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CENTROIDAL MEAN LABELING OF GRAPHS

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ABSTRACT. In this paper the Centroidal mean labeling of cycle containing graphs such as Triangular Ladder TL_n, cycle C_n, Polygonal chain G_{mn}, Square graph P_n^2, $L_n \bigodot K_{1,2}$, Ladder L_n are found.

1. Introduction and preliminaries

Abundant literature exists as of today concerning the structure of graphs admitting a variety of function assigning real numbers to their elements so that given conditions are satisfied. Here we are interested the study of vertex functions $f : V(G) \to A, A \subseteq N$ for which the induced edge function $f^* : E(G) \to N$ is defined as $f^*(uv) = \lceil \frac{2[f(u)^2+f(u)f(v)+f(v)^2]}{3(f(u)+f(v))} \rceil$ or $f^*(uv) = \lfloor \frac{2[f(u)^2+f(u)f(v)+f(v)^2]}{3(f(u)+f(v))} \rfloor$ for every $uv \in E(G)$ are all distinct.

As we know that the notion of mean labeling was introduced in a paper by Somasundaram and Ponraj [7]. A graph G with p vertices and q edges is called a mean graph if there is an injective function f from the vertices of G to $0, 1, 2, 3, 4 \ldots, q$ such that when each edge uv is labeled with $\frac{(f(u)+f(v))}{2}$, if $f(u) + f(v)$ is even, and $\frac{(f(u)+f(v)+1)}{2}$ if $f(u) + f(v)$ is odd, then the resulting edge labels are distinct.

We introduce Centroidal mean labeling of some standard graphs.

Graph: A graph G is a pair (V, E), where V is a nonempty set and E is a set of unordered pairs of elements taken from the set V. A graph which does not contain loops and multiple edges is a simple graph, a finite number of vertices and edges in a graph is a finite graph and undirected with p vertices and q edges. The cardinality of vertex set V of a graph is the order and the cardinality of edge set E is called the size of the graph G. The graph $G - e$ is obtained from G by deleting an edge e.

Sum of the Graphs: The sum G_1+G_2 of two graphs G_1 and G_2 has vertex set $V(G_1)\cup V(G_2)$ and edge set $E(G_1+G_2) = E(G_1)\cup E(G_2)\cup u \in V(G_1)$ *and* $v \in V(G_2)$.

Union of Graph: The union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

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Square Graph: The square graph denoted by G^2 of the graph G has $V(G^2)$ with u, v adjacent in G^2 Whenever $d(u, v) \leq 2$ in the graph G. The detailed survey on graph labeling are found in [2].

For other terminology and notations refer [3].

According to Beineke and Hegde graph labeling serves as a frontier between number theory and structure of graphs. The definitions which are useful to develop this paper are given below.

2. Centroidal mean labeling of a graph

In this section the Centroidal mean labeling of graphs containing cycles such as Triangular Ladder TL_n, cycle C_n, Polygonal chain G_{mn}, Square graph P_n^2 , $L_n \bigodot K_{1,2}$, Ladder L_n are discussed using the following definition.

Definition 2.1. A Graph G with n vertices and m edges is called a Centroidal mean graph if the function $f : V(G) \to A \subseteq N$ to label the vertices $x \in V(G)$ with distinct labels $f(x)$, and each edge $e = x_i x_j$ is labeled with $f^*(x_i x_j) = \left\lceil \frac{2[(f(x_1))^2 + f(x_1)f(x_2) + (f(x_2))^2]}{3(f(x_1)+f(x_2))} \right\rceil$ or $f^*(x_i x_j) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(x_j) + (f(x_j))^2]}{3(f(x_i)+f(x_j))} \right\rfloor$ for every $x_i, x_j \in V(G)$ and $x_i \neq x_j$ are all distinct.

3. Main results

Theorem 3.1. *A Triangular ladder TL_n is Centroidal mean graph.*

Proof. Consider a Triangular ladder TL_n, with n vertices $x_1, x_2, x_3, \ldots, x_n$ as one path and $y_1, y_2, y_3, \ldots, y_n$ as other path.
The function $f : V(TL_n) \to \{1, 2, 3, \ldots, (4n-2)\}$ is defined by $f(x_i) = 4i - 1, \quad 1 \leq i \leq n$ and $f(y_i) = 4i - 3, \quad 1 \leq i \leq n$ such that the induced function $f^* : E(G) \to N$ given by

$$f^*(x_i x_j) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(x_j) + (f(x_j))^2]}{3(f(x_i) + f(x_j))} \right\rfloor$$

for every $x_i, x_j \in V(G)$

The edges $\{x_i, x_{i+1}\}$ are labeled by

$$f^*(x_i x_{i+1}) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(x_{i+1}) + (f(x_{i+1}))^2]}{3(f(x_i) + f(x_{i+1}))} \right\rfloor = 4i+1; \quad 1 \leq i \leq (n-1)$$

The edges $\{y_i, y_{i+1}\}$ are labeled by

$$f^*(y_i y_{i+1}) = \left\lfloor \frac{2[(f(y_i))^2 + f(y_i)f(y_{i+1}) + (f(y_{i+1}))^2]}{3(f(y_i) + f(y_{i+1}))} \right\rfloor = 4i-1; \quad 1 \leq i \leq (n-1)$$

The edges $\{x_i, y_i\}$ are labeled by

$$f^*(x_i y_i) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(y_i) + (f(y_i))^2]}{3(f(x_i) + f(y_i))} \right\rfloor = 4i-2; \quad 1 \leq i \leq n$$

The edges $\{x_i, y_{i+1}\}$ are labeled by

$$f^*(x_i y_{i+1}) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(y_{i+1}) + (f(y_{i+1}))^2]}{3(f(x_i) + f(y_{i+1}))} \right\rfloor = 4i; \quad 1 \leq i \leq (n-1)$$

are all distinct. Hence the Triangular ladder TL_n is Centroidal mean graph. □

Example 3.2. Consider the Triangular Ladder TL_n of $n = 8$. The following figure-1 shows the Centroidal mean labeling of a graph.

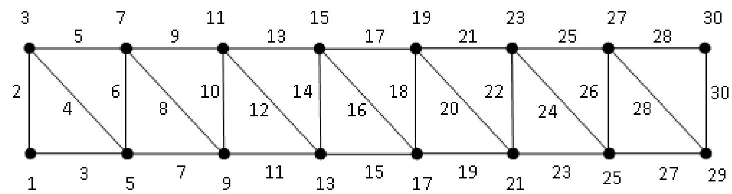

FIGURE 1. Triangular Ladder TL_8

Theorem 3.3. *Any Cycle C_n, $n \geq 3$, is a Centroidal mean graph.*

Proof. Consider a Cycle C_n of length n with vertices $x_1, x_2, x_3, \ldots, x_n$. Define a function $f : V(C_n) \to N$ by $f(x_i) = i$, $1 \leq i \leq n$, Here f is an increasing function on $V(C_n)$, so f^* is also an increasing function on $E(C_n) - \{x_n x_1\}$, for every edge in $E(C_n) - \{x_n x_1\}$ we assign the label

$$f^*(x_i x_j) = \left\lceil \frac{2[(f(x_i))^2 + f(x_i)f(x_j) + (f(x_j))^2]}{3(f(x_i) + f(x_j))} \right\rceil$$

where $x_i, x_j \in V(C_n)$ and $f^*(x_n x_1) = 1$. Such that $f^*(e_i) \neq f^*(e_j)$ for $i \neq j$ and f^* is injective therefore f is a Centroidal mean labeling on C_n.
Hence a Cycle C_n is a Centroidal mean graph. □

Example 3.4. Consider the Cycle of length 4 & 5. The labeling is as shown in figure-2

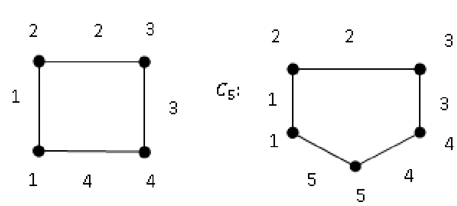

FIGURE 2. Cycle C_4 & C_5

Theorem 3.5. *The Polygonal chain $G_{m,n}$ is a Centroidal mean graph.*

Proof. Consider a Polygonal chain $G_{m,n}$, in which $x_1, x_2, x_4, x_6, \dots x_{n-4}, x_{n-2}, x_{n+1}x_{n-1}, x_{n-3}, x_{n-5}, \dots, x_7, x_3, x_1$ be the first cycle. The second cycle is connected to the first cycle at the vertex x_{n+1}.
Let $x_{n+1}, x_{n+2}, x_{n+4}, \dots, x_{2n+1}, x_{2n-1}, x_{2n-3}, \dots, x_{n+7}, x_{n+5}, x_{n+3}, x_{n+1}$ be the second cycle. The third cycle is connected to the cycle at the vertex x_{2n+1}. The third cycle be $x_{2n+1}, x_{2n+2}, x_{2n+4}, \dots, x_{3n+1}, x_{3n-1}, x_{n-3}, \dots, x_{2n+5}, x_{2n+3}, x_{2n+1}$.
In general r^{th} cycle is connected to the $(r-1)^{th}$ cycle at the vertex x_{rn+1}. Let the r^{th} cycle be $x_{rn+1}, x_{rn+2}, x_{rn+4}, \dots, x_{(r+1)n-4}, x_{(r+1)n-2}, \dots, x_{rn+5}, x_{rn+3}, x_{rn+1}$ and the graph has m cycles.
Define a function $f : V(G_(m,n)) \to \{1, 2, 3, \dots, (q+1)\}$ by $f(x_i) = i$ for $1 \le i \le mn+1$ and $f(x_n) = mn+1$.
Then the label of the edges are done by

$$f^*(x_i x_j) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(x_j) + (f(x_j))^2]}{3(f(x_i) + f(x_j))} \right\rfloor; \quad i \neq j,$$

are all distinct. Hence $G_{m,n}$ is a Centroidal mean graph. □

Example 3.6. The following figure-3 shows the Centroidal mean labeling of polygonal chain G_{mn}.

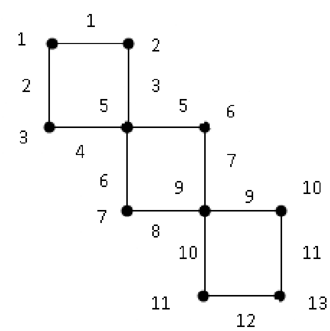

FIGURE 3. Polygonal chain G_{mn}.

Theorem 3.7. *The square graph P_n^2 is a Centroidal mean graph.*

Proof. If a path P_n of n vertices $x_1, x_2, x_3, \dots x_n$ then P_n^2 has n vertices and $(2n-3)$ edges is a graph obtained by joining the vertices whenever $d(u,v) \le 2$.
Define $f : V(P_n^2) \to N$ by
$f(x_i) = 2i - 1, \quad 1 \le i \le (n-1)$ and $f(x_n) = 2n-2$.
The edges are labeled by

$$f^*(x_i x_j) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(x_j) + (f(x_j))^2]}{3(f(x_i) + f(x_j))} \right\rfloor$$

for all $\{x_i x_j\} = e \in E(P_n^2)$ such that $f^*(e_i) \neq f^*(e_j)$ for $i \neq j$ therefore f^* is injective.
Hence P_n^2 is a Centroidal mean graph. □

Example 3.8. Consider a path P_8 with 8 vertices and P_8^2 is a graph obtained by joining the vertices when ever $d(u, v) \leq 2$. The labeling is as shown in the figure-4.

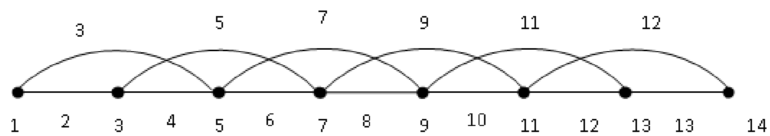

FIGURE 4. square graph P_8^2

Theorem 3.9. *A Graph $L_n \bigodot K_{1,2}$ is a Centroidal mean graph*

Proof. Consider a graph L_n be a Ladder with $u_1, u_2, u_3, \ldots, u_n$ and $v_1, v_2, v_3, \ldots, v_n$ are end vertices of a ladder. Let w_i and x_i be the pendent vertex adjacent to u_i ; y_i and z_i be the pendant vertex adjacent to v_i.

Define a function, $f : V(L_n \bigodot K_{1,2}) \to \{1, 2, \ldots, p+q\}$ by,
$f(u_i) = 13i - 2, \quad 1 \leq i \leq n; \qquad f(v_i) = 13i - 7, \quad 1 \leq i \leq n$
$f(w_i) = 13i - 8, \quad 1 \leq i \leq n; \qquad f(x_i) = 13i - 4, \quad 1 \leq i \leq n$
$f(y_i) = 13i - 12, \quad 1 \leq i \leq n; \qquad f(z_i) = 13i - 11, \quad 1 \leq i \leq n$
Edges are labeled as follows,
The edges $\{u_iu_{i+1}\}$ are labeled by

$$f^*(u_iu_{i+1}) = \left\lfloor \frac{2[(f(u_i))^2 + f(u_i)f(u_{i+1}) + (f(u_{i+1}))^2]}{3(f(u_i) + f(u_{i+1}))} \right\rfloor = 13i+4; \quad 1 \leq i \leq n-1$$

The edges $\{v_iv_{i+1}\}$ are labeled by

$$f^*(v_iv_{i+1}) = \left\lfloor \frac{2[(f(v_i))^2 + f(v_i)f(v_{i+1}) + (f(v_{i+1}))^2]}{3(f(v_i) + f(v_{i+1}))} \right\rfloor = 13i - 1; \quad 1 \leq i \leq n-1$$

The edges $\{u_iw_i\}$ are labeled by

$$f^*(u_iw_i) = \left\lfloor \frac{2[(f(u_i))^2 + f(u_i)f(w_i) + (f(w_i))^2]}{3(f(u_i) + f(w_i))} \right\rfloor = 13i - 6; \quad 1 \leq i \leq n$$

The edges $\{u_ix_i\}$ are labeled by

$$f^*(u_ix_i) = \left\lfloor \frac{2[(f(u_i))^2 + f(u_i)f(x_i) + (f(x_i))^2]}{3(f(u_i) + f(x_i))} \right\rfloor = 13i - 3; \quad 1 \leq i \leq n$$

The edges $\{u_iv_i\}$ are labeled by

$$f^*(u_iv_i) = \left\lceil \frac{2[(f(u_i))^2 + f(u_1)f(v_i) + (f(v_i))^2]}{3(f(u_i) + f(v_i))} \right\rceil = 13i - 5; \quad 1 \leq i \leq n$$

The edges $\{v_iy_i\}$ are labeled by

$$f^*(v_iy_i) = \left\lfloor \frac{2[(f(v_i))^2 + f(v_i)f(y_i) + (f(y_i))^2]}{3(f(v_i) + f(y_i))} \right\rfloor = 13i - 10; \quad 1 \leq i \leq n$$

The edges $\{v_iz_i\}$ are labeled by

$$f^*(v_iz_i) = \left\lceil \frac{2[(f(v_i))^2 + f(v_i)f(z_i) + (f(z_i))^2]}{3(f(v_i) + f(z_i))} \right\rceil = 13i - 9; \quad 1 \leq i \leq n$$

are all distinct.
Hence $L_n \bigodot K_{1,2}$ is a Centroidal mean graph. □

Example 3.10. Consider a Graph $L_4 \bigodot K_{1,2}$. The labeling is as shown in the figure-5.

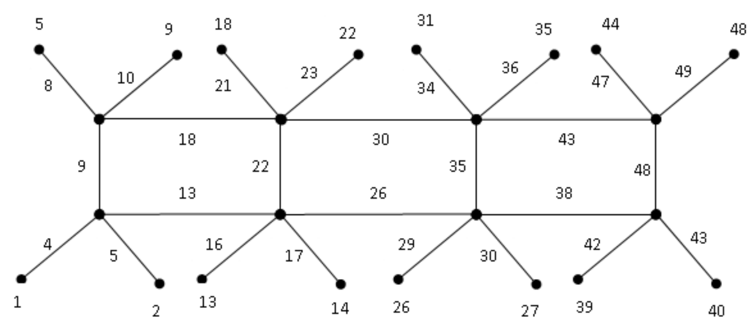

FIGURE 5. Graph $L_4 \bigodot K_{1,2}$

Theorem 3.11. *A Ladder L_n is a Centroidal mean graph.*

Proof. Consider a Ladder L_n. Let $x_1x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ be two paths of length n in the ladder L_n.
Define a function $f : V(TL_n) \to \{1, 2, ..., p + q\}$ by,
$f(x_i) = 6i - 3; \quad 1 \leq i \leq n$
$f(y_i) = 6i - 5; \quad 1 \leq i \leq n$
Edges are labeled by,

$$f^*(x_ix_{i+1}) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(x_{i+1}) + (f(x_{i+1}))^2]}{3(f(x_i) + f(x_{i+1}))} \right\rfloor; \quad 1 \leq i \leq (n-1)$$

,

$$f^*(x_iy_i) = \left\lfloor \frac{2[(f(x_i))^2 + f(x_i)f(y_i) + (f(y_i))^2]}{3(f(x_i) + f(y_i))} \right\rfloor; \quad 1 \leq i \leq n$$

and

$$f^*(y_iy_{i+1}) = \left\lfloor \frac{2[(f(y_i))^2 + f(y_i)f(y_{i+1}) + (f(y_{i+1}))^2]}{3(f(y_i) + f(y_{i+1}))} \right\rfloor; \quad 1 \leq i \leq (n-1)$$

are all distinct.

We observe that f is a Centroidal mean labeling and the ladder L_n is Centroidal mean graphs. □

Example 3.12. Consider a ladder L_5. The labeling is as shown in the figure-6.

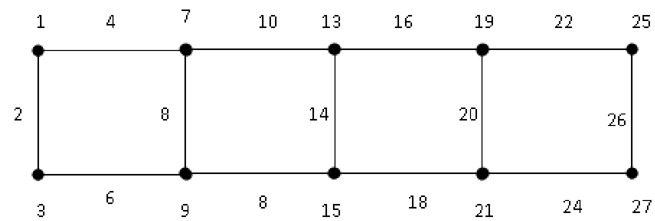

Figure 6. Ladder Graph L_5

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