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Inverse and Disjoint Connected Cototal Domination number of the Jump Graph of a Graph

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Abstract- Let $D \subseteq V[J(G)]$ be a connected cototal dominating set of $J(G)$, if $< V[J(G)] - D > \neq \emptyset$ contain a dominating set D' such that $< V[J(G)] - D' >$ has no isolated vertex and $< D' >$ is connected, then D' is the inverse connected cototal dominating set of $J(G)$ with respect to D. The minimum cardinality of a minimal inverse connected cototal dominating set is termed as, the inverse connected cototal domination number, denoted by $\gamma_{cct}[J(G)]$. Exact values of some standard graphs, bounds and the relationship of this parameter with other graph theoretic graph parameters are evaluated.
Keywords: Inverse Domination number of the jump graph of a graph, Inverse Connected Cototal Dominating set of a jump graph, Inverse Connected Cototal Domination number of a jump graph, Disjoint Connected Cototal Domination number of a jump graph.

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1. INTRODUCTION:

All graphs G(p,q) considered here are simple,finite, connected, undirected with order p and size q. For all other notations and terminologies we refer [1].

A graph whose vertex set is the edge set of a graph G is called as a **line graph L(G)**. Two vertices are adjacent in L(G) if and only if the corresponding edges are adjacent in G. The graph defined on the edge set E(G) where two vertices are adjacent if and only if the corresponding edges in G are non-adjacent is referred as **Jump graph $J(G)$** of the graph G. Thus, jump graph is the complement of line graph. Hence the isolated vertices of G, if it exist, has no part in $J(G)$.

A non-empty subset D of the vertex set V[$J(G)$] is a dominating set of $J(G)$, if every vertex not in D is adjacent to atleast one vertex in D. The minimum cardinality of a minimal dominating set of $J(G)$ is the dominating set of $J(G)$ and the cardinality is the domination number of $J(G)$, denoted by $\gamma\ [J(G)]$. Imposing restrictions on the dominating set D, various domination parameters have been defined. When $< D >$ is connected then D is a connected dominating set of $J(G)$ and the minimum cardinality of a minimal connected dominating set is the connected domination number, $\gamma_c[J(G)]$.

Restriction on the complement set $\{ V[J(G)] - D\}$ of the jump graph define many parameters. If there exist a dominating set D' in $\{ V[J(G)] - D \}$ then D' is the inverse dominating set with respect to the dominating set D.When the induced subgraph of the inverse dominating set D' is connected, then D' is the inverse connected dominating set of G, denoted by $\gamma_c^{-1}(G)$.

A dominating set D is a cototal dominating set of $J(G)$ if $< V[J(G)] - D > \neq \emptyset$ contains no isolated vertex. The minimal cototal dominating set with minimum cardinality is the cototal domination number of $J(G)$, denoted as $\gamma_{ct}[J(G)]$.

The disjoint domination number, $\gamma\gamma(G)$ of G is the minimum cardinality of two disjoint dominating sets in G [9].

In this paper, we discussed about the inverse connected cototal domination number of a jump graph and the disjoint connected cototal domination number of a jump graph have been carried.

Here, we have considered, simple connected graph G of size $|E| = q \geq 4$.

2. Inverse Connected Cototal Domination number of the Jump Graph of a Graph:

Definition: 2.1.

Let $D \subseteq V[J(G)]$ be a connected cototal dominating set of the jump graph of G. In $[V[J(G)] - D]$, if there exists a dominating set D' such that $< V[J(G)] - D' >$ contains no isolated vertex and $< D' >$ is connected, then D' is the inverse connected cototal dominating set of $J(G)$ with respect to D. The minimum cardinality of D' is the inverse connected cototal domination number, denoted by $\gamma_{cct}^{-1}\ [J(G)]$.

Definition: 2.2

If the cardinality of D' is maximum with respect to the minimality condition, then D' is known as the upper inverse connected cototal dominating set. Thus, $|D'| = \Gamma_{cct}[J(G)]$ is called the upper inverse connected cototal domination number.

Theorem: 2.3.

Exact values of Some Standard graphs.

(i). For $p \geq 6$, $\gamma_{cct}^{-1}\left[J(P_p)\right] = 2$

(ii).For $p \geq 6$, $\gamma_{cct}^{-1}\left[J(C_p)\right] = 2$

(iii).For $p \geq 6$, $\gamma_{cct}^{-1}\left[J(K_p)\right] = 3$

(iv). For $p = p_1 + p_2$, $\gamma_{cct}^{-1}\left[J(K_{p_1,p_2})\right] = \begin{cases} 3, & p_1 = 2, p_2 \geq 4 \\ 2, & p_1, p_2 \geq 3 \end{cases}$

(v). For $p \geq 6$, $\gamma_{cct}^{-1}\left[J(W_p)\right] = \begin{cases} 3, & p = 6 \\ 2, & p > 6 \end{cases}$

(vi). For $p \geq 4$, $\gamma_{cct}^{-1}\left[J(P_p \circ K_1)\right] = 2,$

(vii). For $p \geq 4$, $\gamma_{cct}^{-1}\left[J(C_p \circ K_1)\right] = 2$

(viii). For the Spider graph of $K_{1,n}$, $n > 3$, $\gamma_{cct}^{-1}\left[J(K_{1,n})\right] = 2$

(ix). For a Fan graph ,$F_p = P_{P-1} + K_1$, $\gamma_{cct}^{-1}[J(G)] = \begin{cases} 3, & p = 5 \\ 2, & p > 5 \end{cases}$

(x). For a Friendship graph $F_p, p \geq 3$, $\gamma_{cct}^{-1}[J(G)] = 2$

(xi). For Petersen graph, G =(10,15), $\gamma_{cct}^{-1}[J(G)] = 2.$

3. Bounds of $\gamma_{cct}^{-1}[J(G)]$:

Theorem: 3.1.

The connected cototal domination number of the jump graph $J(G)$ of G is

$$\gamma_{cct}^{-1}[J(G)] \geq 2.$$

Theorem: 3.2.

If the inverse connected cototal dominating set exist for the jump graph $J(G)$ of G, then, $\gamma_{cct}^{-1}[J(G)] \geq 2$.

Proof:

Let D be the connected cototal dominating set of the jump graph of G, then $|D| = \gamma_{cct}[J(G)] \geq 2$. If there exist a connected cototal dominating set D' in $[V[J(G)] - D]$ then, $|D'| \geq |D|$. Thus, $\gamma_{cct}^{-1}[J(G)] \geq 2$.

Theorem:3.3.

Let G (p,q) be any connected graph, then the inverse connected cototal dominating set exist for the jump graph $J(G)$ only if $q \geq 6$.

Proof:

Let D be the connected cototal dominating set of $J(G)$, then by theorem 3.1, $|D| = \gamma_{cct}[J(G)] \geq 2$ and $[V[J(G)] - D]$ is a non-empty without isolated vertex. If $[V[J(G)] - D]$, contain a connected cototal dominating set D', then D' is called the inverse connected cototal dominating set of $J(G)$ and by theorem 3.2, $|D'| = \gamma_{cct}^{-1}[J(G)] \geq 2$. Thus, the existence of inverse connected cototal dominating set D' implies that $[V[J(G)] - D']$ is a non-empty set containing no isolates. This means $|V[J(G)] - D'| \geq 2$.

Theorem :3.4.

The inverse connected cototal dominating set does not exist for the jump graph of a connected graph G, if G contains an edge with $\deg(e_i) = q - 2$, i= 1to q.

Proof:

Let there exist an edge e_i in the simple connected G, such that $\deg(e_i) = q - 2$. Then in the jump graph the vertex v_i' corresponding to e_i will be a pendent vertex. But every pendent vertex is a member of the connected cototal dominating set along with its support vertex. Thus there exists no inverse connected dominating set in $J(G)$.

Observation:3.5.

The inverse connected cototal dominating set does not exist for all the jump graphs of G.

Theorem:3.6.

If the inverse connected cototal dominating set exist, then,

$$\gamma_{cct}[J(G)] \leq \gamma_{cct}^{-1}[J(G)]$$

Proof:

Let D' be the inverse connected cototal dominating set of $J(G)$ then D' is also the connected cototal dominating set.

Theorem:3.7.

Let $J(G)$ be the jump graph of G with the inverse connected cototal dominating set, then,

$$\gamma_{cct}[J(G)]+\gamma_{cct}^{-1}[J(G)] \leq q.$$ Bound is sharp for W_5.

Proof:

We have considered G to be a simple connected graph, hence, $|E(G)| = q$. Then in $J(G)$, $|V[J(G)]| = |E(G)| = q$. By Ore[1], the theorem follows.

For the Wheel graph on 5 vertices, equality holds.

Relation between Inverse connected cototal domination of $J(G)$ with other graph theoretic Parameters:

Theorem: 3.8.

Let G be a connected graph and $J(G)$ be its jump graph, then
$\gamma^{-1}[J(G)] \leq \gamma_{ct}^{-1}[J(G)] \leq \gamma_{cct}^{-1}[J(G)]$. Bound is sharp for P_p, C_p.

Proof:

Every inverse connected cototal dominating set of $J(G)$ is the inverse cototal dominating set of $J(G)$and also it is the inverse dominating set of $J(G)$.

Theorem:3.9.

For the jump graph of a connected graph G, $\gamma_c^{-1}[J(G)] \leq \gamma_{cct}^{-1}[J(G)]$.

Theorem:3.10. (Inverse Domination Chain)

Let G be a connected graph and $J(G)$ be its jump graph, then

$$\gamma^{-1}[J(G)] \leq \gamma_c^{-1}[J(G)] \leq \gamma_{ct}^{-1}[J(G)] \leq \gamma_{cct}^{-1}[J(G)].$$

Theorem:3.11.

Let $\beta_1(G)$ denote the edge independence number of a connected graph G, then

$$\gamma_{cct}^{-1}[J(G)] \leq \beta_1(G).$$

Proof:

Let E= $(e_1, e_2, \ldots\ldots e_q)$ be the edge set of G(p,q), q ≥ 6. Let D be the connected cototal domination number of $J(G)$ and let S= $(e_1, e_2, \ldots\ldots e_n)$ denote the maximum edge independent set of G with respect to D. Then in $J(G)$ the vertex set of the corresponding edges of the set S form a connected induced sub graph which is also a dominating set of $J(G)$. Hence, by the choice of q and S , it is apparent that $\{V[J(G)] - D'\}$ is non –empty and connected.

Theorem:3.12.[4]

For the jump graph J(G) with inverse connected cototal dominating set , $\gamma_{cct}^{-1}[J(G)] \leq q - \Delta'(G)$. Equality holds for $G \cong K_{2,p}, p \geq 4$.

Theorem: 3.13[5]

If $G \cong T$, and diam(T) not less than 4, then, $\gamma_{cct}^{-1}[J(G)] = 2$

Proof:

Consider a tree T having diameter greater than 3, otherwise, the jump graph of T will have isolated vertices. Let D be the connected cototal dominating set . Thus in T, with respect to connected cototal dominating set there exists vertices v_j and v_k with maximum distance between them. In $J(T)$, the vertices corresponding to the edges e_j and e_k adjacent to v_j and v_k form the minimum connected cototal dominating set of $J(T)$ with respect to connected cototal dominating set.

4. Disjoint Connected Cototal Domination number of the Jump Graph of a graph:

Definition:4.1.[4]

Let D_1 and D_2 be two disjoint connected cototal dominating sets of $J(G)$ of G. Then the minimum cardinality of the union of two disjoint minimal connected cototal dominating set of $J(G)$is called the disjoint connected cototal domination number, denoted by, $\gamma_{cct}\gamma_{cct}[J(G)]$.

(i.e) $\gamma_{cct}\gamma_{cct}[J(G)] = mini\ \{|D_1| + |D_2|\}$.

The two disjoint connected cototal dominating set whose union has the cardinality $\gamma_{cct}\gamma_{cct}[J(G)]$ is called $\gamma_{cct}\gamma_{cct}[J(G)]$ – pair.

Theorem:4.2.

Exact values of $\boldsymbol{\gamma_{cct}\gamma_{cct}[J(G)]}$ – for some standard graphs:

1. $\gamma_{cct}\gamma_{cct}[J(P_p)] = 4, \quad p \geq 6$
2. $\gamma_{cct}\ \gamma_{cct}[J(C_p)] = 4, \quad p \geq 6$
3. $\gamma_{cct}\ \gamma_{cct}[J(K_p)] = 6, \quad p \geq 6$
4. $\gamma_{cct}\gamma_{cct}[J(G)] = \begin{cases} 8, & for \quad G \cong K_{m,n},\ m = 2, n \geq 4 \\ 6, & for\ G \cong K_{m,n},\ m, n \geq 3 \end{cases}$
5. $\gamma_{cct}\gamma_{cct}[J(W_p)] = \begin{cases} 8, & p = 5 \\ 6, & p = 6 \\ 4 & p \geq 7 \end{cases}$
6. $\gamma_{cct}\ \gamma_{cct}[J(G)] = 4,\ G \cong P_p \circ K_1.\ p \geq 4$
7. $\gamma_{cct}\gamma_{cct}[J(G)] = 4,\ G \cong C_p \circ K_1$
8. For petersen graph, $\gamma_{cct}\gamma_{cct}[J(G)] = 4$.

Theorem:4.3.

Let $J(G)$ be the jump graph of a graph G, then
$\gamma_{cct}\gamma_{cct}[J(G)] \leq q$. Equality holds for $K_{2,4}$ and W_5.

Proof:

Let $D_1\ and\ D_2$ be two disjoint connected cototal dominating set of $J(G)$. Since $|V[J(G)]| = |E(G)| = q$, the theorem follows.

Exact value(Theorem 4.2) proves the equality.

Theorem: 4.4.

Let G be a connected graph, then, every disjoint connected cototal dominating set of $J(G)$is the disjoint total dominating set of $J(G)$.

(i.e) $\gamma_{cct}\gamma_{cct}[J(G)] = \gamma_t\gamma_t[J(G)]$, where γ_t is the total dominating set of G.

Proof:

Let $D_1\ and\ D_2$ be two disjoint dominating set of J(G), with $< D_1 >$ and $< D_2 >$ are connected. If $< V[J(G)] - D_1 >$ and $< V[J(G)] - D_2 >$ contain no isolated vertices then $D_1\ and\ D_2$ are the connected cototal dominating set of $J(G)$. Thus the two disjoint dominating set $D_1\ and\ D_2$ are the total dominating set of J(G).

Theorem: 4.5.

For any connected graph G, $2\,\gamma_{cct}[J(G)] \leq \gamma_{cct}\gamma_{cct}[J(G)]$.

Equality holds for the standard graphs given in Theorem 4.2.

Proof:

For a connected graph G, if its jump graph contains two disjoint connected cototal dominating sets $D_1\ and\ D_2$ then, $|D_1| \leq |D_2|$, since both the sets are minimum cardinality sets.

Equality holds for the graphs given in theorem 4.2

Theorem: 4.6.

For the jump graph $J(G)$ of the graph G, with inverse connected cototal dominating set,

$$2\,\gamma_{cct}[J(G)] \leq \gamma_{cct}[J(G)] + \gamma_{cct}^{-1}[J(G)]$$

Equality holds for $P_p, C_p, K_p (p \geq 6)$.

Theorem: 4.7.[4]

For the connected graph G(p,q), $\gamma\gamma[J(G)] \leq \gamma_{cct}\gamma_{cct}[J(G)]$.

Bounds are sharp for $P_p, C_p\ and\ K_p(\ p \geq 6)$.

Theorem: 4.8.

Let $J(G)$ be the jump graph of G, then, $\gamma_{ct}\gamma_{ct}[J(G)] \leq \gamma_{cct}\gamma_{cct}[J(G)]$.

Theorem: 4.9.

Let $G \cong T$, then for $J(T)$ with $diam(T) > 3$, we have

$$\gamma\gamma[J(T)] = \gamma_{ct}\gamma_{ct}[J(T)] = \gamma_{cct}\gamma_{cct}[J(T)].$$

Proof:

For a tree graph T, let $J(T)$ has $\gamma_{cct}^{-1}[J(T)]$ −set. This implies we have two disjoint connected cototal dominating set in $J(T)$. Moreover, for a tree T with $diam(T) > 3$, there exists two disjoint pair of vertices in $J(T)$ corresponding to two disjoint pair of edges (e_1, e_{q-1}) and (e_2, e_q) which form the minimal dominating set of $J(T)$. Also $< V[J(T)] - D_1 >$ and $< V[J(T)] - D_2 >$ has no isolated vertices. Thus $D_1 \, and \, D_2$ are the two disjoint cototal dominating set of $J(T)$. By the choice of T, $< D_1 >$ and $< D_2 >$ is connected. Thus D_1 and D_2 are the disjoint connected cototal dominating set.

Definition: 4.10.[4]

The jump graph $J(G)$ of a connected graph G is $\gamma_{cct}\gamma_{cct}$ −minimum if $\gamma_{cct}\gamma_{cct}[J(G)] = 2\gamma_{cct}[J(G)]$.

Definition:4.11.[4]

The jump graph $J(G)$ of a connected graph G is $\gamma_{cct}\gamma_{cct}[J(G)]$ −maximum if $\gamma_{cct}\gamma_{cct}[J(G)] = q$.

Definition:4.12.[4]

The jump graph $J(G)$ of a connected graph G is $\gamma_{cct}\gamma_{cct}[J(G)]$ −strong if $\gamma_{cct}\gamma_{cct}[J(G)] = 2\,\gamma_{cct}[J(G)] = q$.

Example:4.13.

(i). For p≥6, all the standard graphs given in theorem. 4.2, are $\gamma_{cct}\gamma_{cct}$ −minimum.

(ii). The graphs, $K_{2,4}$ and W_5 are $\gamma_{cct}\gamma_{cct}[J(G)]$ −maximum

(iii). When G is either $K_{2,4}$ or W_5 ,then G is $\gamma_{cct}\gamma_{cct}[J(G)]$ − strong.

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