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Dynamical Influence of Heat and Mass Transfer on Unsteady Visco-Elastic Fluid on Blood Flow through an Artery: Effects of Chemical Reaction

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Abstract- The heat and mass transfer phenomena of blood always play a key role to the understanding and development of arterial diseases in general and the heat flow together with the transport of macromolecules with dissolved gases through the arterial wall influence more on the growth and development of atherogenetic process. An analysis is made on the unsteady MHD flow of heat and mass transfer in visco-elastic fluid on blood flow through an artery with the effect of chemical reaction. The non-linear unsteady flow phenomenon is governed by the Navier-Stokes equations while those of heat and mass transfer are controlled by the heat conduction and the convection diffusion equations respectively. All these equations together with the appropriate boundary condition describing the present biomechanical problem are solved analytically by using Bessel function. The effects of various parameters on velocity, temperature and concentration are presented graphically and discussed.
Keywords: Visco-elastic fluid, MHD flow, Chemical Reaction, Heat and Mass transfer, Bessel function.

1. INTRODUCTION

Heat is continuously generated in the human body by metabolic processes and exchanged with the environment and among internal organs by conduction, convection, evaporation and radiation. Transport of heat by the circulatory system makes heat transfer in the body through blood flow which has two main mechanisms inside a tissue is through conduction, meaning that the gradient in temperature within the tissue itself drives the blood flow, and through convection of thermal energy by the perfusing blood. The mass transport refers to the movement of atherogenic molecules that is blood borne components, such as oxygen and low density lipoprotenis from flowing blood into the arterial walls. Mass transfer occurs in many processes, such as absorption, evaporation, drying, precipitations, membrane filtration. There are many important technological problems that concern the flow of chemically-reacting fluid mixtures. Many biological fluid systems are examples of such mixtures. For example, blood is a complex mixture of plasma, proteins, cells and a variety of other chemicals which is modelled usually in a homogenized sense as a single constituent fluid.

The study of Heat and Mass transfer on blood flow has become quite interesting to many researchers for clinical or experimental point of view and they have analyzed the Heat and mass transfer of Visco-elastic fluid with chemical reaction in blood flow characteristics. Victor et al.[1] analysed steady state heat transfer to blood flowing in a tube. Barozzi et al.[2]calculated heat transfer in the entrance region considering the rhelogical properties of the blood stream and a cell free peripheral plasma layer at the vessel wall. Imeda et al .[3] investigated the modelling of the non linear Pulsatile blood flow in arteries. Misra et al.[4] have been derived effect of thermal radiation on MHD flow of blood and heat transfer in a permeable capillary in stretching motion.Craciunescu et al.[5] developed The pulsatile blood flow effects on temperature in regid vessels.Wany[6] have obtained heat transfer to blood flow in a small tube. Chato[8] discussed heat transfer to blood cells.Lagendijk [7] developed the influence of blood Flow in large vessels on the temperature distribution in hyperthermia. Olajuwon et al.[11] have considersed effets of thermal radiation and hall current on Heat and Mass Transfer of unsteady MHD flow of viscoelastic micropolar fluid through a porous medium. Devika et al.[9] have obtained MHD oscillatory flow of a viscoelastic fluid in a porous channel with chemical reaction . Patil P et.al[12] have considered the effects of chemical reaction on the free convective flow of a polar fluid through a porous medium in the presence of internal heat generation Mohamed et al.[13] discussed influence of chemical reaction and thermal radiation and heat and mass transfer in MHD micropolar flow over a vertical moving porous plate in a porous medium with heat generation.

The object of this research is to investigate heat and mass transfer on unsteady visco-elastic fluid on blood flow through an artery with chemical reaction. In this research the governing equation of the problem contain a system of partial differential equation which are transformed by non-dimensional system of non-linear partial differential equation's. The obtained non-linear partial differential equation are solved analytically by Bessel function, the results of this research are discussed for the different values of the well-known dimensionless parameters and are shown graphically.

NOMENCLATURE

u Axial velocity
r Radius of artery
t Time
T Fluid temperature
C Concentration
C_p Specific heat at constant pressure
B_0 Magnetic flux density
k_r Chemical reaction
C_w Concentration at wall
C_o Initial concentration
T_w Temperature at the wall
T_0 Initial temperature
P Pressure
k Thermal conductivity
K Porous medium permeability coefficient
N Radiation parameter
D The coefficient of mass diffusivity
q Radiative heat flux
Q Metabolic heat flux
S_c Schmidt number
D_a Darcy number
R_e Reynolds number
P_e Pecklet number
J Chemical reaction parameter
G_r Grashof number
G_c Modified Grashof number
E Heat source parameter

Greek symbols

ρ Fluid density
γ Kinematic viscosity coefficient
θ Fluid temperature
β_T Coefficient of thermal expansion
β_C Coefficient of mass expansion
σ_e Conductivity of the fluid

2. MATHEMATICAL FORMULATION

Let us consider an unsteady flow of a viscous incompressible and electrically conducting Visco-elastic fluid (blood) in arteries with porous medium, subjected to a constant transverse magnetic field B_0 in the presence of thermal and concentration effects on velocity. The governing equations of motion of the circular tube of the artery under the Boussinesq incompressible fluid mode are given by

Momentum equation

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+\gamma_1\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right]+\gamma_2\frac{\partial}{\partial t}\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right]-\frac{\gamma_1}{k}u-\frac{\sigma_e B_0^2 u}{\rho}+g\beta_T\left(T-T_0\right)+g\beta_C\left(C-C_0\right) \quad (1)$$

Energy equation

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right]-\frac{1}{\rho C_p}\frac{\partial q}{\partial r}+\frac{Q}{\rho C_p}\left(T-T_0\right) \quad (2)$$

Concentration equation

$$\frac{\partial C}{\partial t}=D\left[\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right]-K_r\left(C-C_0\right) \quad (3)$$

We assume that the blood flow has high viscous with relatively low density and radiative heat flux is

$$\frac{\partial q}{\partial r}=4\alpha^2\left(T-T_0\right)$$

The boundary conditions for velocity, temperature and concentration fields are given as follows

$$\left.\begin{array}{l} \textit{Initial condition} \\ u=0, T=T_0, C=C_0 \; at\, t=0 \\ \textit{Symmetric condition} \\ \dfrac{\partial u}{\partial r}=0, \dfrac{\partial T}{\partial r}=0, \dfrac{\partial C}{\partial r}=0 \; at\, r=0 \\ \textit{Slip condition} \\ u=\lambda \dfrac{\partial u}{\partial r}, T=T_w, \mathrm{C}=C_w \; at\, r=R_0 \end{array}\right\} \tag{4}$$

Let us introducing the non-dimensional variables as follows

$$\left.\begin{array}{l} z^*=\dfrac{z}{a}, r^*=\dfrac{r}{R_0}, u^*=\dfrac{u}{U}, t^*=\dfrac{tU}{a} \\ C^*=\dfrac{C-C_0}{C_w-C_0}, \theta^*=\dfrac{T-T_0}{T_w-T_0}, P^*=\dfrac{aP}{\rho\gamma_1 U}, D_a=\dfrac{k}{a^2} \\ H^2=\dfrac{a^2\sigma_e B_0^2}{\rho\gamma_1}, R_e=\dfrac{Ua}{\gamma_1}, \gamma=\dfrac{\gamma_2 U}{\gamma_1 a} \\ G_r=\dfrac{g\beta_T\left(T_w-T_0\right)a^2}{\gamma_1 U}, G_c=\dfrac{g\beta_T\left(C_w-C_0\right)a^2}{\gamma_1 U} \\ S_c=\dfrac{D}{aU}, P_e=\dfrac{Ua\rho C_p}{k}, N^2=\dfrac{4\alpha^2 a^2}{k}, S^2=\dfrac{1}{Da} \\ E=\dfrac{Qa^2}{k}, J=\dfrac{K_r a}{U} \end{array}\right\} \tag{5}$$

Using above non-dimensional quantities in equation (1), (2) and (3) and eliminating $*$, we get

$$\mathrm{R}_e \frac{\partial u}{\partial t}=-\frac{\partial P}{\partial z}+\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)+\gamma\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right)-S^2 u-H^2 u+G_r\theta+G_c C \tag{6}$$

$$P_e\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}-N^2\theta+E\theta \tag{7}$$

$$\frac{\partial C}{\partial t}=S_c\left[\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}\right]-JC \tag{8}$$

The non-dimensional boundary conditions and eliminating * from equation (4) are

$$\left.\begin{array}{l} \textit{Initial conditions} \\ u=0, \theta=0, C=0 \; at\, t=0 \\ \textit{Symmetric conditions} \\ \dfrac{\partial u}{\partial r}=0, \dfrac{\partial\theta}{\partial r}=0, \dfrac{\partial C}{\partial r}=0 \; at\, r=0 \\ \textit{Slip conditions} \\ u=\lambda\dfrac{\partial u}{\partial r}, \theta=1, C=1 \; at\, r=1 \end{array}\right\} \tag{9}$$

3. Method of solution

Let us consider the solution of velocity, temperature and concentration are

$$u(r,t)=u(r)e^{\text{int}},\theta(r,t)=\theta(r)e^{\text{int}},C(r,t)=C(r)e^{\text{int}} \tag{10}$$

By using equations (8) and (10) becomes

The solution of the equations (8) as

$$\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}-k_1^2 C=0 \quad \text{where } k_1^2=\frac{in+J}{S_c} \tag{11}$$

By using Bessel function, we get $C=AJ_0\left(ik_1 r\right)$ (12)

Using boundary condition (4) in the equation (12), we get $C(r)=\frac{1}{J_0\left(ik_1\right)}J_0\left(ik_1 r\right)$ (13)

The solution of the concentration is $C(r,t)=\frac{J_0\left(ik_1 r\right)}{J_0\left(ik_1\right)}e^{\text{int}}$ (14)

By using equations (7) and (10) becomes

The solution of the equation (7) as

$$\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}-k_2^2\theta=0, \text{ where } k_2^2=\sqrt{N^2-E+inP_e} \tag{15}$$

By using Bessel function, we get the solution is $\theta=AJ_0\left(ik_2 r\right)$ (16)

Using boundary condition (4) in the equation (16), we get

$$\theta(r)=\frac{1}{J_0\left(ik_2\right)}J_0\left(ik_2 r\right) \tag{17}$$

The solution of the fluid temperature is $\theta(r,t)=\frac{J_0\left(ik_2 r\right)}{J_0\left(ik_2\right)}e^{\text{int}}$ (18)

By using equations (6) and (10) becomes

The solution of the equation (6) as

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-k_3^2 u=-\frac{1}{1+\gamma\, in}\left[P+Gr\theta+GcC\right] \quad \text{where } k_3=\sqrt{\frac{S^2+H^2+inR_e}{1+\gamma\, in}} \tag{19}$$

By using Bessel function, we get the complementary function $C_1 J_0\left(ik_3 r\right)+C_2 y_0\left(ik_3 r\right)$ (20)

By using method of variation parameter, we get particular integral.

Wronskin= $w=\begin{vmatrix} J_0\left(ik_3 r\right) & Y_0\left(ik_3 r\right) \\ -J_1\left(ik_3 r\right)ik_3 & -Y_1\left(ik_3 r\right)\left(ik_3\right) \end{vmatrix}=\frac{2}{\pi r}$

$$A_1=\frac{\pi}{2\left(1+in\gamma\right)}\int r\, y_0\left(ik_3 r\right)\left[P+G_r\theta+G_c C\right]dr$$

$$B_1=\frac{-\pi}{2\left(1+in\gamma\right)}\int r\, J_0\left(ik_3 r\right)\left[P+G_r\theta+G_c C\right]dr$$

The solution of the velocity is

$$u=C_1 J_0\left(ik_3 r\right)+C_2\left(ik_3 r\right)+A_1 J_0\left(ik_3 r\right)+B_1 Y_0\left(ik_3 r\right) \tag{21}$$

Using boundary condition equations (9) and (21), we get $C_2=0$ and

$$C_1 = \frac{1}{J_0\left(ik_3\right)+\lambda ik_3 J_1\left(ik_3\right)}\left[-A_1\left(1\right)\left[J_0\left(ik_3\right)+ik_3 J_1\left(ik_3\right)\right]-B_1\left(1\right)\left[J_0\left(ik_3\right)-ik_3 Y_1\left(ik_3\right)\right]\right] \quad (22)$$

The solution of the velocity distribution is

$$u\left(r,t\right)=\left\{\frac{J_0\left(ik_3 r\right)}{J_0\left(ik_3\right)+\lambda ik_3 J_1\left(ik_3\right)}\begin{bmatrix}-A_1\left(1\right)\left[J_0\left(ik_3\right)+ik_3 J_1\left(ik_3\right)\right]\\ -B_1\left(1\right)\left[J_0\left(ik_3\right)-ik_3 Y_1\left(ik_3\right)\right]\end{bmatrix}+A_1 J_0\left(ik_3 r\right)+B_1 Y_0\left(ik_3 r\right)\right\}e^{\text{int}} \quad (23)$$

4. RESULTS AND DISCUSSION

The objective of the present study is to understand and bring out the effects of heat and mass transfer of viscoelastic fluid on blood flow through an artery**.** Figure-1 depict the concentration of blood flow decreases when the chemical parameter value increases.Fig.2. illustrates that the concentration of the blood increases as the increasing of schmidt number values. From Figure 3 it is noted that the temperature of blood flow increases as the increasing values of heat source parameter. Figure 4 shows that The temperature of blood flow increases as the increasing values of radiation parameter. Figure 5 shows that the temperature of blood flow increases when the increasing values of pecklet number parameter.Fig.6 presents that the velocity of the blood increases as Hartmann number values increases. Fig.7, illustrates that The velocity of the blood increases as chemical parameter increases. Fig. 8 it is noted that The velocity of the blood decreases as the modified Grashof Number increases. Fig. 9 depict that The velocity of the blood increases as Grashof Number increases. Fig. 10 shows the velocity of the blood decreases as Pecklet Number increases.

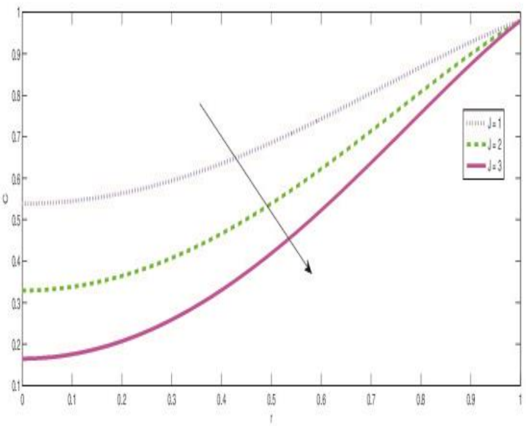

Fig.1 Concentration of the blood versus Radius of artery with different values of chemical parameter J with n=2, t=0.4,a=1, D=2,K=2

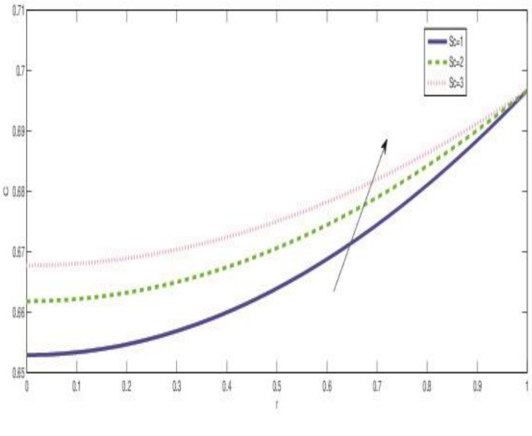

Fig.2. Concentration of the blood versus Radius of artery with different values of Schmidt number S_c with n=2,t=0.4, B_O=0.5,ρ=1,σ=0.5,a=1, γ=2,λ=0.5,D=2, K=2.

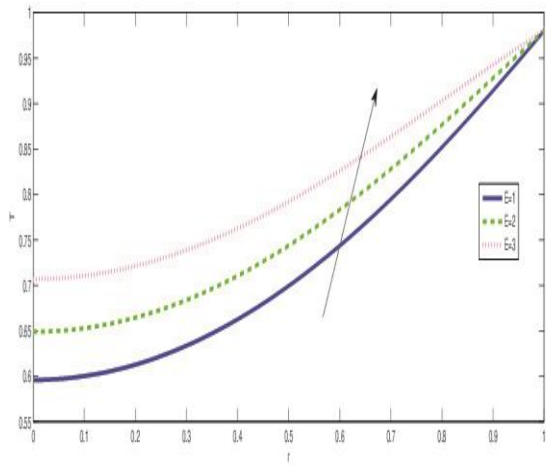

Fig.3. Temperature of the blood versus Radius of artery with different values of Heat source parameter E with n=2, t=0.4, E=2,k=2,α=2, C_p=2;D=2,

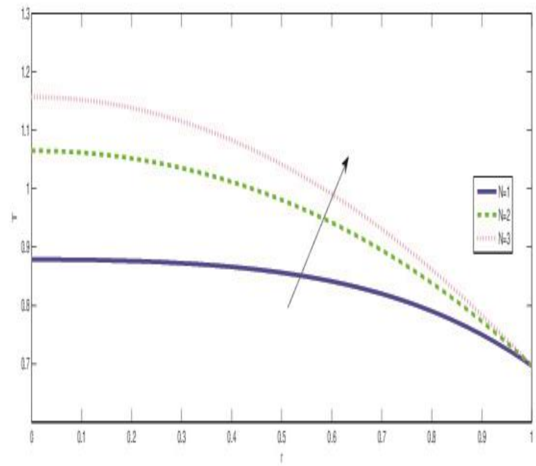

Fig.4. Temperature of the blood versus Radius of artery with different values of Radiation parameter N with n=2, t=0.4, E =2,k=2,α=2, C_p=2.

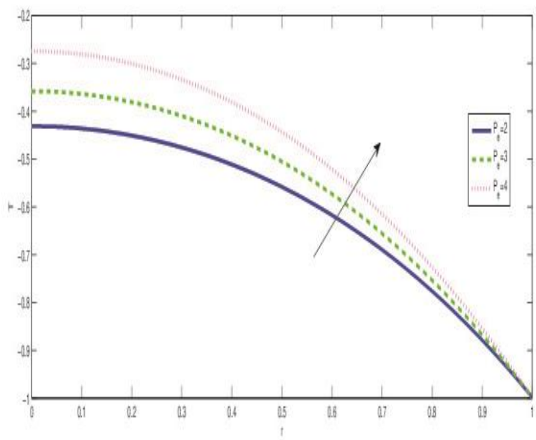

Fig.5. Temperature of the blood versus Radius of artery with Pecklet number P_e with n=2, t=0.4 ,E=2,k=2,α=2, C_p=2.

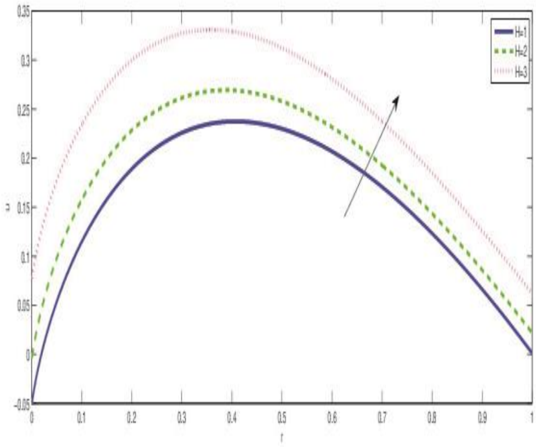

Fig.6. Velocity of the blood versus Radius of artery with different values of Hartmann number H with $n=2, t=0.4, B_O=0.5, \rho=1, \sigma=0.5, a=1, \gamma=2, \lambda=0.5, R_e=1, E=2, k=2, K=2, \alpha=2, C_p=2; D=2, G_r=2, G_c=3$

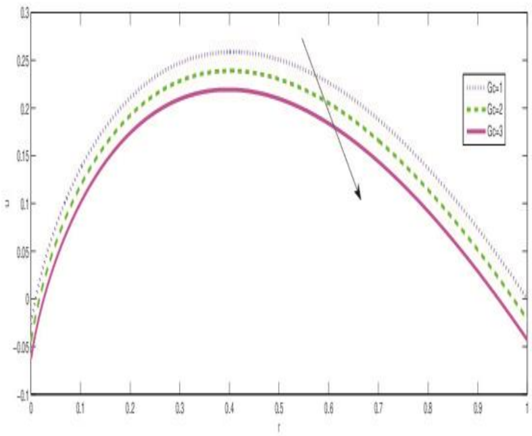

Fig.7. Velocity of the blood versus Radius of artery with different values of modified Grashof number G_c with $n=2$, $t=0.4$, $B_O=0.5, \rho=1, \sigma=0.5, a=1, \gamma=2, \lambda=0.5, R_e=1, E=2, k=2, K=2, \alpha=2, C_p=2; D=2, G_r=2$.

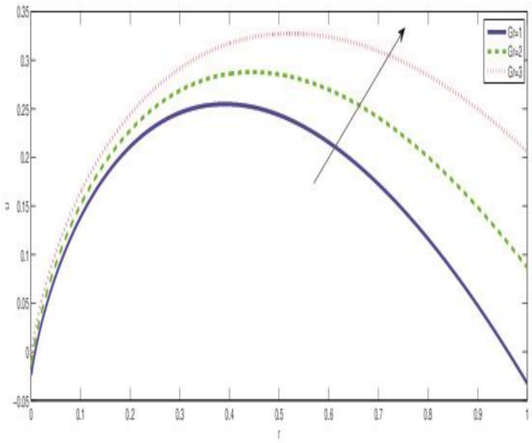

Fig.8. Velocity of the blood versus Radius of artery with different values of Grahof number G_r with $n=2$, $t=0.4, B_O=0.5$, $\rho=1, \sigma=0.5, a=1, \gamma=2, \lambda=0.5, R_e=1, E=2, k=2, K=2, \alpha=2, C_p=2; D=2, G_r=2, G_c=3$

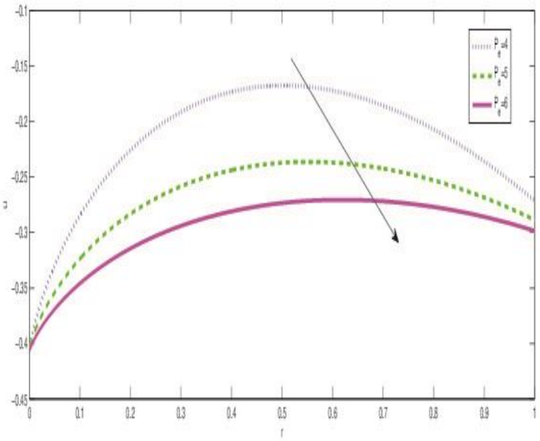

Fig.9. Velocity of the blood versus Radius of artery with different values of Peclet number P_e with n=2, t=0.4,B_O=0.5, ρ=1,σ=0.5,a=1, γ=2,λ=0.5,R_e=1 , E=2, k=2,K=2,α=2, C_p=2;D=2,G_r=2,G_c=3.

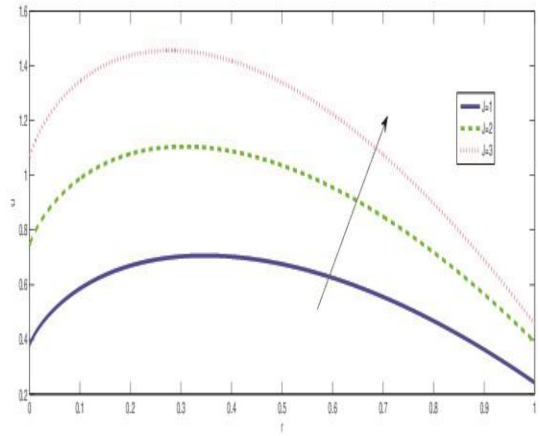

Fig.10. Velocity of the blood versus Radius of artery with different values of chemical parameter J with n=2, ρ=1, σ=0.5,a=1, γ=2,λ=0.5,R_e=1 ,E=2, k=2,K=2,α=2, C_p=2;D=2,G_r=2,G_c=3.

5.CONCLUSION

The purpose of finding the concentration,temperature and velocity distribution and relavent computational work has been performed for various parameters encountered in the present analysis. The main results of the present investigation are as follows

- By increasing J, the concentraction of the blood flow decreased.
- By increasing S_c , the concentraction of the blood flow increased.
- By increasing P_e,E,N, the temperature profile on the blood flow increased.
- By increasing H,J,G_r, the velocity profile of the blood flow increases.
- By increasing G_c,P_e, the velocity distribution of the blood flow decreases.

It is concluded that the heat and mass transfer to the velocity field of the blood stream. The concentration and temperature of the fluid increases, then the velocity of the blood flow decreases. The heat and mass transport highly convection because of the low diffusion coefficients of the principal constituents governing transportation of the blood. Blood is maintained in a delicate balance by a variety of chemical reactions, some that aid its coagulation and others its dissolution.

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