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Sequences of Diophatine Triples for k-Jacobsthal and k-Jacobsthal Lucas With Property $D(-2)^{n-r}$ And $D(-2)^{n-r}\ (k^2+8)$

G.Srividhya[1] and T.Ragunathan[2]
1. Assistant professor, Department of Mathematics, Government Arts College, Trichy-22, Tamilnadu, India. e-mail : vkm292011@hotmail.com
2. Assistant professor, Department of Mathematics, PGP College of Arts and Science, Namakkal. Tamilnadu, India. e-mail : math_ragu@rediffmail.com

Abstract- This paper concerns with Sequences of Diophantine triples (a,b,c) with $D(-2)^{n-r}$ for k - Jacobsthal and also Sequences of Diophantine triples (a,b,c) with $D(-2)^{n-r}\ (k^2+8)$ for k - Jacobsthal Lucas.

Keywords: Diophantine triple, k -Jacobsthal, k - Jacobsthal Lucas.

1. INTRODUCTION

Let n be a given nonzero integer,A set of m positive integers $\{a_1, a_2, a_3 \ldots\ldots a_m\}$ is said to have the property $D(n), n\epsilon z-\{0\}$
$if\ a_i a_j + n$, a perfect square for all $1 \le i \le j \le m$ and such a set is called a Diophantine m-tuples with property $D(n)$. Many mathematician considered the construction of different formulations of Diophantine triples with property $D(n)$.
In Sequence I we find Diophantine triple for k-Jacobsthal numbers with the property $(-2)^{n-r}$.

In sequence II we find Diophantine triple k- Jacobsthal Lucas with property $(-2)^{n-r}\ (k^2+8)$.

2. METHOD OF ANALYSIS

Sequence 1

An attempt is made to form a sequence of Diophantine triples $(a,b,c),(b,c,d),(c,d,e)$….. with the property $D(-2)^{n-r}$ k- Jacobsthal

Case 1

Let $a = j_{k,n-r}$ and $b = j_{k,n+r}$
Let c be any non – zero integer.

Consider

$$ac + (-2)^{n-r} = p^2$$

Which yields

$$j_{k,n-r}\, c + (-2)^{n-r} = p^2 \quad (1)$$

as well

$$\mathrm{b}c + (-2)^{n-r} = q^2$$

gives

$$j_{k,n+r} c + (-2)^{n-r} = q^2 \quad (2)$$

by some algebra,

$$j_{k,n+r} p^2 - j_{k,n-r}\, q^2 = (-2)^{n-r}(j_{k,n+r} - j_{k,n-r}) \quad (3)$$

with the linear transformations

$$p = X + j_{k,n-r} T$$
$$q = X + j_{k,n+r} T$$

and T = 1 we have

$$X = j_{k,n} \quad \text{and} \quad p = j_{k,n} + j_{k,n-r} T$$

From (1)

$$c = j_{k,n-r} + 2j_{k,n} + j_{k,n+r}$$

hence (a, b, c) is the Diophatine triple with the property $D(-2)^{n-r}$.

Case II

Let $b = j_{k,n+r}$ and $c = j_{k,n-r} + 2j_{k,n} + j_{k,n+r}$
Let d be any non – zero integer.

Consider

$$bd + (-2)^{n-r} = \beta^2$$

Which yields

$$(j_{k,n+r})d + (-2)^{n-r} = \beta^2 \tag{4}$$

as well

$$cd + (-2)^{n-r} = \gamma^2$$

gives

$$(j_{k,n-r} + 2j_{k,n} + j_{k,n+r})d + (-2)^{n-r} = \gamma^2 \tag{5}$$

by some algebra,

$$c\beta^2 - b\gamma^2 = (c-b)(-2)^{n-r} \tag{6}$$

with the linear transformations

$$\beta = X + bT$$
$$\gamma = X + cT$$

and T = 1 we have

$X = j_{k,n+r} + j_{k,n}$ and $\beta = 2j_{k,n+r} + j_{k,n}$

From (4)

$$d = j_{k,n-r} + 4j_{k,n} + 4j_{k,n+r}$$

hence (b, c, d) is the Diophatine triple with the property $D(-2)^{n-r}$

Case III

Let $c = j_{k,n-r} + 2j_{k,n} + j_{k,n+r}$ and $d = j_{k,n-r} + 4j_{k,n} + 4j_{k,n+r}$

Let e be any non – zero integer.

Consider

$$ce + (-2)^{n-r} = \delta^2$$

Which yields

$$(j_{k,n-r} + 2j_{k,n} + j_{k,n+r})e + (-2)^{n-r} = \delta^2 \tag{7}$$

as well

$$de + (-2)^{n-r} = \theta^2$$

gives

$$(j_{k,n-r} + 4j_{k,n} + 4j_{k,n+r})e + (-2)^{n-r} = \theta^2 \tag{8}$$

by some algebra,

$$d\delta^2 - c\theta^2 = (d-c)(-2)^{n-r} \tag{9}$$

with the linear transformations

$$\delta = X + cT$$
$$\theta = X + dT$$

and T = 1 we have

$X = 2j_{k,n+r} + 3j_{k,n} + j_{k,n-r}$ and $\delta = 2(j_{k,n+r} + 2j_{k,n} + j_{k,n-r}) + j_{k,n} + j_{k,n+r}$

From (7)

$$e = 4(j_{k,n-r} + 2j_{k,n} + j_{k,n+r}) + 4(j_{k,n} + j_{k,n+r}) + j_{k,n+r}$$

hence (c, d, e) is the Diophatine triple with the property $D(-2)^{n-r}$.

From all the above cases $(a,b,c),(b,c,d),(c,d,e)$….. will form a sequence of Diophantine triples.

n	r	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}$.
3	2	$1,k^4+6k^2+4,k^4+6k^2+4+1+2((k^2+2)$	$k^4+6k^2+4,k^4+6k^2+4+1+2((k^2+2),4(k^4+6k^2+4)+1+4((k^2+2),$	$k^4+6k^2+4+1+2((k^2+2),4(k^4+6k^2+4)+1+4((k^2+2),4(\quad k^4+6k^2+4+1+2((k^2+2)+k^4+6k^2+4+4(k^4+6k^2+4+(k^2+2)$	$D(-2)$
	3	$0,k^5+8k^3+12k,k^5+8k^3+12k+2(k^2+2)$	$k^5+8k^3+12k,k^5+8k^3+12k+2(k^2+2),4(k^5+8k^3+12k)+4(k^2+2)$	$k^5+8k^3+12k+2(k^2+2),\ 4(k^5+8k^3+12k)+4(k^2+2),\quad 4(k^5+8k^3+12k+2(k^2+2))+k^5+8k^3+12k+4(k^5+8k^3+12k+k^2+2)$	$D(1)$
4	2	$k,k^5+8k^3+12k,k^5+8k^3+12k+k+2(k^3+4k))$	$k^5+8k^3+12k,k^5+8k^3+12k+k+2(k^4+4k),4(k^5+8k^3+12k)+k+4(k^3+4k))$	$k^5+8k^3+12k+k+2(k^4+4k),4(k^5+8k^3+12k)+k+4(k^3+4k),4(k^5+8k^3+12k+k+2(k^4+4k)+(k^5+8k^3+12k+4(k^5+8k^3+12k+(k^3+4k))$	$D(4)$
	3	$1,k^6+10k^4+24k^2+8,(k^6+10k^4+24k^2+8+1+2(k^3+4k))$	$k^6+10k^4+24k^2+8,(k^6+10k^4+24k^2+8+1+2(k^3+4k)),4(k^6+10k^4+24k^2+8+1+4(k^3+4k))$	$(k^6+10k^4+24k^2+8+1+2(k^3+4k)),4(k^6+10k^4+24k^2+8+1+4(k^3+4k)),4(k^6+10k^4+24k^2+8+1+2(k^3+4k)+(k^6+10k^4+24k^2+8)+4(k^6+10k^4+24k^2+8+(k^3+4k))$	$D(-2)$
5	2	$k^2+2,k^6+10k^4+24k^2+8,(k^6+10k^4+24k^2+8+k^2+2+2(k^4+6k^2+4)$	$(k^6+10k^4+24k^2+8),(k^6+10k^4+24k^2+8+k^2+2+2(k^4+6k^2+4),4(k^6+10k^4+24k^2+8)+k^2+2+4(k^4+6k^2+4)$	$(k^6+10k^4+24k^2+8+k^2+2+2(k^4+6k^2+4),4(k^6+10k^4+24k^2+8)+k^2+2+4(k^4+6k^2+4),4(k^6+10k^4+24k^2+8+k^2+2+2(k^4+6k^2+4)+(k^6+10k^4+24k^2+8)+4((k^6+10k^4+24k^2+8+(k^4+6k^2+4))$	$D(-8)$
	3	$k,(k^7+12k^5+40k^3+32k),((k^7+12k^5+40k^3+32k+k+2(k^4+6k^2+4))$	$(k^7+12k^5+40k^3+32k),((k^7+12k^5+40k^3+32k+k+2(k^4+6k^2+4)),(4(k^7+12k^5+40k^3+32k)+k+4(k^4+6k^2+4))$	$((k^7+12k^5+40k^3+32k+k+2(k^4+6k^2+4)),(4(k^7+12k^5+40k^3+32k)+k+4(k^4+6k^2+4)),(4(k^7+12k^5+40k^3+32k)+k+2(k^4+6k^2+4)+(k^7+12k^5+40k^3+32k)+4((k^7+12k^5+40k^3+32k+k^4+6k^2+4))$	$D(4)$

When $k\epsilon N,\ r\geq 2$Some numerical examples are tabulated

n	r	k	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}$.
3	2	1	(1,11,18)	(11,18,57)	(18,57,139)	$D(-2)$
	3	1	(0,21,27)	(21,27,96)	(27,96,225)	$D(1)$

n	r	k	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}$
4	2	1	(1,21,32)	(21,32,105)	(32,105,243)	$D(4)$
	3	1	(1,43,54)	(43,54,193)	(54,193,451)	$D(-2)$

n	r	k	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}$..
5	2	1	(3,43,69)	(43,69,219)	(69,219,531)	$D(8)$
	3	1	(1,85,108)	(85,108,385)	(108,385,901)	$D(4)$

Sequence II

Simillarly the same procedure maynbe applied for k-Jacobsthal Lucas and can be verified as $a = \hat{j}_{k,n-r}$, $b = \hat{j}_{k,n+r}$, $c = (\hat{j}_{k,n-r} + 2\hat{j}_{k,n} + \hat{j}_{k,n+r}$, $d = \hat{j}_{k,n-r} + 4\hat{j}_{k,n} + 4\hat{j}_{k,n+r}$ and
$e = 4(\hat{j}_{k,n-r} + 2\hat{j}_{k,n} + \hat{j}_{k,n+r}) + 4(\hat{j}_{k,n} + \hat{j}_{k,n+r}) + \hat{j}_{k,n+r}$

From all the above $(a,b,c), (b,c,d), (c,d,e)$….. will form a sequence of Diophantine triples.

n	r	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}$.(k^2+8)
3	2	$k, k^5+10k^3+20k, (k+2(k^3+6k)+k^5+10k^3+20k)$	$k^5+10k^3+20k, (k+2(k^3+6k)+k^5+10k^3+20k), (k+4(k^3+6k)+4(k^5+10k^3+20k))$	$(k+2(k^3+6k)+k^5+10k^3+20k), (k+4(k^3+6k)+4(k^5+10k^3+20k)$,$(4(k+2(k^3+6k)+k^5+10k^3+20k)+4(k^3+6k+k^5+10k^3+20k)+(k^5+10k^3+20k))$	$D(-2)(k^2+8)$
	3	$2, k^6+12k^4+36k^2+16, (2+2(k^3+6k)+k^6+12k^4+36k^2+16$	$k^6+12k^4+36k^2+16, (2+2(k^3+6k)+k^6+12k^4+36k^2+16)$, $(2+4(k^3+6k)+4(k^6+12k^4+36k^2+16))$	$(2+2(k^3+6k)+k^6+12k^4+36k^2+16), (2+4(k^3+6k)+4(k^6+12k^4+36k^2+16)$,$(4(2+2(k^3+6k)+k^6+12k^4+36k^2+16)+4(k^3+6k+k^6+12k^4+36k^2+16)+k^6+12k^4+36k^2+16))$	$D(1)(k^2+8)$
4	2	$k^2+4, k^6+12k^4+36k^2+16, (k^2+4+2(k^4+8k^2+8+k^6+12k^4+36k^2+16)$	$k^6+12k^4+36k^2+16, (k^2+4+2(k^4+8k^2+8+k^6+12k^4+36k^2+16)$, $(k^2+4+4(k^4+8k^2+8+4(k^6+12k^4+36k^2+16))$	$(k^2+4+2(k^4+8k^2+8+k^6+12k^4+36k^2+16)$, $(k^2+4+4(k^4+8k^2+8+4(k^6+12k^4+36k^2+16)$,$(4((k^2+4+2(k^4+8k^2+8+k^6+12k^4+36k^2+16)+4(k^4+8k^2+8+k^6+12k^4+36k^2+16)+k^6+12k^4+36k^2+16))$	$D(4)(k^2+8)$
	3	$k, k^7+14k^5+56k^3+56k, (k+2(k^4+8k^2+8)+k^7+14k^5+56k^3+56k)$	$k^7+14k^5+56k^3+56k, (k+2(k^4+8k^2+8)+k^7+14k^5+56k^3+56k)$, $(k+4(k^4+8k^2+8)+4(k^7+14k^5+56k^3+56k))$	$(k+2(k^4+8k^2+8)+k^7+14k^5+56k^3+56k)$, $\left(k+4(k^4+8k^2+8)+4(k^7+14k^5+56k^3+56k)\right), (4(k+2(k^4+8k^2+8)+k^7+14k^5+56k^3+56k)+4(k^4+8k^2+8+k^7+14k^5+56k^3+56k)+k^7+14k^5+56k^3+56k))$	$D(-2)(k^2+8)$
5	2	$k^3+6k, k^7+14k^5+56k^3+56k, (k^3+6k+2(k^5+10k^3+20k)+k^7+14k^5+56k^3+56k)$	$k^7+14k^5+56k^3+56k, (k^3+6k+2(k^5+10k^3+20k)+k^7+14k^5+56k^3+56k)$, $(k^3+6k+4(k^5+10k^3+20k)+4(k^7+14k^5+56k^3+56k))$	$(k^3+6k+2(k^5+10k^3+20k)+k^7+14k^5+56k^3+56k)$, $(k^3+6k+4(k^5+10k^3+20k)+4(k^7+14k^5+56k^3+56k)), (4(k^3+6k+2(k^5+10k^3+20k)+k^7+14k^5+56k^3+56k)+4(k^5+10k^3+20k)+k^7+14k^5+56k^3+56k)$+ $k^7+14k^5+56k^3+56k))$	$D(-8)(k^2+8)$

	3	$k^2+4, k^8+16k^6+80k^4+128k^2+32, (k^2+4+2(k^5+10k^3+20k)+k^8+16k^6+80k^4+128k^2+32)$	$k^8+16k^6+80k^4+128k^2+32, (k^2+4+2(k^5+10k^3+20k)+k^8+16k^6+80k^4+128k^2), (k^2+4+4(k^5+10k^3+20k)+4(k^8+16k^6+80k^4+128k^2+32))$	$(k^2+4+2(k^5+10k^3+20k)+k^8+16k^6+80k^4+128k^2), (k^2+4+4(k^5+10k^3+20k)+4(k^8+16k^6+80k^4+128k^2+32)), (4(k^2+4+2(k^5+10k^3+20k)+k^8+16k^6+80k^4+128k^2+32)+4(k^5+10k^3+20k+k^8+16k^6+80k^4+128k^2+32)+k^8+16k^6+80k^4+128k^2+32$	$D(4)$ (k^2+8)

When $k\epsilon N,\ r\geq 2$Some numerical examples are tabulated

n	r	k	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}.(k^2+8)$
3	2	1	(1,31,46)	(31,46,153)	(46,153,367)	$D(-2).9$
	3	1	(2,65,81)	(65,81,290)	(81,290,677)	$D(1).9$

n	r	k	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}.(k^2+8)$
4	2	1	(5,65,104)	(65,104,333)	(104,333,809)	$D(4).9$
	3	1	(1,127,162)	(127,162,577)	(162,577,1351)	$D(-2).9$

n	r	k	(a,b,c)	(b,c,d)	(c,d,e)	$D(-2)^{n-r}.(k^2+8)$
5	2	1	(7,127,196)	(127,196,639)	(196,639,1543)	$D(-8).9$
	3	1	(5,257,324)	(257,324,1157)	(324,1157,2705)	$D(4).9$

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