed or random size. Similarly, retrial queues with Bernoulli feedback have been widely used to model problems in telephone switching systems, telecommunication networks and computer networks.

In this paper, we have suggested an optimal N-policy for $M^X/G/1$ machining system with Bernoulli feedback, state-dependent rates and general setup time. Supplementary variable technique has been employed to establish the state equations of the model.

By suitable selecting an optimal value of the decision variable N, the total system cost can be minimized. The work done can provide valuable insight to maintainability engineers for designing cost-effective and efficient models of real time systems arising in the fields of telecommunication, data communications, manufacturing systems etc.

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