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Majorization Properties for a new Subclass of Multivalent Functions

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Abstract- The object of this paper we investigated the majorization property of the subclass $\psi^{m,n,b,\delta,p,q,A,B}$
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1. INTRODUCTION AND DEFINITIONS

Definition 1.1

Let A_p denote the class of function of $f(z)$ of p- valent analytic function

$$f(z) = z^p + \sum_{j=p+1}^{\infty} a_j z^j \qquad (1.1)$$

Which is an analytic in the open unit disc $U = \{z : z \in C \text{ and } |z| < 1\}$.

Definition 1.2

Let two functions f and g are analytic in U, *we* say that $f(z)$ is subordinate to $g(z)$ if there exist a function $w(z)$ analytic in U satisfying $w(0) = 0$ and $|w(z)| < 1$ such that $f(z) = g(w(z))$ .It is denoted by $f(z) \prec g(z)$.(by Schwarz lemma [2]) (1.2)

Definition 1.3

Let f and g be the analytics functions of the open unit disc $U = \{z : z \in C, |z| < 1\}$.

If f is said to be majorized by g in U ,then we write $f(z) << g(z)$, $z \in U$. (1.3)

If there exist a function ϕ ,analytic in U such that $|\phi(z)| \le 1$ and $f(z) = \phi(z) g(z)$, $z \in U$ (1.4)

It might be noted here that the condition (1.2) is firmly identified with the idea of quasi subordination between analytic functions.

Definition 1.4

Let A_p denote the class of function of $g(z)$ of p- valent analytic function

$$g(z) = z^p + \sum_{j=p+1}^{\infty} c_j z^j$$

(1.5)

Which is an analytic in the open unit disc $U = \{z : z \in C \text{ and } |z| < 1\}$.

Then the Hadamard product $(f * g)$ defined by if and only if $(f * g)(z) = z^p + \sum_{j=p+1}^{\infty} a_j c_j z^j$ (1.6)

Definition 1.5

Let $f^{(q)}(z)$ be the q[th] order differential operator, where

$$f^{(q)}(z) = \frac{p!}{(p-q)!} z^{p-q} + \sum_{j=p+1}^{\infty} \frac{j!}{(j-q)!} a_j z^{j-q}, \qquad (1.7)$$

$p > q,\ p \in N,\ q \in N \cup \{0\},\ z \in A_p$

Definition 1.6

Let

$$D_\delta^n f^{(q)}(z) = \frac{(p - \delta q)^n p!}{(p-q)!} z^{p-q} + \sum_{j=p+1}^{\infty} \left((p + (j-p)\delta) - \delta q\right)^n \frac{j!}{(j-q)!} a_j z^{j-q} \qquad (1.8)$$

Letting $\delta = 1$,we get Frasin [4] differential operator.
Letting $p = 1$,we get Al-Oboudi [1] differential operator.
Letting $\delta = 1, q = 0$,we get Salagean [5] differential operator.

And it is very clear that, $D_\delta^0 f^{(0)}(z) = f(z)$, $D_\delta^1 f^{(0)}(z) = p(1-\delta) f(z) + \delta z f'(z)$ and $D_\delta^n f^{(0)}(z) = D_\delta^n f(z)$

2. MAJORIZATION PROBLEM FOR THE CLASS $\psi^{m,n,b,\delta,p,q,A,B}$

Definition 2.1

A function $\psi^{m,n,b,\delta,p,q,A,B}$ denoted the subclass of A_p consisting of functions $f(z)$ which is satisfy the inequality

$$\mathrm{Re}\left(1+\frac{1}{b}\left(\frac{D_\delta^m f^q(z)}{D_\delta^n f^q(z)}-(p-q)^{m-n}\right)\right) \prec h(z)$$

$(z \in U,\ p \in N,\ n,q \in N_0 = N \cup \{0\},$

,(2.1) $b \in C - \{0\},$.

$$\left|b(A-B)-(p-q)^{m-n}\right| \le (p-q)^{m-n})$$

Theorem 2.2

Let the function $f \in A_p$ and suppose that $g \in \psi^{m,n,b,\delta,p,q,A,B}$. If $D^n f^q(z)$ is majorized by $D^n g^q(z)$ in U ,then $\left|D^m f^q(z)\right| \le \left|D^m g^q(z)\right|$, $|z| \le r_0$

Where

$r_0 = r_0(m,n,b,\delta,p,q,A,B)$

$$= \frac{k-\sqrt{k^2-4(p-q)^{m-n}\left|b(A-B)+B(p-q)^{m-n}\right|}}{2\left|b(A-B)+B(p-q)^{m-n}\right|} \qquad (2.2)$$

$(z \in U,\ p \in N,\ n,q \in N_0 = N \cup \{0\},$

$b \in C - \{0\},$

$$k = 2+(p-q)^{m-n} + \left|b(A-B)+B(p-q)^{m-n}\right|) \qquad (2.3)$$

Proof

Since $g \in \psi^{m,n,b,\delta,p,q,A,B}$, we find (2.1) that

$$h(z) = 1+\frac{1}{b}\left(\frac{D^m f^q(z)}{D^n f^q(z)}-(p-q)^{m-n}\right), \qquad (2.4)$$

then

$$\mathrm{Re}\, h(z) > 0 \qquad (2.5)$$

and

$$h(z) = \frac{1+Aw(z)}{1+Bw(z)} \qquad (2.6)$$

Using (2.4) and (2.6) we get,

$$1+\frac{1}{b}\left(\frac{D^m f^q(z)}{D^n f^q(z)}(p-q)^{m-n}\right) = \frac{1+Aw(z)}{1+Bw(z)} \qquad (2.7)$$

$$\frac{D^n f^q(z)}{D^m f^q(z)} = \frac{1+Bw(z)}{(p-q)^{m-n}-(b(A-B)+B(p-q)^{m-n}w(z))} \qquad (2.8)$$

$$\left|D^n f^q(z)\right| \le \frac{1+B|z|}{(p-q)^{m-n}-\left|b(A-B)+B(p-q)^{m-n}\right||z|}\left|D^m f^q(z)\right| \qquad (2.9)$$

Where $|w(z) \le |z||$

Since $D^n f^q(z)$ is majorized by $D^n g^q(z)$,from (1.7) we have

$$D^m f^q(z) = z\phi'(z) D^n g^q(z) + \phi(z) D^m g^q(z) \qquad (2.10)$$

And ϕ satisfying the inequality

$$|\phi'(z)| \le \frac{1-|\phi(z)|^2}{1-|z|^2} \qquad (2.11)$$

And setting $|z| = r, |\phi(z)| = \rho,\ (0 \le \rho \le 1)$

$$\therefore \left|D^n f^q(z)\right| \le \frac{\varphi(\rho)}{(1-r^2)\left(\begin{array}{l}(p-q)^{m-n}\\ -\left|\begin{array}{l}b(A-B)\\ +B(p-q)^{m-n}\end{array}\right| r\end{array}\right)}\left|D^m g^q(z)\right| \qquad (2.12)$$

$$\varphi(\rho) = r(1+|B|r) - r\rho^2(1+|B|r) + \rho(1-r^2)\left((p-q)^{m-n}-\left|b(A-B)+B(p-q)^{m-n}\right|r\right) \qquad (2.13)$$

Takes on its maximum value at $\rho = 1$ with $r_0 = r_0(m,n.b,\delta,p,q,A,B)$

Furthermore if $0 \le \sigma \le r_0(m,n.b,\delta,p,q,A,B)$, $0 \le \rho \le 1$

$$\psi(\rho) = \sigma(1+|B|\sigma) - \sigma\rho^2(1+|B|\sigma) + \rho(1-\sigma^2)\left(\begin{array}{l}(p-q)^{m-n}\\ -\left|b(A-B)+B(p-q)^{m-n}\right|\sigma\end{array}\right) \qquad (2.14)$$

So, this is increasing function on the interval, $0 \le \rho \le 1$

$$\psi(\rho) \le \psi(1) = (1-\sigma^2)\left(\begin{array}{l}(p-q)^{m-n}\\ -\left|b(A-B)+B(p-q)^{m-n}\right|\sigma\end{array}\right)$$

Hence by setting $\rho = 1$ in (2.12), we conclude the theorem holds true for $|z| \le r_0(m,n,b,\delta,p,q,A,B)$ which is smallest root of the equation

$$(p-q)^{m-n} - r(\left|b(A-B)+(p-q)^{m-n}\right|+2) - r^2((p-q)^{m-n}+2|B|) + r^3\left|b(A-B)+B(p-q)^{m-n}\right| = 0$$

Its complete the proof of the theorem.

Letting $m = j, n = l, b = \gamma, \delta = 1, A = 1, \& B = -1$ theorem (3.2) gives the result of Pranay Goswami and M.K. Aouf. [6]

Corollary 2.3

Let the function $f \in A_p$ and suppose that $g \in S_{p,q}(\gamma)$. If $f^q(z)$ is majorized by $g^q(z)$ in U, then $\left|f^q(z)\right| \le \left|g^q(z)\right|$, $|z| \le r_0$

Where

$$r_0 = r_0(p,q;\gamma) = \frac{\lambda - \sqrt{\lambda^2 - 4(p-q)^{j-l}\left|2\gamma-(p-q)^{j-l}\right|}}{2\left|2\gamma-(p-q)^{j-l}\right|}$$

$(\lambda = 2 + (p-q)^{j-l} + \left|2\gamma - (p-q)^{j-l}\right|, j > l, p, j \in \mathbb{N},$
$q, l \in \mathbb{N}_0 \; \gamma \in \mathbb{C} - \{0\}).$

Letting
$m = 1, n = 0, b = \gamma, \delta = 1, p = 1, q = 0, A = 1, \& B = -1$
theorem (3.2) gives the result of Altintas et al. [7]

Corollary 2.4

Let the function $f \in A_p$ and suppose that $g \in S(\gamma)$. If $f(z)$ is majorized by $g(z)$ in U, then $\left|f'(z)\right| \le \left|g'(z)\right|$, $|z| \le r_0$

Where

$$r_0 = r_0(\gamma) = \frac{3 - |2\gamma - 1| - \sqrt{9 + 2|2\gamma-1| + |2\gamma-1|^2}}{2|2\gamma-1|}$$

Letting
$m = 1, n = 0, b = 1, \delta = 1, p = 1, q = 0, A = 1, \& B = -1$
theorem (3.2) gives the result of MacGregor. [8]

Corollary 2.5

Let the function $f \in A_p$ and suppose that $g \in S^* = S^*(0)$. If $f(z)$ is majorized by $g(z)$ in U, then $\left|f'(z)\right| \le \left|g'(z)\right|$, $|z| \le 2 - \sqrt{3}$.

3. CONCLUSION

In this paper, we investigate a majorization problems for the class $\psi^{m,n,b,p,q,A,B}$, we give some values of the parameters of the class we obtained some special well known cases of our main results.

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