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Generalized rb Sets for Real and Imaginary Domains

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Abstract. This paper dealt with the Regular b-open sets in topological spaces and we extend the same for the real domain and inverse regular b-open sets for the complex domain.

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1. Introduction

Regular open sets and b-open sets have been introduced and investigated by Steiner [7] and Andrijevic [2], respectively. They generalized closed sets, semi-generalized closed sets, regular generalized closed sets, semi-closed sets, pre-closed sets, α-closed sets. Al-Omari and Noorani [1] investigated the class of generalized b-closed sets and obtained some of its fundamental properties. We introduce the concepts of rb-open sets for identifying the inverse operator in any space (real domain or an imaginary domain).

2. Preliminaries

In this section, we present some basic definitions and fundamental results which will be useful for further discussion.

Definition 2.1. A subset M of a space X is said to be regular closed if $M = Cl(Int(M))$.

Definition 2.2. A subset M of a space X is said to be b-closed if $Int(Cl(M)) \cap Cl(Int(M)) \subset M$.

Definition 2.3. A subset M of a space X is said to be

(1) generalized closed if $Cl(M) \subset U$ whenever $M \subset U$ and U is open in X,
(2) generalized semi-closed if $scl(M) \subset U$ whenever $M \subset U$ and U is open in X,
(3) semi-generalized closed if $scl(M) \subset U$ whenever $M \subset U$ and U is semi-open in X,
(4) regular-generalized closed if $Cl(M) \subset U$ whenever $M \subset U$ and U is regular-open in X,
(5) generalized b-closed if $bcl(M) \subset U$ whenever $M \subset U$ and U is open in X.
(6) f-generalized closed if $cl(M) \subset U$ whenever $M \subset u$ and U is open in X.

The complements of the above mentioned closed sets are their respective open sets. These operators are used in the real domain and the corresponding complements are related to regular inverse operators which are used in the complex domain. The intersection of all regular closed sets of X containing M is called the regular closure. The grneralized inverse, of M is the union of all regular open sets contained in M.

Definition 2.4. The inverse of the generalized rb - inverse open form of the n^{th} kind denoted by f_M^{-1} is defined as, if $f_M v(k) = u(k)$, then

$$v(k) = f_L^{-1}u(k) + c_{(n-1)j}\frac{k_{\ell_{n-1}}^{(n-1)}}{(n-1)!\ell_{n-1}^{n-1}} + c_{(n-2)j}\frac{k_{\ell_{n-2}}^{(n-2)}}{(n-2)!\ell_{n-2}^{n-2}} + \cdots + c_{2j}\frac{k_{\ell_2}^{(2)}}{(2)!\ell_2^2} + c_{1j}\frac{k}{\ell_1} + c_{0j}, \quad (1)$$

where c_{ij}'s are constants. In general

$$f_M^{-m} = f_M^{-1}(f_M^{-(m-1)}).$$

3. Inverse of Generalized rb - closed operator of The n^{th} Kind

In this section, we define the generalized form of rb - inverse open and closed space and present the general formula to find the sums and general partial sums of a given sequence. Also we find the partial sums of the products of n consecutive terms of known coefficients.

Definition 3.1. If $\ell \in (0, \infty)$, then the rb-inverse operator f_ℓ^{-1} is defined as, if

$$f_\ell(v(k)) = u(k),$$

then

$$v(k) = f_\ell^{-1}(u(k)) + c_j, \tag{2}$$

where c_j is a constant, where $j = k - \left[\frac{k}{\ell}\right]\ell$.

Theorem 3.2. *If m, n are positive integers, ℓ is a real coefficient, and $m > n\ell$, then*

(i) $(k-(n-1)\ell)^m - (n-1)(k-(n-2)\ell)^m + \cdots + (-1)^{n-1}k^m$

$$= \frac{1}{n}\sum_{r=1}^{m} S_r^m \ell^{m-r} k_{\underline{M}}^{(r)}. \tag{3}$$

(ii)
$$f_{\underline{L}}^{-1} k_\ell^{(m)} = \frac{k_{\underline{L}}^{(m+2n-1)}}{n(m+1)(m+2)\cdots(m+2n-1)\ell^{2n-1}} + c_{(n-1)j}\left(\frac{k_\ell^{(n-1)}}{(n-1)!\ell^{n-1}}\right)$$
$$+ c_{(n-2)j}\left(\frac{k_\ell^{(n-2)}}{(n-2)!\ell^{n-2}}\right) + \cdots + c_{2j}\left(\frac{k_\ell^{(2)}}{2!\ell^2}\right) + c_{1j}\left(\frac{k_\ell^{(1)}}{\ell}\right) + c_{0j}. \tag{4}$$

(iii)
$$k_\ell^{(m)} = \frac{1}{n}\sum_{r=1}^{m} S_r^m \ell^{m-r} f^{-1}_{f_\ell^{-(n-1)}} k_{\underline{M}}^{(r)}. \tag{5}$$

Theorem 3.3. *If ℓ is a positive real and $j = k - \left[\frac{k}{\ell}\right]\ell$, then*

$$f_\ell^{-1} u(k) = \sum_{r=1}^{\left[\frac{k}{\ell}\right]} u(k - r\ell) + c_j, \tag{6}$$

where c_j is constant.

Theorem 3.4. *If $n \in \mathbb{N}(1)$, $\ell \in (0, \infty)$ and $k \in [0, \infty)$, then*

$$f_{\underline{M}}^{-1} u(k)\Big|_j^k\Big|_{(n-1)\ell+j}^k = f_\ell^{-n} u(k) = \sum_{r=n}^{\left[\frac{k}{\ell}\right]} \frac{(r-1)^{(n-1)}}{(n-1)!} u(k - r\ell). \tag{7}$$

Proof. Since

$$f_\ell\left\{\sum_{r=1}^{\left[\frac{k}{\ell}\right]} u(k-r\ell)\right\} = \sum_{r=1}^{\left[\frac{k}{\ell}\right]+1} u(k+\ell-r\ell) - \sum_{r=1}^{\left[\frac{k}{\ell}\right]} u(k-r\ell) = u(k),$$

by Definition 3.1, we can obtain

$$f_\ell^{-1}u(k)\Big|_j^k = \sum_{r=1}^{\left[\frac{k}{\ell}\right]} u(k-r\ell). \tag{8}$$

Since $f_{\ell,\ell}^{-1} = f_\ell^{-1}(f_\ell^{-1})$, by taking f_ℓ^{-1} on both sides of (8) and again applying (8), we get

$$f_{\ell,\ell}^{-1}u(k)\Big|_j^k\Big|_{\ell+j}^k = \sum_{r=2}^{\left[\frac{k}{\ell}\right]} (r-1)u(k-r\ell).$$

□

The following theorem is the formula for finding the general partial sums of a given sequence.

Theorem 3.5. *Let* $k \in [n\ell_n, \infty)$, $\ell_i \geq \ell_{i-1}$, $u_i(k) = \Delta_{\ell_i}^{-1}(u_{i-1}(k) - u_{i-1}(j_{i-1}))$, *for* $i = 2, 3, \ldots, n$ *and* $u_1(k) = f_{\ell_1}^{-1}u(k)$ *with* $j_i = k - \left[\frac{k}{\ell_i}\right]\ell_i$, *for* $i = 1, 2, \ldots, n$. *Then*

$$\sum_{r_n=1}^{r_n^*}\sum_{r_{n-1}=1}^{r_{n-1}^*}\cdots\sum_{r_1=1}^{r_1^*} u(k - r_n\ell_n - r_{n-1}\ell_{n-1} - \cdots - r_2\ell_2 - r_1\ell_1) = u_n(k) - u_n(j_n)$$

$$-\{U_{1.2\ldots n} + U_{2.3\ldots n} + \cdots + U_{(n-1)n},\}$$

where

$$r_n^* = \left[\frac{k}{\ell_n}\right], r_{n-1}^* = \left[\frac{k-r_n\ell_n}{\ell_{n-1}}\right], \ldots, r_1^* = \left[\frac{k-r_n\ell_n-\cdots-r_2\ell_2}{\ell_1}\right],$$

$$U_{i.(i+1).\cdots(n-1).n} = \sum_{r_n=1}^{r_n^*}\sum_{r_{n-1}=1}^{r_{n-1}^*}\cdots\sum_{r_{i+1}=1}^{r_{i+1}^*} u_i(j_{ir_{i+1}r_{i+2}\cdots r_n})$$

and

$$j_{ir_{i+1}r_{i+2}\cdots r_n} = (k - r_n\ell_n - \cdots - r_{i+1}\ell_{i+1}) - \left[\frac{k - r_n\ell_n - \cdots - r_{i+1}\ell_{i+1}}{\ell_i}\right]\ell_i,$$

for $i = 1, 2, \cdots, (n-1)$.

Proof. From the equation (6) and replacing k by j_1, we obtain

$$z_1(k) = f_{\ell_1}^{-1} u(k)\Big|_{j_1}^{k} = \sum_{r_1=1}^{\left[\frac{k}{\ell_1}\right]} u(k - r_1\ell_1) = u_1(k) - u_1(j_1), \tag{9}$$

where $u_1(k) = f_{\ell_1}^{-1} u(k)$. Taking $f_{\ell_2}^{-1}$ on both sides and then replacing k by j_2, we obtain where $u_1(k) = f_{\ell_1}^{-1} u(k)$. Taking $f_{\ell_2}^{-1}$ on both sides and then replacing k by j_2, we obtain

$$z_2(k) = f_{\ell_2}^{-1} z_1(k)\Big|_{j_2}^{k} = \sum_{r_2=1}^{\left[\frac{k}{\ell_2}\right]} z_1(k - r_2\ell_2) = u_2(k) - u_2(j_2) - U_{12}, \tag{10}$$

where $u_2(k) = f_{\ell_1}^{-1}(u_1(k) - u_1(j_1))$ and $U_{12} = \sum_{r_2=1}^{\left[\frac{k}{\ell_2}\right]} u_1(j_{1r_2})$ with $j_{1r_2} = (k - r_2\ell_2) - [\frac{k-r_2\ell_2}{\ell_1}]\ell_1$. Again applying $f_{\ell_3}^{-1}$ on both sides and then replacing k by j_3, we find

$$z_3(k) = f_{\ell_3}^{-1} z_2(k)\Big|_{j_3}^{k} = \sum_{r_3=1}^{\left[\frac{k}{\ell_3}\right]} z_2(k - r_3\ell_3) = u_3(k) - u_3(j_2) - \{U_{123} + U_{12}\}, \tag{11}$$

where $u_3(k) = f_{\ell_2}^{-1}(u_2(k) - u_2(j_2))$, $U_{123} = \sum_{r_3=1}^{\left[\frac{k}{\ell_3}\right]} \sum_{r_2=1}^{\left[\frac{k-r_3\ell_3}{\ell_2}\right]} u_1(j_{1r_2r_3})$,
$j_{1r_2r_3} = (k - r_3\ell_3 - r_2\ell_2) - \Big[\dfrac{k - r_3\ell_3 - r_2\ell_2}{\ell_1}\Big]\ell_1$, $U_{23} = \sum_{r_3=1}^{\left[\frac{k}{\ell_3}\right]} u_2(j_{2r_3})$ and
$j_{2r_3} = (k - r_3\ell_3) - [\frac{k-r_3\ell_3}{\ell_2}]\ell_2$.
The proof follows from (9), (10), (11) and repeating this process n times. □

The following is the statement of the Theorem 3.5 when $n = 2$.

Corollary 3.6. *Let $\ell_2 \geq \ell_1$, $k \in [2\ell_2, \infty)$ and $u(k) = k$. Then*

$$\sum_{r_2=1}^{\left[\frac{k}{\ell_2}\right]} \sum_{r_1=1}^{\left[\frac{k-r_2\ell_2}{\ell_1}\right]} (k - r_2\ell_2 - r_1\ell_1) = u_2(k) - u_2(j_2) - U_{12}, \tag{12}$$

where

$$u_2(k) = \frac{k_{\ell_2}^{(3)}}{6\ell_1\ell_2} + \frac{(\ell_2 - \ell_1)k_{\ell_2}^{(2)}}{4\ell_1\ell_2} - \frac{(j_1)_{\ell_1}^{(2)} k_{\ell_2}^{(1)}}{2\ell_1\ell_2}$$

is obtained from

$$u_1(k) = \frac{k_{\ell_1}^{(2)}}{2\ell_1} - \frac{(j_1)_{\ell_1}^{(2)}}{2\ell_1},$$

$$U_{12} = \sum_{r_2=1}^{\left[\frac{k}{\ell_2}\right]} u_1(j_{1r_2})$$

and

$$j_{1r_2} = (k - r_2\ell_2) - \left[\frac{k - r_2\ell_2}{\ell_1}\right]\ell_1.$$

Theorem 3.7. *Assume that $f(k) = q(k) - p(k) - p(k-\ell) \geq 0$ for all $k \in [\ell, \infty)$ and $f(k) \neq 0$ for infinitely many k. Then every solution is bounded on $[\ell, \infty)$ if and only if*

$$\sum_{r=1}^{\infty}\sum_{s=1}^{r} \frac{f(s)}{p(r)} < \infty. \tag{13}$$

Proof. Let all solutions of (13) be bounded. For $k_1 \in [\ell, \infty)$, we define a solution of $\alpha^{-\left[\frac{k}{\ell}\right]}u(k)$) by setting

$$u(k_1 + j) = (\ell + j)\alpha^{-\left[\frac{k_1}{\ell}\right]-1}$$

and

$$u(k_1 + j + \ell) = (2\ell + j)\alpha^{-\left[\frac{k_1}{\ell}\right]}$$

for $0 \leq j < \ell$. Thus,

$$f_{\alpha(\ell)}u(k_1 + j + \ell) = \ell\alpha^{-\left[\frac{k_1}{\ell}\right]} > 0$$

Considerr=

$$p(k_1 + j + \ell)f_{\alpha(\ell)}u(k_1 + j + \ell) = \alpha p(k_1 + j)f_{\alpha(\ell)}u(k_1 + j)$$
$$+\alpha f(k_1 + j + \ell)u(k_1 + j + \ell)$$
$$> \alpha f(k_1 + j + \ell)u(k_1 + j + \ell) \geq 0.$$

Since $p(k) > 0, \alpha > 0$ and $f(k) \geq 0$, implies that $f_{\alpha(\ell)}u(k_1 + j + \ell) \geq 0$ which yields $u(k_1 + j + 2\ell) \geq 0$. Thus, by induction $u(k) \geq (\ell + j)\alpha^{-\left[\frac{k}{\ell}\right]-1}$ and $f_{\alpha(\ell)}u(k) > 0$ for all $k \in [k_1, \infty)$. Expanding this, we have

$$u(k_1 + j + n\ell + \ell) = \alpha u(k_1 + j + \ell) + (\alpha - 1)\sum_{r=1}^{n-1} u(k_1 + j + (r+1)\ell)$$

$$+\sum_{r=1}^{n}\frac{\alpha p(k_1+j)f_{\alpha(\ell)}u(k_1+j)}{p(k_1+j+r\ell)}$$

$$+\sum_{r=1}^{n}\frac{1}{p(k_1+j+r\ell)}\sum_{s=1}^{r}\alpha f(k_1+j+s\ell)u(k_1+j+s\ell), \tag{14}$$

it follows that

$$u(k_1+j+n\ell+\ell)\geq(2\ell+j)+\sum_{r=1}^{n}\frac{\alpha p(k_1+j)f_{\alpha(\ell)}u(k_1+j)}{p(k_1+j+r\ell)}$$

$$+\sum_{r=1}^{n}\sum_{s=1}^{r}\frac{\alpha f(k_1+j+s\ell)u(k_1+j+s\ell)}{p(k_1+j+r\ell)}$$

from which it is clear that if $u(k)$ is bounded, then (13) must be satisfied as α is finite.

Conversely, let $u(k)$ be an unbounded solution of (13) so that here exists a $k_2\in[\ell,\infty)$ such that $u(k)>0$ and $f_{\alpha(\ell)}u(k)>0$ for all $k\in[k_2,\infty)$. Then we get

$$f(k)=\frac{f_{\alpha(\ell)}(\frac{1}{\alpha}p(k-\ell)f_{\alpha(\ell)}u(k-\ell))}{u(k)}$$

$$\geq\frac{\frac{1}{\alpha}p(k)f_{\alpha(\ell)}u(k)}{u(k)}-\frac{p(k-\ell)f_{\alpha(\ell)}u(k-\ell)}{\alpha u(k-\ell)},\ k\in[k_2+\ell,\infty)$$

which yields

$$\frac{1}{p(k_2+j)}\sum_{r=1}^{n}\alpha f(k_2+j+r\ell)+\frac{p(k_2+j+n\ell)f_{\alpha(\ell)}u(k_2+j)}{p(k_2+j+n\ell)u(k_2+j)}$$

$$\geq\frac{f_{\alpha(\ell)}u(k_2+j+n\ell)}{u(k_2+j+n\ell)}$$

and hence

$$\sum_{r=1}^{n}\sum_{s=1}^{r}\frac{\alpha f(k_2+j+s\ell)}{p(k_2+j+r\ell)}+\frac{p(k_2+j)f_{\alpha(\ell)}u(k_2+j)}{u(k_2+j)}\sum_{r=1}^{n}\frac{1}{p(k_2+j+r\ell)}$$

$$\geq\sum_{r=1}^{n}\frac{f_{\alpha(\ell)}u(k_2+j+r\ell)}{u(k_2+j+r\ell)}. \tag{15}$$

For $k_2+j+r\ell\leq t\leq k_2+j+r\ell+\ell$, define

$$\rho_\alpha(t)=\ell u(k_2+j+r\ell)+[t-(k_2+j+r\ell)]f_{\alpha(\ell)}\frac{1}{\alpha}u(k_2+j+r\ell). \tag{16}$$

Then $\rho'_\alpha(t) = f_{\alpha(\ell)}\frac{1}{\alpha}u(k_2+j+r\ell)$ and $\rho_\alpha(t) \geq \ell u(k_2+j+r\ell)$. Hence, we have

$$\frac{f_{\alpha(\ell)}u(k_2+j+r\ell)}{\ell u(k_2+j+r\ell)} \geq \frac{\alpha\rho'_\alpha(t)}{\rho_\alpha(t)}. \tag{17}$$

From (16) and integrating (17) from $k_2+j+r\ell$ to $k_2+j+r\ell+\ell$, we get

$$\frac{f_{\alpha(\ell)}u(k_2+j+r\ell)}{u(k_2+j+r\ell)} \geq \alpha[\ln u(k_2+j+r\ell+\ell) - \ln u(k_2+j+r\ell)] \tag{18}$$

which yields

$$\sum_{r=1}^{n}\frac{f_{\alpha(\ell)}u(k_2+j+r\ell)}{u(k_2+j+r\ell)} \geq \alpha\sum_{r=1}^{n}[\ln u(k_2+j+r\ell+\ell) - \ln u(k_2+j+r\ell)]. \tag{19}$$

Hence, for $0 \leq j < \ell$ and $n = [\frac{k-k_2}{\ell}]$, we have

$$\sum_{r=1}^{n}\frac{f_{\alpha(\ell)}u(k_2+j+r\ell)}{u(k_2+j+r\ell)} \geq \alpha[\ln u(k_2+j+n\ell+\ell) - \ln u(k_2+j+\ell)]. \tag{20}$$

Since $f(k) \neq 0$ for infinitely many k, there exists $0 \leq j < \ell$ and $m \geq 1$ such that $k_3 \geq k_2+j+\ell$ and $f(k_3) \neq 0$, which yields

$$\sum_{r=1}^{\infty}\sum_{s=1}^{r}\frac{\alpha f(s)}{p(r)} \geq \sum_{t=k_3}^{\infty}\frac{\alpha f(k_3)}{p(k_2+j+r\ell)}. \tag{21}$$

From our assumptions, (15), (20) and (21), we get a contradiction

$$\alpha[\ln u(k_2+j+n\ell) - \ln u(k_2+j+\ell)] < \infty \ as\ n \to \infty.$$

□

4. Conclusion

The theory incorporated in this paper related to Regular open closed and open sets in the Real and Imaginary domain were expressed in the well defined manner. We can extend the applications of these theoritical concepts into the Circuit Analysis and Numberical Analysis.

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