Odd-Even Graceful Labeling in Duplicate Graphs of certain Path and Star Related Graphs

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Abstract

In this paper, we prove that the extended duplicate graphs of star graph, bistar graph, double star graph, path graph, comb graph and twig graph admit odd-even graceful labeling labeling.

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Key Words: Graph labeling, duplicate graph, odd-even graceful labeling.

1 Introduction

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). In the intervening years various labeling of graphs have been investigated in over 2000 papers [1]]. The concept of duplicate graph was introduced by Sampath kumar and he proved many results on it[2]. E. Sampath kumar introduced the concept of duplicate graph and studied the charactarization of the the duplicate graphs[2]. Odd Even graceful was introduced by R Sridevi, S Navaneethakrishnan, A Nagarajan, K Nagarajan[3]. Thirusangu, Ulaganathan and Selvam, have proved that the duplicate graph of a path graph P_m is Cordial [6]. Thirusangu, Ulaganathan and Vijaya kumar have proved that the duplicate graph of Ladder graph L_m, $m \geq 2$, is cordial , total cordial and prime cordial[5]. Vijaya kumar, Ulaganathan and Thirusangu, proved the existence of 3- Equitable and 3 - Cordial Labeling in Duplicate Graph of Some Graphs[6].

2 MAIN RESULTS

Definition 1. Let $G\left(V, E\right)$ be a graph with p vertices and q edges. A function $f : V \rightarrow \{1, 3, 5, ..., 2q + 1\}$ is called an odd-even graceful labeling, when each edge uv is assigned the label $|f\left(u\right) - f\left(v\right)|$ such that the resulting edge labels are $\{2, 4, 6, ..., 2q\}$. A graph which admits an odd-even graceful labelingis called an odd-evengraceful graph. [3].

2.1 Algorithm - OEGS

V$\leftarrow \{v_1, v_2, ..., v_m, v_1^{'}, v_2^{'}, ..., v_m^{'}\}$.

E$\leftarrow \{e_1, e_2, ..., e_m, e_1^{'}, e_2^{'}, ..., e_{m-1}^{'}\}$.

For $1 \leq k \leq \ m - 1$

$v_k \leftarrow 2k - 1$, $v_k^{'} \leftarrow 2m - 2k - 1$;

Theorem 2. *The extended duplicate graph of the star graph $EDG(S_m)$, $m \geq 2$ admits odd - even graceful labeling.*

Proof. Let $\{v_1, v_2, ..., v_m, v_1^{'}, v_2^{'}, ..., v_m^{'}\}$ and $\{e_1, e_2, ..., e_m, e_1^{'}, e_2^{'}, ..., e_{m-1}^{'}\}$ be the set of vertices and the edges of the $EDG\left(S_m\right)$. Using the algorithm OEGS, the $2m$ vertices are labeled using 1, 3, 5,..., $4m - 1(= 2q + 1)$. Using the induced function $f^{*}(uv) = |f\left(u\right) - f\left(v\right)|$, the $m - 1$ edges namely $e_{m-1}^{'}$, $e_{m-2}^{'}$, $e_{m-3}^{'}$,..., $e_2^{'}$, $e_1^{'}$ receive labels 2, 4, 6,..., $2m - 4$, $2m - 2$ respectively, the edge e_{2m} receives label $2m$ and the $m - 1$ edges namely e_1, e_2, e_3,..., e_{m-2}, e_{m-1} receive labels $2m + 2$, $2m + 4$, $2m + 6$,..., $4m - 4$, $4m - 2(= 2q)$. Thus the $2m - 1$ edges are labeled with 2, 4, 6,...,

$4m-2(=2q)$. Hence, the extended duplicate graph of the star graph $EDG(S_m)$, $m \geq 2$ admits odd-even graceful labeling. □

2.2 Algorithm - OEGBS

V← $\{v_1, v_2, ..., v_{2m+2}, v_1^{'}, v_2^{'}, ..., v_{2m+2}^{'}\}$

E← $\{e_1, e_2, ..., e_{2m+2}, e_1^{'}, e_2^{'}, ..., e_{2m+1}^{'}\}$

Fix $v_1 \leftarrow 1$, $v_1^{'} \leftarrow 4m+5$;

For $1 \leq k \leq m+1$

$v_{k+1}^{'} \leftarrow 8m-2k+9$;

For $1 \leq k \leq m$

$v_{m+k+2}^{'} \leftarrow 6m-2k+7$;

For $1 \leq k \leq 2m+1$

$v_{k+1} \leftarrow 4m-2k+5$;

Theorem 3. *The extended duplicate graph of the bistar graph $EDG\,(BS_m)$, $m \geq 2$ admits odd - even graceful labeling.*

Proof. Let $\{v_1, v_2, ..., v_{2m+2}, v_1^{'}, v_2^{'}, ..., v_{2m+2}^{'}\}$ and $\{e_1, e_2, ..., e_{2m+2}, e_1^{'}, e_2^{'}, ..., e_{2m+1}^{'}\}$ be the set of vertices and the edges of $EDG\,(CB_m)$. Using the algorithm OEGBS, $4m+4$ vertices are labeled using 1, 3, 5,..., $8m+7(=2q+1)$. Using the induced function f^* defined in theorem 2, the $m+1$ edges namely $e_1, e_2, ..., e_m, e_{m+1}$ receive labels $8m+6(=2q)$, $8m+4$, $8m+2$,..., $6m+8$, $6m+6$ respectively, the m edges namely $e_{2m+1}^{'}$, $e_{2m}^{'}$, $e_{2m-1}^{'}$,...,$e_{m+3}^{'}$, $e_{m+2}^{'}$ receive labels $6m+4$, $6m+2$, $6m$,..., $4m+8$, $4m+6$ respectively, the m edges namely e_{m+2}, e_{m+3}, e_{m+4},..., e_{2m}, e_{2m+1} receive labels $4m+2$, $4m$, $4m-2$,..., $2m+6$, $2m+4$ respectively, the m edges namely $e_{m+1}^{'}$, $e_m^{'}$, $e_{m-1}^{'}$,..., $e_2^{'}$, $e_1^{'}$ receive labels $2m+2$, $2m$, $2m-2$,..., 4,2 respectively and the edge e_{2m+2} receives label $4m+4$. Thus the $4m+3$ edges are labeled with 2, 4, 6,..., $8m+6(=2q)$. Hence the extended duplicate graph of the bistar graph $EDG\,(BS_m)$, $m \geq 2$ admits odd - even graceful labeling.

□

2.3 Algorithm - OEGDS

V← $\{v_1, v_2, ..., v_{2m+1}, v_1^{'}, v_2^{'}, ..., v_{2m+1}^{'}\}$

E← $\{e_1, e_2, ..., e_{2m+2}, e_1^{'}, e_2^{'}, ..., e_{2m}^{'}\}$

Fix $v_1 \leftarrow 1$. For $1 \leq k \leq 2m$

$v_{k+1} \leftarrow 6m - 2k + 3;$

For $1 \leq k \leq \ m+1$

$v^{'}_{m-k+2} \leftarrow 8m - 2k + 5;$

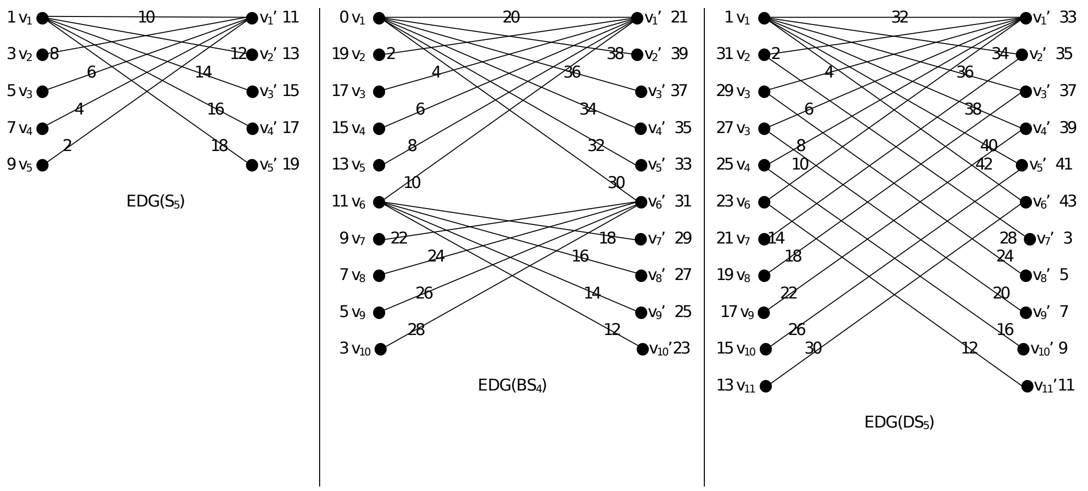
For $1 \leq k \leq \ m+1$

$v^{'}_{m+k+1} \leftarrow 2k + 1;$

Theorem 4. *The extended duplicate graph of the double star graph $EDG\,(DS_m)$, $m \geq 2$ admits odd - even graceful labeling.*

Proof. Let $\{v_1, v_2, ..., v_{2m+1}, v^{'}_1, v^{'}_2, ..., v^{'}_{2m+1}\}$ and $\{e_1, e_2, ..., e_{2m+1}, e^{'}_1, e^{'}_2, ..., e^{'}_{2m}\}$ be the set of vertices and the edges of $EDG\,(DS_m)$. Using the algorithm OEGDS, the $4m+2$ vertices are labeled using 1, 3, 5,..., $8m+3(=\ 2q+1)$. Using the induced function f^* defined in theorem 2, the m edges namely $e^{'}_1$, $e^{'}_2$, $e^{'}_3$,..., $e^{'}_{m-1}$, $e^{'}_m$ receive labels 2, 4, 6,..., $2m-2$, $2m$ respectively, the m edges namely e_{2m}, e_{2m-1}, e_{2m-2},...,e_{m+2}, e_{m+1} receive labels $2m+2$, $2m+6$, $2m+10$,..., $6m-6$, $6m-2$ respectively, the m edges namely $e^{'}_{m+1}$, $e^{'}_{m+2}$, $e^{'}_{m+3}$,..., $e^{'}_{2m-1}$, $e^{'}_{2m}$ receive labels $2m+4$, $2m+8$, $2m+12$,..., $6m-4$, $6m$ respectively, the m edges namely e_1, e_2, e_3,..., e_{m-1}, e_m receive labels $6m+4$, $6m+6$, $6m+8$,..., $8m$,$8m+2$ respectively and the edge e_{2m+1} receives the label$6m+2$. Thus the $4m+1(=\ 2q)$ edges are labeled with 2, 4, 6,..., $8m+2(=\ 2q)$. Hence the extended duplicate graph of the double star graph $EDG\,(DS_m)$, $m \geq 2$ admits odd - even graceful labeling.

Illustration:



captionFigure 1. Odd-Even Graceful labeling in $EDG\,(S_5)\,, EDG\,(BS_4)\,andEDG\,(DS_5)$ □

2.4 Algorithm - OEGP

V$\leftarrow \{v_1, v_2, ..., v_{m+1}, v_1^{'}, v_2^{'}, ..., v_{m+1}^{'}\}$

E$\leftarrow \{e_1, e_2, ..., e_{m+1}, e_1^{'}, e_2^{'}, ..., e_m^{'}\}$

Case (i): when m is odd

For $1 \leq k \leq \dfrac{m+1}{2}$

$v_{2k} \leftarrow 2k-1$, $v_{2k-1} \leftarrow 2m-2k+3$

$v_{2k}^{'} \leftarrow 2m+2k+1$, $v_{2k-1}^{'} \leftarrow 4m-2k+5$

Case (ii): when m is even

For $1 \leq k \leq \dfrac{m}{2}$

$v_{2k} \leftarrow 2k-1$, $v_{2k-1} \leftarrow 2m-2k+3$

$v_{2k}^{'} \leftarrow 2m+2k+1$, $v_{2k+1}^{'} \leftarrow 4m-2k+5$;

Theorem 5. *The extended duplicate graph of the path graph $EDG\,(P_m)$, $m \geq 2$ admits odd - even graceful labeling.*

Proof. Let $\{v_1, v_2, ..., v_{m+1}, v_1^{'}, v_2^{'}, ..., v_{m+1}^{'}\}$ and $\{e_1, e_2, ..., e_{m+1}, e_1^{'}, e_2^{'}, ..., e_m^{'}\}$ be the set of vertices and the edges of the $EDG\,(P_m)$.

Case (i): when m is odd

Using the algorithm OEGP, the $2m+2$ vertices are labeled using 1, 3, 5,..., $4m+3(=2q+1)$. Using the induced function f^* defined in theorem 2, the m edges namely, e_1 $e_2^{'}$, e_3, $e_4^{'}$, e_5, $e_6^{'}$,..., e_{m-2}, $e_{m-1}^{'}$,e_m receive labels 2, 4, 6, 8, 10, 12,..., $2m-4$, $2m-2$, $2m$ respectively, the m edges namely $e_1^{'}$, e_2, $e_3^{'}$, e_4, $e_5^{'}$, e_6,..., $e_{m-2}^{'}$,e_{m-1}, $e_m^{'}$ receive labels $4m+2(=2q)$, $4m$, $4m-2$, $4m-4$, $4m-6$, $4m-8$,..., $2m+8$, $2m+6$, $2m+4$ respectively and the edge e_{m+1} receives the label $2m+2$. Thus the $2m+1(=q)$ edges are labeled with 2, 4, 6,..., $4m+2(=2q)$.

Case (ii): when m is even

Using the algorithm OEGP, the $2m+2$ vertices are labeled using 1, 3, 5,..., $4m+3(=2q+1)$. Using the induced function f^* defined in theorem 2, the m edges namely, e_1 $e_2^{'}$, e_3, $e_4^{'}$, e_5, $e_6^{'}$,..., e_{m-3}, $e_{m-2}^{'}$,e_{m-1},$e_m^{'}$ receive labels 2, 4, 6, 8, 10, 12,..., $2m-6$, $2m-4$, $2m-2$, $2m$ respectively, the m edges namely $e_1^{'}$, e_2, $e_3^{'}$, e_4, $e_5^{'}$, e_6,..., $e_{m-3}^{'}$,e_{m-2}, $e_{m-1}^{'}$, e_m receive labels $4m+2(=2q)$, $4m$, $4m-2$, $4m-4$, $4m-6$, $4m-8$,..., $2m+10$, $2m+8$, $2m+6$, $2m+4$ respectively and the edge e_{m+1} receives the label $2m+2$. Thus the $2m+1(=q)$ edges are labeled with 2, 4, 6,..., $4m+2(=2q)$. Hence the extended duplicate graph of the path graph $EDG\,(P_m)$, $m \geq 2$ admits odd - even graceful labeling. □

2.5 Algorithm - OEGCB

V$\leftarrow \{v_1, v_2, ..., v_{2m}, v_1^{'}, v_2^{'}, ..., v_{2m}^{'}\}$

E$\leftarrow \{e_1, e_2, ..., e_{2m}, e_1^{'}, e_2^{'}, ..., e_{2m-1}^{'}\}$

Case (i): when m is odd

Fix $v_1 \leftarrow 1$, $v_1^{'} \leftarrow 4m+1$

For $1 \leq k \leq \dfrac{m-1}{2}$

$v_{4k} \leftarrow 4k-1$, $v_{4k+1} \leftarrow 4k+1$, $v_{4k-2} \leftarrow 4m-4k+3$, $v_{4k-1} \leftarrow 4m-4k+1$,

$v_{4k}^{'} \leftarrow 4m-4k+7$, $v_{4k+1}^{'} \leftarrow 4m-4k+9$, $v_{4k-2}^{'} \leftarrow 8m-4k+3$, $v_{4k-1}^{'} \leftarrow 8m-4k+1$,;

Case (ii): when m is even

Fix $v_1 \leftarrow 1$, $v_1^{'} \leftarrow 4m+1$

For $1 \leq k \leq \dfrac{m-2}{2}$

$v_{4k} \leftarrow 4k-1$, $v_{4k+1} \leftarrow 4k+1$;

$v_{4k}^{'} \leftarrow 4m+4k-1$, $v_{4k+1}^{'} \leftarrow 4m+4k+1$

For $1 \leq k \leq \dfrac{m}{2}$

$v_{4k-2} \leftarrow 4m-4k+3$, $v_{4k-1} \leftarrow 4m-4k+1$;

$v_{4k-2}^{'} \leftarrow 8m-4k+3$, $v_{4k-1}^{'} \leftarrow 8m-4k+1$

For $k = 2m$ $v_k \leftarrow 2m$, $v_k^{'} \leftarrow 6m+1$;

Theorem 6. *The extended duplicate graph of the comb graph $EDG\,(CB_m)$, $m \geq 2$ admits odd - even graceful labeling.*

Proof. Let $\{v_1, v_2, ..., v_{2m}, v_1^{'}, v_2^{'}, ..., v_{2m}^{'}\}$ and $\{e_1, e_2, ..., e_{2m}, e_1^{'}, e_2^{'}, ..., e_{2m-1}^{'}\}$ be the set of vertices and the edges of the $EDG\,(CB_m)$.

Case (i): when m is odd

Using the algorithm OEGCB, the $4m$ vertices are labeled using 1, 3, 5,..., $8m-1(=2q+1)$. Using the induced function f^* defined in theorem 2, the $2m-2$ edges namely $e_1^{'}$, $e_2^{'}$, e_3, e_4, $e_5^{'}$, $e_6^{'}$, e_7, e_8,..., $e_{2m-5}^{'}$, $e_{2m-4}^{'}$, e_{2m-3}, e_{2m-2} receive labels 2, 4, 6, 8, 10, 12, 14, 16,..., $4m-10$, $4m-8$, $4m-6$, $4m-4$ respectively, the 3 edges $e_{2m-1}^{'}$, e_{2m}, e_{2m-1} receive labels$4m-2$, $4m$, $4m+2$ respectively and the $2m-2$ edges namely $e_{2m-2}^{'}$, $e_{2m-3}^{'}$, e_{2m-4}, e_{2m-5},..., $e_4^{'}$, $e_3^{'}$, e_2, e_1 receive labels $4m+4$, $4m+6$, $4m+8$, $4m+10$,..., $8m-8$, $8m-6$, $8m-4$, $8m-2$ respectively. Thus the

$4m-1(=q)$ edges are labeled with 2, 4, 6, 8, ..., $8m-2(=2q)$.

Case (ii): when m is even

Using the algorithm OEGCB, the $4m$ vertices are labeled using 1, 3, 5,..., $8m-1(=2q+1)$. Using the induced function f^* defined in theorem 2, the $2m-2$ edges namely $e_1^{'}$, $e_2^{'}$, e_3, e_4, $e_5^{'}$, $e_6^{'}$, e_7, e_8,..., $e_{2m-7}^{'}$, $e_{2m-6}^{'}$, e_{2m-5}, e_{2m-4} receive labels 2, 4, 6, 8, 10, 12, 14, 16,..., $4m-14$, $4m-12$, $4m-10$, $4m-8$, $4m-6$, $4m-4$ respectively, the 2 edges e_{2m-1}, e_{2m} receive labels $4m-2$, $4m$ respectively and the edge $e_{2m-1}^{'}$ receives label $4m+2$ and the $2m-2$ edges namely e_{2m-2}, e_{2m-3}, $e_{2m-4}^{'}$, $e_{2m-5}^{'}$,e_{2m-6}, e_{2m-7},..., $e_4^{'}$, $e_3^{'}$, e_2, e_1 receive labels $4m+4$, $4m+6$, $4m+8$, $4m+10$,..., $8m-8$, $8m-6$, $8m-4$, $8m-2$ respectively. Thus the $4m-1(=q)$ edges are labeled with 2, 4, 6, 8, ..., $8m-2(=2q)$.

Hence, the extended duplicate graph of the comb graph $EDG(CB_m)$, $m \geq 2$ admits odd - even graceful labeling. □

2.6 Algorithm - OEGT

V$\leftarrow \{v_1, v_2, ..., v_{3m+2}, v_1^{'}, v_2^{'}, ..., v_{3m+2}^{'}\}$

E$\leftarrow \{e_1, e_2, ..., e_{3m+2}, e_1^{'}, e_2^{'}, ..., e_{3m+1}^{'}\}$

Case (i): when m is odd

Fix $v_1 \leftarrow 6m+3$, $v_2 \leftarrow 1$, $v_1^{'} \leftarrow 12m+7$, $v_2^{'} \leftarrow 6m+5$

For $1 \leq k \leq \dfrac{m+1}{2}$

$v_{6k-3} \leftarrow 6m-6k+7$, $v_{6k-2} \leftarrow 6m-6k+5$, $v_{6k-1} \leftarrow 6m-6k+3$;

$v_{6k-3}^{'} \leftarrow 12m-6k+11$, $v_{6k-2}^{'} \leftarrow 12m-6k+9$, $v_{6k-1}^{'} \leftarrow 12m-6k+7$;

For $1 \leq k \leq \dfrac{m-1}{2}$

$v_{6k} \leftarrow 6k-3$, $v_{6k+1} \leftarrow 6k-1$, $v_{6k+2} \leftarrow 6k+1$;

$v_{6k}^{'} \leftarrow 6m+6k+1$, $v_{6k+1}^{'} \leftarrow 6m+6k+3$, $v_{6k+2}^{'} \leftarrow 6m+6k+7$;

Case (ii): when m is even

Fix $v_1 \leftarrow 6m+3$, $v_2 \leftarrow 1$, $v_1^{'} \leftarrow 12m+7$, $v_2^{'} \leftarrow 6m+5$

For $1 \leq k \leq \dfrac{m}{2}$

$v_{6k-3} \leftarrow 6m-6k+7$, $v_{6k-2} \leftarrow 6m-6k+5$, $v_{6k-1} \leftarrow 6m-6k+3$, $v_{6k} \leftarrow 6k-3$, $v_{6k+1} \leftarrow 6k-1$, $v_{6k+2} \leftarrow 6k+1$;

$v_{6k-3}^{'} \leftarrow 12m-6k+11$, $v_{6k-2}^{'} \leftarrow 12m-6k+9$, $v_{6k-1}^{'} \leftarrow 12m-6k+7$, $v_{6k}^{'} \leftarrow 6m+6k+1$, $v_{6k+1}^{'} \leftarrow 6m+6k+3$, $v_{6k+2}^{'} \leftarrow 6m+6k+5$;

Theorem 7. *The extended duplicate graph of the twig graph $EDG\,(T_m)$, $m \geq 2$ admits odd - even graceful labeling.*

Proof. Let $\{v_1, v_2, ..., v_{3m+2}, v_1^{'}, v_2^{'}, ..., v_{3m+2}^{'}\}$ and $\{e_1, e_2, ..., e_{3m+2}, e_1^{'}, e_2^{'}, ..., e_{3m+1}^{'}\}$ be the set of vertices and the edges of the $EDG\,(T_m)$.

Case (i): when m is odd

Using the algorithm OEGT, the $6m+4$ vertices are labeled using 1, 3, 5,..., $12m+7(=2q+1)$. Using the induced function f^* defined in theorem 2, the $3m-3$ edges namely e_2, e_3, e_4, $e_5^{'}$, $e_6^{'}$,$e_7^{'}$, e_8, e_9, e_{10}, $e_{11}^{'}$, $e_{12}^{'}$,$e_{13}^{'}$,..., $e_{3m-7}^{'}$, $e_{3m-6}^{'}$,$e_{3m-5}^{'}$, e_{3m-4}, e_{3m-3}, e_{3m-2} receive labels $12m+4$, $12m+2$, $12m$, $12m-2$, $12m-4$, $12m-6$, $12m-8$, $12m-10$, $12m-12$, $12m-14$, $12m-16$,..., $6m+22$, $6m+20$, $6m+18$, $6m+16$, $6m+14$,$6m+12$ respectively, the $3m-3$ edges namely $e_2^{'}$, $e_3^{'}$,$e_4^{'}$, e_5, e_6, e_7, $e_8^{'}$, $e_9^{'}$,$e_{10}^{'}$, e_{11}, e_{12}, e_{13},..., e_{3m-7}, e_{3m-6}, e_{3m-5}, $e_{3m-4}^{'}$, $e_{3m-3}^{'}$,$e_{3m-2}^{'}$ receive labels 2, 4, 6, 8, 10, 12, 14, 16,18, 20, 22, 24,..., $6m-14$, $6m-12$, $6m-10$, $6m-8$, $6m-6$, $6m-4$ respectively, the 6 edges namely $e_{3m-1}^{'}$, $e_{3m}^{'}$,$e_{3m+1}^{'}$, e_{3m-1}, e_{3m}, e_{3m+1} receive labels $6m-2$, $6m$, $6m+2$, $6m+10$, $6m+8$, $6m+6$ respectively and the 3 edges namely e_1, $e_1^{'}$, e_{3m+2} receive labels 2, $12m+6(=2q)$, $6m+4$ respectively. Thus the $6m+3(=q)$ edges are labeled with 2, 4, 6, 8, ..., $12m+6(=2q)$.

Case(ii): when m is even

Using the algorithm OEGT, the $6m+4$ vertices are labeled using 1, 3, 5,..., $12m+7(=2q+1)$. Using the induced function f^* defined in theorem 2, the $3m$ edges namely e_2, e_3, e_4, $e_5^{'}$, $e_6^{'}$,$e_7^{'}$, e_8, e_9, e_{10}, $e_{11}^{'}$, $e_{12}^{'}$,$e_{13}^{'}$,..., , e_{3m-4}, e_{3m-3}, e_{3m-2}, $e_{3m-1}^{'}$, $e_{3m}^{'}$,$e_{3m+1}^{'}$ receive labels $12m+4$, $12m+2$, $12m$, $12m-2$, $12m-4$, $12m-6$, $12m-8$, $12m-10$, $12m-12$, $12m-14$, $12m-16$,..., $6m+16$, $6m+14$, $6m+12$, $6m+10$, $6m+8$,$6m+6$ respectively, the $3m$ edges namely $e_2^{'}$, $e_3^{'}$,$e_4^{'}$, e_5, e_6, e_7, $e_8^{'}$, $e_9^{'}$,$e_{10}^{'}$, e_{11}, e_{12}, e_{13},..., $e_{3m-4}^{'}$, $e_{3m-3}^{'}$,$e_{3m-2}^{'}$, e_{3m-1}, e_{3m}, e_{3m+1}, receive labels 2, 4, 6, 8, 10, 12, 14, 16,18, 20, 22, 24,..., $6m-2$, $6m$, $6m+2$ and the 3 edges namely e_1, $e_1^{'}$, e_{3m+2} receive labels 2, $12m+6(=2q)$, $6m+4$ respectively. Thus the $6m+3(=q)$ edges are labeled with 2, 4, 6, 8, ..., $12m+6(=2q)$.

Hence, the extended duplicate graph of the twig graph $EDG\,(T_m)$, $m \geq 2$ admits odd - even graceful labeling.

Illustration:

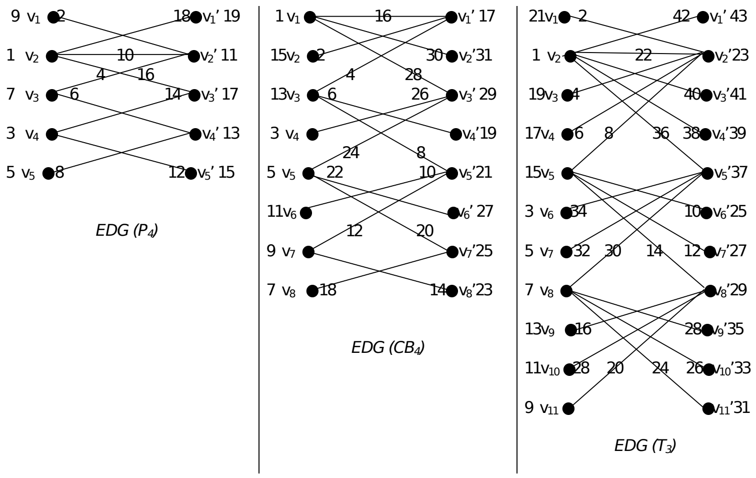


Figure 2. Odd-Even Graceful labeling in $EDG\,(P_5)\,, EDG\,(CB_4)\,andEDG\,(T_3)$ □

3 Conclusion

We have proved that extended duplicate graphs of star graph, bistar graph, double star graph, path graph, comb graph and twig graph admit odd - even graceful labeling.

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