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Reliable sampled data control for Stabilization of non-linear multi-agent system with mixed delay and Markovian jumping parameters

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Abstract: The problem of stabilization and consensus of nonlinear multiagent system with mixed dealy and Markovian jumping parameters using both sampled-data and reliable control are presented in this paper. We discuss about three issues of the non-linear MASs using a protocol 1) to derive stability condition 2) to achieve the consensus creteria 3) to calculate the controller gain matrix. These three creteria are obtained by employing a relevent Lyapunov Krasovskii functional and derivating it by using Jensen's inequality, integral inequality technique with Kronecker product properties, we get a linear matrix inequality form which can be solved by wellknown MATLAB LMI toolbox. Terminally, numerical example is provided to illustrate the effectiveness of theoretical results.

Keywords: Reliable, sampled data, nonlinear, Markovian jump.

1 Introduction

For the past few years, the problem of linear and non-linear multi-agent systems is an interesting topic attracted by many literature's due to its plentiful applications in various fields like unmanned autonomous vehicles, large-scale sensor networks, spacecraft formation flying and so on [1]-[4]. A major problem in this multi-agent systems is the consensus problem that is the agreement of a group of agents on their states of leader or leaderless by interaction. The main motive of the consensus protocol is to assure the intelligible performance of multiagents by implementing suitable feedback control signals and by incorporating communication or network topologies and its applied in various fields such as vehicle systems, groups of mobile autonomous agent, networked control systems [5],[6]. To mention the communication protocols between the multiagents, graph theory has been effectively used for the modeling of networks and the derivation of stability conditions via Lyapunov stability theory [7],[8].

We known that time-delay is frequently causes unwanted signal like oscillations and noises of the system so it is very important to study them which can be discussed in [9]. Especially multiagent system with mixed time delay system can be studied in [10]. However lots of controls are used by researchers, specially in sampled-data control its states suffer successive impulses at fixed times. The sampled-data system is a combination of both continuous time and discrete time signals. In sampled-data control systems, control signals are in constants during sampling intervals and are allowed to change only at sampling instants. Because of this reason, the control signals in sampled-data control systems have stepwise form and these discontinuous signals cannot be applied directly to stabilize control systems. Consensus of multi-agent system with sampled data control are examined in [11],[13].

Designing the reliable control with multi-agent systems has been attracted since practical systems often have actuator failures. To stabilize the systems against actuator failures or to design fault-tolerant control

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systems, the well known reliable control system is used. The actuator failure model contains scaling factor with upper and lower bounds to the signal which can be measured. In [14],[15] reliable control with multiagent system is invesigated. Markovian jump systems is used to gives an essential results for the system model which are affected by random switching behaviour. Lots of interesting results are exists for Markovian jumping parameters in multiagent systems because of its applications in different fields which are examined in [16],[17]. Consensus problem for linear multi-agent system with both sampled -data and reliable control by using Wirtinger-based integral inequality can be examined in [18].

Motivated by this above disscussion, in this paper, stabilization and consensus problem for mixed delayed non-linear multi-agent systems with both sampled-data and reliable control along with Markovian jumping parameters will be studied.

$Notations$: The notations used in this paper are standard. $\mathbb{R}^n$ denotes n dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. The script $*$ denotes the symmetric term in the matrix. The transpose and inverse for the matrix $\mathcal{A}$ is denoted by $\mathcal{A}^T$ and $\mathcal{A}^{-1}$, respectively. $\mathcal{I}$ is the identity matrix with appropriate dimension, $\mathcal{X} > 0$ is symmetric positive definite matrix, $\mathcal{A}\otimes\mathcal{B}$ denotes the Kronecker product of matrices $\mathcal{A}$ and $\mathcal{B}$.

2 Problem description and preliminaries

Before we enter in to the main part of this section we must know about some basic concepts of the algebraic graph theory. It is very important to examine the problem of multi-agent system. A graph is used to indicate the interconnection of agents with one another. It may be directed or undirected graph. In this paper we consider the undirected graph $\mathscr{G} = (\mathscr{V},\ \mathscr{E},\ \mathscr{A})$ with order N, i.e., where N is the number of agents, the vertex set $\mathscr{V} = \{v_1, v_2, ..., v_N\}$ represents the set of all agents, the edge set $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$ represents the connection between the agents, and $\mathscr{A} = [a_{ij}]_{N\times N}$ is the adjacency matrix with $a_{ij} > 0$ if $(v_i, v_j) \in \mathscr{E}$ and $a_{ij} = 0$ otherwise. The neighbour set of v_i can be represented as $\mathscr{N}_i = \{j : (v_i, v_j) \in \mathcal{E}\}$. Let $\mathscr{D} = diag\{deg(1), deg(2), ..., deg(i)\}$ be the degree matrix of the undirected graph $\mathscr{G}$ with entries $deg(i) = \sum_{j\in\mathscr{N}_i} a_{ij}$. Then the Laplacian matrix of $\mathscr{G}$ can be expressed as $\mathcal{L} = \mathscr{D} - \mathscr{A}$.

Now we consider a mixed delayed non-linear multiagent system with Markov jumping parameter consists of N agents, is represented as follows:

$$\begin{aligned}\dot{x}_i(t) &= \mathcal{A}_1(r(t))x_i(t) + \mathcal{A}_2(r(t))f(x_i(t)) + \mathcal{A}_3(r(t))f(x_i(t-\tau(t))) + \mathcal{A}_4(r(t))\int_{t-\tau(t)}^{t} f(x_i(s))ds \\ &\quad + \mathcal{B}(r(t))u_i(t),\ \ i = 1, ..., N, \qquad (1)\end{aligned}$$

where N is the number of agents, $x_i(t) \in \mathbb{R}^n$ is a state of agent i, $u_i(t) \in \mathbb{R}^m$ is a consensus protocol, $r(t)(t \geq 0)$ is continuous time Markovian process taking the values from the finite set space $\mathbb{S}$ with transition probability matrix $\Pi \triangleq \Pi_{pq}$ is given by

$$Pr(r(t+\triangle(t)) = q|r(t) = p) = \begin{cases} \Pi_{pq}\triangle(t) + o(\triangle(t)),\ if\ q \neq p, \\ 1 + \Pi_{pq}\triangle(t) + o(\triangle(t)),\ if\ q = p, \end{cases}$$

where $\triangle(t) > 0,\ lim_{\triangle(t)\to 0}\frac{o(\triangle(t))}{\triangle(t)} = 0$ and $\Pi_{pq} \geq 0$ is the transition rate from mode p at time t to mode q at time $t+\triangle(t)$ if $p \neq q$ and $\Pi_{pp} = -\sum_{q=1,q\neq p}^{N}\Pi_{pq}\forall p \in \mathbb{S}$. $\mathcal{A}_1(r(t)),\ \mathcal{A}_2(r(t)),\ \mathcal{A}_3(r(t)),\ \mathcal{A}_4(r(t)),\ \mathcal{B}(r(t))$ are known constant matrices. For our convienence, we denote the terms $\mathcal{A}_1(r(t)),\ \mathcal{A}_2(r(t)),\ \mathcal{A}_3(r(t)),\ \mathcal{A}_4(r(t)),\ \mathcal{B}(r(t))$ are as $\mathcal{A}_{1p},\ \mathcal{A}_{2p},\ \mathcal{A}_{3p},\ \mathcal{A}_{4p},\ \mathcal{B}_p$ respectively. The control input is denoted as

$$u_i(t) = -\sum_{j\in N_i} a_{ij}(x_j(t) - x_i(t)),\ i = 1, ..., N \qquad (2)$$

where a_{ij} are the interconnection weights defining

$$\begin{cases} a_{ij} > 0, \ if \ agent \ i \ isconnected \ to \ agent \ j \\ a_{ij} = 0, \ otherwise \end{cases}$$

The actuator fault is considered in this paper,

$$u_i^F(t) = \mathcal{R}u_i(t) \tag{3}$$

where $\mathcal{R}$ is the actuator fault with $\mathcal{R} = diag\{r_1, r_2, ..., r_n\}, \ 0 \leq r_m \leq 1, \ m = 1, 2, ...n.$

We assumed that the updating instant time is denoted by t_k and it satisfies $0 = t_0 < t_1 < .... < t_k < .... < lim_{k\to\infty} t_k = +\infty$. The sampling interval is defined as $t_{k+1} - t_k = \eta_k \leq \eta$ for any integer $k \geq 0$, where $\eta > 0$ represents the largest sampling interval. Define $x_i(t_k) = x_i(t - h(t))$ with $h(t) = t - t_k, \ 0 \leq h(t) \leq h$ for $t \neq t_k$. Then, combining eqn.(2) and eqn.(3) we get,

$$u_i(t) = -\mathcal{R} \sum_{j \in N_i} a_{ij}(x_j(t - h(t)) - x_i(t - h(t))). \tag{4}$$

The eqn.(1) becomes,

$$\begin{aligned} \dot{x}_i(t) &= \mathcal{A}_{1p}x_i(t) + \mathcal{A}_{2p}f(x_i(t)) + \mathcal{A}_{3p}f(x_i(t - \tau(t))) + \mathcal{A}_{4p}\int_{t-\tau(t)}^{t} f(x_i(s))ds \\ &+ \mathcal{B}_p\mathcal{R}\sum_{j \in N_i} a_{ij}(x_j(t - h(t)) - x_i(t - h(t))), \ \ i = 1, ..., N, \end{aligned} \tag{5}$$

By using the Kronecker product properties, the above equation can be written as

$$\begin{aligned} \dot{x}(t) &= (I_N \otimes \mathcal{A}_{1p})x(t) + (I_N \otimes \mathcal{A}_{2p})f(x(t)) + (I_N \otimes \mathcal{A}_{3p})f(x(t - \tau(t))) + (I_N \otimes \mathcal{A}_{4p})\int_{t-\tau(t)}^{t} \\ & f(x(s))ds + (\mathcal{L} \otimes \mathcal{B}_p)\mathcal{K}\mathcal{R}(x(t - h(t)), \ \ i = 1, ..., N, \end{aligned} \tag{6}$$

The following assumption, definition, lemmas are important to prove our results.

Assumption 2.1. [19] For any $j \in 1, 2, ..., n, \ f_j(0) = 0$ and their exist constants $\Lambda_j^- \ and \ \Lambda_j^+$ such that

$$\Lambda_j^- \leq \frac{f_j(\alpha_1) - f_j(\alpha_2)}{\alpha_1 - \alpha_2} \leq \Lambda_j^+ \ \ \forall \ \alpha_1 \neq \alpha_2.$$

Definition 2.2. [20] The stabilization of multi-agent system (6) is said to be achieved if, for any given initial conditions, the following holds:

$$lim_{t\to\infty}|x_i(t)| = 0, \ \forall \ i = 1, ..., N.$$

Definition 2.3. [21] The consensus of system (1) is said to be achieved asymptotically in the sense of mean-square if, for each agent $i \in \{1, 2, ..., N\}$ there is a local state feedback u_i of $\{x_j : j \in \mathscr{N}_i\}$ such that the closed-loop system (6) satisfies

$$lim_{t\to\infty}\mathbb{E}\{\|x_i(t) - x_j(t)\|^2\} = 0.$$

Lemma 2.4 (Jensen's inequality). *[22] For any constant matrix $M \in \mathbb{R}^{n\times n}, \ M^T = M > 0$, scalars $\alpha, \ \beta$ with $\alpha > \beta$ and vector $x : [\beta, \alpha] \to \mathbb{R}^n$, such that the following integrations are well defined, then*

$$-(\alpha - \beta)\int_{\beta}^{\alpha} x^T(s)Mx(s)ds \leq -\bigg(\int_{\beta}^{\alpha} x(s)ds\bigg)^T M\bigg(\int_{\beta}^{\alpha} x(s)ds\bigg).$$

Lemma 2.5. *[23]For given symmetric positive definite matrix $Y > 0$ and any differentiable function x in $[a,b] \to \mathbb{R}^n$, the following inequality holds:*

$$\int_a^b \dot{x}(s) Y \dot{x}(s) ds \geq \frac{3}{4(b-a)} \Omega^T R \Omega,$$

where $\Omega = (x^T(b),\ x^T(a), \frac{1}{b-a}\int_a^b x^T(s)ds),\ R = \begin{bmatrix} 5Y & 2Y & -7Y \\ * & 4Y & -6Y \\ * & * & 13Y \end{bmatrix}.$

3 Main results

Theorem 3.1. *For given scalars $h > 0,\ \mu > 0,\ \tau > 0$, then the stabilization and consensus criteria for the MASs (6) and (1) is achieved respectively if there exists a positive definite matrices $\mathcal{P}_1,\ \mathcal{P}_2,\ \mathcal{Q}_n,\ (n = 1,2,...,7)$, diagonal matrices $\Gamma_1,\ \Gamma_2$ and any matrix $\mathcal{S}$, with appropriate dimensions such that the following linear matrix inequality exists:*

$$\Xi = \begin{bmatrix} \Omega & \Psi \\ * & \Upsilon \end{bmatrix} < 0, \tag{7}$$

where,

$$\Omega = \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & -\frac{3}{2}(I_N \otimes \mathcal{Q}_2) & -\frac{3}{2}(I_N \otimes \mathcal{Q}_2) & 0 \\ * & \Omega_{2,2} & 0 & 0 & 0 \\ * & * & -3(I_N \otimes \mathcal{Q}_3) & 0 & 0 \\ * & * & * & -3(I_N \otimes \mathcal{Q}_6) - (I_N \otimes \mathcal{Q}_5) & 0 \\ * & * & * & * & -\Lambda_1\Gamma_2 - (1-\mu)(I_N \otimes \mathcal{Q}_1) \end{bmatrix},$$

$$\Psi = \begin{bmatrix} 0 & \Lambda_2\Gamma_1 & 0 & \frac{21}{4}(I_N \otimes \mathcal{Q}_3) & \frac{21}{4}(I_N \otimes \mathcal{Q}_6) & 0 \\ (L \otimes \mathcal{B})\mathcal{R}\mathcal{X} & (I_N \otimes \mathcal{A}_{2p}\mathcal{S}) & \mathcal{S}(I_N \otimes \mathcal{A}_{3p}) & 0 & 0 & (I_N \otimes \mathcal{A}_{4p}\mathcal{S}) \\ 0 & 0 & 0 & \frac{9}{2}\mathcal{Q}_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{9}{2}(I_N \otimes \mathcal{Q}_6) & 0 \\ 0 & 0 & \Lambda_2\Gamma_2 & 0 & 0 & 0 \end{bmatrix},$$

$\Upsilon = diag\{-(1-h)(I_N \otimes \mathcal{Q}_4),\ -\Gamma_1 + \tau^2(I_N \otimes \mathcal{Q}_7),\ -\Gamma_2,\ -\tau^2(I_N \otimes \mathcal{Q}_2) - \frac{45}{4}(I_N \otimes \mathcal{Q}_3),\ -\frac{45}{4}(I_N \otimes \mathcal{Q}_6),\ -(I_N \otimes \mathcal{Q}_7)\}, \Omega_{1,1} = \sum_{q=1}^{N} \Pi_{pq}(I_N \otimes \mathcal{P}_q) + (I_N \otimes \mathcal{Q}_1) + \tau^2(I_N \otimes \mathcal{Q}_2) + (I_N \otimes \mathcal{Q}_4) + (I_N \otimes \mathcal{Q}_5) - \frac{15}{4}(I_N \otimes \mathcal{Q}_3) - \frac{15}{4}(I_N \otimes \mathcal{Q}_6) - \Gamma_1\Lambda_1, \Omega_{1,2} = (I_N \otimes \mathcal{P}_p) + (I_N \otimes \mathcal{A}_{1p}\mathcal{S}),\ \Omega_{2,2} = h^2(I_N \otimes \mathcal{Q}_6) + \tau^2(I_N \otimes \mathcal{Q}_3) - (I_N \otimes \mathcal{S}).$

Proof: Let us consider the Lyapunov Krasovskii functional as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t),$$

where,

$$V_1(t) = x^T(t)(I_N \otimes \mathcal{P}_p)x(t),$$
$$V_2(t) = \int_{t-\tau_1(t)}^{t} x^T(s)(I_N \otimes \mathcal{Q}_1)x(s)ds,$$
$$V_3(t) = \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} x^T(s)(I_N \otimes \mathcal{Q}_2)x(s)dsd\theta + \tau \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)(I_N \otimes \mathcal{Q}_3)\dot{x}(s)dsd\theta,$$
$$V_4(t) = \int_{t-h(t)}^{t} x^T(s)(I_N \otimes \mathcal{Q}_4)x(s)ds + \int_{t-h}^{t} x^T(s)(I_N \otimes \mathcal{Q}_5)x(s)ds,$$

$$V_5(t) = h\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)(I_N\otimes\mathcal{Q}_6)\dot{x}(s)dsd\theta,$$
$$V_6(t) = \tau\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}f^T(x(s))(I_N\otimes Q_7)f(x(s))dsd\theta.$$

Calculating the derivatives of $V(t)$ we get,

$$\dot{V}_1(t) = 2x^T(t)(I_N\otimes\mathcal{P}_p)\dot{x}(t) + \sum_{q=1}^{N}\Pi_{pq}x^T(t)(I_N\otimes\mathcal{P}_q)x(t), \tag{8}$$
$$\dot{V}_2(t) = x^T(t)(I_N\otimes\mathcal{Q}_1)x(t) - (1-\mu)x^T(t-\tau(t))\mathcal{Q}_1x(t-\tau(t)), \tag{9}$$
$$\dot{V}_3(t) = \tau^2x^T(t)(I_N\otimes\mathcal{Q}_2)x(t) - \tau\int_{t-\tau}^{t}x^T(s)(I_N\otimes\mathcal{Q}_2)x(s)ds + \tau^2\dot{x}^T(t)(I_N\otimes\mathcal{Q}_3)\dot{x}(t) - \tau\int_{t-\tau}^{t}\dot{x}^T(s)$$
$$(I_N\otimes\mathcal{Q}_3)\dot{x}(s)ds, \tag{10}$$
$$\dot{V}_4(t) = x^T(t)((I_N\otimes\mathcal{Q}_4)+(I_N\otimes\mathcal{Q}_5))x(t) - (1-h)x^T(t-h(t))(I_N\otimes\mathcal{Q}_4)x(t-h(t))$$
$$- x^T(t-h)(I_N\otimes\mathcal{Q}_5)x(t-h), \tag{11}$$
$$\dot{V}_5(t) = h^2\dot{x}^T(t)(I_N\otimes\mathcal{Q}_6)\dot{x}(t) - h\int_{t-h}^{t}\dot{x}^T(s)(I_N\otimes\mathcal{Q}_6)\dot{x}(s)ds, \tag{12}$$
$$\dot{V}_6(t) = \tau^2f^T(x(t))(I_N\otimes\mathcal{Q}_7)f(x(t)) - \tau\int_{t-\tau(t)}^{t}f^T(x(s))(I_N\otimes\mathcal{Q}_7)f(x(s))ds. \tag{13}$$

By using Jensen's inequality,

$$-\tau\int_{t-\tau}^{t}x^T(s)\Big(I_N\otimes\mathcal{Q}_2\Big)x(s)ds \leq -\Big(\int_{t-\tau}^{t}x(s)ds\Big)^T(I_N\otimes\mathcal{Q}_2)\Big(\int_{t-\tau}^{t}x(s)ds\Big), \tag{14}$$
$$-\tau\int_{t-\tau(t)}^{t}f^T(x(s))(I_N\otimes\mathcal{Q}_7)f(x(s))ds \leq -\Big(\int_{t-\tau(t)}^{t}f(x(s))ds\Big)^T(I_N\otimes\mathcal{Q}_7)\Big(\int_{t-\tau(t)}^{t}f(x(s))ds\Big). \tag{15}$$

By using Lemma ,

$$-\tau\int_{t-\tau}^{t}\dot{x}^T(s)(I_N\otimes\mathcal{Q}_3)\dot{x}(s)ds \leq -\frac{3}{4}\Theta_1^T\Pi(I_N\otimes\mathcal{Q}_3)\Theta_1, \tag{16}$$
$$-h\int_{t-h}^{t}\dot{x}^T(s)(I_N\otimes\mathcal{Q}_6)\dot{x}(s)ds \leq \Theta_2^T\Pi(I_N\otimes\mathcal{Q}_6)\Theta_2, \tag{17}$$

where,

$$\Theta_1 = [x(t)\ x(t-\tau)\ \frac{1}{\tau}\int_{t-\tau}^{t}x^T(s)ds],\ \Theta_2 = [x(t)\ x(t-h)\ \frac{1}{h}\int_{t-h}^{t}x^T(s)ds],\ \Pi = \begin{bmatrix} 5 & 2 & -7 \\ * & 4 & -6 \\ * & * & 13 \end{bmatrix}.$$

For any positive diagonal matrices Γ_1 and Γ_2, from Assumption we have,

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T\begin{bmatrix} -\Lambda_1\Gamma_1 & \Lambda_2\Gamma_1 \\ * & -\Gamma_1 \end{bmatrix}\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \geq 0, \tag{18}$$
$$\begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t)) \end{bmatrix}^T\begin{bmatrix} -\Lambda_1\Gamma_2 & \Lambda_2\Gamma_2 \\ * & -\Gamma_2 \end{bmatrix}\begin{bmatrix} x(t-\tau(t)) \\ f(x(t-\tau(t)) \end{bmatrix} \geq 0. \tag{19}$$

For any appropriate dimensional matrix $\mathcal{S}$, the following equation hold:

$$\dot{x}^T(t)(I_N \otimes \mathcal{S})[-\dot{x}(t) + (I_N \otimes \mathcal{A}_{1p})x(t) + (I_N \otimes \mathcal{A}_{2p})f(x(t)) + (I_N \otimes \mathcal{A}_{3p})f(x(t-\tau(t))) + (I_N \otimes \mathcal{A}_{4p}) \int_{t-\tau(t)}^{t} f(x(s))ds + (\mathcal{L} \otimes \mathcal{B}_p)\mathcal{K}\mathcal{R}(x(t-h(t))] = 0. \tag{20}$$

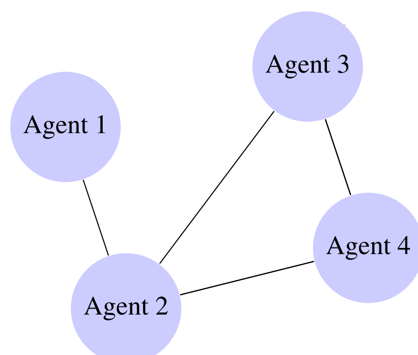
From eqn(8) to (20) we get,

$$\dot{V}(t) \leq \xi^T(t)\Xi\xi(t), \tag{21}$$

where $\xi(t) = [x(t)\ \ \dot{x}(t)\ \ x(t-\tau)\ \ x(t-h)\ \ x(t-\tau(t))\ \ x(t-h(t))\ \ f(x(t))\ \ f(x(t-\tau(t)))\ \ \frac{1}{\tau}\int_{t-\tau}^{t} x(s)ds$ $\frac{1}{h}\int_{t-h}^{t} x(s)ds\ \ \int_{t-\tau(t)}^{t} f(x(s))ds]$, we get $\dot{V}(t) \leq 0$ which leads to $lim_{t\to\infty}|x_i(t)| = 0$, therefore by Definition 2.2 the closed loop system (6) is stable and by Definition 2.3 the multiagent system (1) achieves consensus in mean square. This completes the proof.

4 Numerical examples

Here we present the numerical example to show the effectiveness of theoretical results.
Now we consider 4 agents i.e., $N = 4$ the communication among the agents are represented through the following graph $\mathcal{G}$.



$fig\ 1\ The\ interconnection\ topology\ graph\ \mathcal{G}$

By using this graph, the Laplacian matrix is obtained as $\mathcal{L} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$.

Example 4.1. Consider a mixed delayed closed loop multiagent system with Markovian jumping parameter is as follows:

$$\dot{x}(t) = (I_N \otimes \mathcal{A}_{1p})x(t) + (I_N \otimes \mathcal{A}_{2p})f(x(t)) + (I_N \otimes \mathcal{A}_{3p})f(x(t-\tau(t))) + (I_N \otimes \mathcal{A}_{4p}) \int_{t-\tau(t)}^{t} f(x(s))ds + (\mathcal{L} \otimes \mathcal{B})\mathcal{K}\mathcal{R}(x(t-h(t)), \tag{22}$$

with the parameter as,
Mode 1:

$\mathcal{A}_{11} = \begin{bmatrix} -0.5 & 0 \\ 0 & -1.8 \end{bmatrix}$, $\mathcal{A}_{21} = \begin{bmatrix} -2.6 & 1.8 \\ 0.5 & 0.01 \end{bmatrix}$, $\mathcal{A}_{31} = \begin{bmatrix} -0.5 & 1.1 \\ 2.3 & -0.7 \end{bmatrix}$, $\mathcal{A}_{41} = \begin{bmatrix} 1 & 0.1 \\ 0.4 & 3.7 \end{bmatrix}$,
$\mathcal{B}_1 = \begin{bmatrix} 1.3 & 0 \\ -1 & 1.3 \end{bmatrix}$,

Mode 2:

$$\mathcal{A}_{12} = \begin{bmatrix} -2.5 & 0 \\ 0 & -1.7 \end{bmatrix}, \mathcal{A}_{22} = \begin{bmatrix} 1.4 & 0.8 \\ 0.05 & 1.7 \end{bmatrix}, \mathcal{A}_{32} = \begin{bmatrix} 0.6 & 1.8 \\ 1.5 & 0 \end{bmatrix}, \mathcal{A}_{42} = \begin{bmatrix} 1 & 0.4 \\ 0.2 & 1 \end{bmatrix},$$
$$\mathcal{B}_2 = \begin{bmatrix} 1.4 & 0 \\ -1 & 1.4 \end{bmatrix}.$$

The other matrices are taken as

$$\Lambda_1 = \Lambda_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \mathcal{R} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

and the remaining parameters are set as $\mu = 0.3,\ \tau = 0.4,\ h = 0.6$
with the transition probability matrix $\Pi = \begin{bmatrix} -2 & 2 \\ 3 & -3 \end{bmatrix}$.
By solving LMIs in Theorem 3.1 we get,

$$\mathcal{P}_1 = \begin{bmatrix} 0.0742 & -0.1262 \\ -0.1262 & 0.4729 \end{bmatrix}, \mathcal{P}_2 = \begin{bmatrix} 0.4047 & -0.0957 \\ -0.0957 & 0.3065 \end{bmatrix}, \mathcal{Q}_1 = \begin{bmatrix} 0.1829 & -0.1264 \\ -0.1264 & 0.2648 \end{bmatrix},$$
$$\mathcal{Q}_2 = \begin{bmatrix} 0.8875 & -0.3993 \\ -0.3993 & 1.1424 \end{bmatrix}, \mathcal{Q}_3 = \begin{bmatrix} 0.1288 & 0.0192 \\ 0.0192 & 0.1548 \end{bmatrix}, \mathcal{Q}_4 = \begin{bmatrix} 0.4264 & -0.2396 \\ -0.2396 & 0.5939 \end{bmatrix},$$
$$\mathcal{Q}_5 = \begin{bmatrix} 0.2911 & -0.1757 \\ -0.1757 & 0.4041 \end{bmatrix}, \mathcal{Q}_6 = \begin{bmatrix} 0.0489 & 0.0054 \\ 0.0054 & 0.0648 \end{bmatrix}, \mathcal{Q}_7 = \begin{bmatrix} 2.9417 & -2.0088 \\ -2.0088 & 9.0892 \end{bmatrix},$$
$$\Gamma_1 = \begin{bmatrix} 7.5053 & 0 \\ 0 & 7.5053 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 4.9091 & 0 \\ 0 & 4.9091 \end{bmatrix}, \mathcal{S} = \begin{bmatrix} 0.1379 & -0.0397 \\ -0.0397 & 0.1560 \end{bmatrix},$$
$$\mathcal{X} = \begin{bmatrix} 0.5041 & 0.7590 \\ 0.7590 & 2.6012 \end{bmatrix}.$$

The control gain matrix is given by $\mathcal{K} = \mathcal{S}^{-1}\mathcal{X}$ as $\mathcal{K} = \begin{bmatrix} 0.4063 & -0.1555 \\ -0.1883 & 0.6059 \end{bmatrix}$.

5 Conclusion

This paper concerns the Markovian jumping nonlinear multiagent system with mixed delay using reliable sampled data control. To get stability condition for the multi-agent system, we choose the relevant Lyapunov Krasovskii functional and constructing the suitable lemmas such as new integral inequality, Jensen's inequality, then the result can be obtain in the form of linear matrix inequalities. At last a numerical example is given to show the effectiveness of the proposed theoretical results.

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