Mohand Transform of Error Function

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Abstract: The solutions of many advanced engineering problems like Fick's second law, heat and mass transfer problems, vibrating beams problems contains error and complementary error function. When we use any integral transform to solve these types of problems, it is very necessary to know the integral transform of error function. In this article, we find the Mohand transform of error and complementary error functions. To demonstrate the usefulness of Mohand transform of error function, some numerical applications are considered in application section for solving improper integrals which contain error function. It is pointed out that Mohand transform give the exact solution of improper integral which contains error function without any tedious calculation work.

Keywords: Mohand transform, Error function, Complementary error function, Improper integral.
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1. INTRODUCTION:

Integral transforms are highly efficient for solving many advance problems of science and engineering such as radioactive decay problems, heat conduction problems, problem of motion of a particle under gravity, vibration problems of beam, electric circuit problems and population growth problems. Many researchers applied different integral transforms (Laplace transform [1-2], Fourier transform [2], Mahgoub transform [3-11, 41-43], Kamal transform [12-18, 44-45], Aboodh transform [19-24, 46-48], Mohand transform [25-28, 49-52], Elzaki transform [34-36, 53-55], Shehu transform [37-38, 56] and Sumudu transform [39, 57-58]) and solved differential equations, delay differential equations, partial differential equations, integral equations, integro-differential equations and partial integro-differential equations. Sudhanshu et al. [29-33, 40] discussed the comparative study of Mohand and other transforms (Laplace transform, Kamal transform, Elzaki transform, Aboodh transform, Sumudu transform and Mahgoub transform).

Error function occurs frequently in probability, physics, statistics, mathematics and many engineering problems like heat conduction problems, vibrating beams problems etc. Mathematically error and complimentary error functions are defined by [59-64]

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{1}$$

and

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt \tag{2}$$

In 2017, Mohand and Mahgoub [49] defined a new integral transform "Mohand transform'' of the function $F(t)$ for $t \geq 0$ as

$$M\{F(t)\} = v^2\int_0^\infty F(t)e^{-vt}dt = R(v), k_1 \leq v \leq k_2 \tag{3}$$

where operator M is called the Mohand transform operator.

The main purpose of the present article is to determine Mohand transform of error function and explain the importance of Mohand transform of error function by giving some numerical applications in application section of this paper.

2. SOME USEFUL PROPERTIES OF MOHAND TRANSFORM:

2.1 Linearity property [9-11]:

If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of $[aF_1(t) + bF_2(t)]$ is given by $[aR_1(v) + bR_2(v)]$, where a, b are arbitrary constants.

Proof: By the definition of Mohand transform, we have

$$M\{F(t)\} = v^2\int_0^\infty F(t)e^{-vt}dt$$

$\Rightarrow M\{aF_1(t)+bF_2(t)\}$
$= v^2\int_0^\infty [aF_1(t)+bF_2(t)]e^{-vt}dt$

$\Rightarrow M\{aF_1(t)+bF_2(t)\}$
$= av^2\int_0^\infty F_1(t)e^{-vt}dt + bv^2\int_0^\infty F_2(t)e^{-vt}dt$

$\Rightarrow M\{aF_1(t)+bF_2(t)\} = aM\{F_1(t)\}+bM\{F_2(t)\}$

$\Rightarrow M\{aF_1(t)+bF_2(t)\} = aR_1(v)+bR_2(v)$,

where a, b are arbitrary constants.

2.2 Change of scale property [10-11]:

If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $F(at)$is given by $aR\left(\frac{v}{a}\right)$.

Proof: By the definition of Mohand transform, we have

$M\{F(at)\} = v^2\int_0^\infty F(at)e^{-vt}dt$ (4)

Put $at = p \Rightarrow adt = dp$ in equation(4), we have

$$M\{F(at)\} = \frac{v^2}{a}\int_0^\infty F(p)e^{\frac{-vp}{a}}dp$$

$$\Rightarrow M\{F(at)\} = a\left[\frac{v^2}{a^2}\int_0^\infty F(p)e^{\frac{-vp}{a}}dp\right]$$

$$\Rightarrow M\{F(at)\} = aR\left(\frac{v}{a}\right)$$

2.3 Shifting property [11]:

If Mohand transform of function $F(t)$ is $R(v)$ then Mohand transform of function $e^{at}F(t)$is given by $\frac{v^2}{(v-a)^2}R(v-a)$.

Proof: By the definition of Mohand transform, we have

$$M\{e^{at}F(t)\} = v^2\int_0^\infty e^{at}F(t)e^{-vt}dt$$

$$= v^2\int_0^\infty F(t)e^{-(v-a)t}dt$$

$$= \frac{v^2}{(v-a)^2}(v-a)^2\int_0^\infty F(t)e^{-(v-a)t}dt$$

$$= \frac{v^2}{(v-a)^2}R(v-a)$$

2.4 Mohand transform of the derivatives of the function $F(t)$ [7-8, 10]:

If $M\{F(t)\} = R(v)$ then

a) $M\{F'(t)\} = vR(v) - v^2F(0)$
b) $M\{F''(t)\} = v^2R(v) - v^3F(0) - v^2F'(0)$
c) $M\{F^{(n)}(t)\} = v^nR(v) - v^{n+1}F(0) - v^nF'(0) - \cdots\ldots - v^2F^{(n-1)}(0)$

2.5 Mohand transform of integral of a function $F(t)$:

If $M\{F(t)\} = R(v)$ then $M\left\{\int_0^t F(t)dt\right\} = \frac{1}{v}R(v)$

Proof: Let $G(t) = \int_0^t F(t)dt$. Then $G'(t) = F(t)$ and $G(0) = 0$.

Now by the property of Mohand transform of the derivative of function, we have

$$M\{G'(t)\} = vM\{G(t)\} - v^2G(0) = vM\{G(t)\}$$

$$\Rightarrow M\{G(t)\} = \frac{1}{v}M\{G'(t)\} = \frac{1}{v}M\{F(t)\}$$

$$\Rightarrow M\{G(t)\} = \frac{1}{v}R(v)$$

$$\Rightarrow M\left\{\int_0^t F(t)dt\right\} = \frac{1}{v}R(v)$$

2.6 Mohand transform of function $tF(t)$:

If $M\{F(t)\} = R(v)$ then $M\{tF(t)\} = \left[\frac{2}{v} - \frac{d}{dv}\right]R(v)$

Proof: By the definition of Mohand transform, we have

$$M\{F(t)\} = v^2\int_0^\infty F(t)e^{-vt}dt = R(v)$$

$$\Rightarrow \frac{d}{dv}R(v) = 2v\int_0^\infty F(t)e^{-vt}dt + v^2\int_0^\infty (-t)F(t)e^{-vt}dt$$

$$\Rightarrow \frac{d}{dv}R(v) = \frac{2}{v}.v^2\int_0^\infty F(t)e^{-vt}dt - v^2\int_0^\infty tF(t)e^{-vt}dt$$

$$\Rightarrow \frac{d}{dv}R(v) = \frac{2}{v}R(v) - M\{tF(t)\}$$

$$\Rightarrow M\{tF(t)\} = \left[\frac{2}{v} - \frac{d}{dv}\right]R(v)$$

2.7 Convolution theorem for Mohand transforms [11]:

If Mohand transform of functions $F_1(t)$ and $F_2(t)$ are $R_1(v)$ and $R_2(v)$ respectively then Mohand transform of their convolution $F_1(t)*F_2(t)$ is given by

$$M\{F_1(t)*F_2(t)\} = \frac{1}{v^2}M\{F_1(t)\}M\{F_2(t)\}$$

$\Rightarrow M\{F_1(t)*F_2(t)\} = \frac{1}{v^2}R_1(v)R_2(v)$, where $F_1(t)*F_2(t)$ is defined by

$$F_1(t)*F_2(t) = \int_0^t F_1(t-x)F_2(x)dx$$

$$= \int_0^t F_1(x)F_2(t-x)dx$$

3. MOHAND TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [7-11]:

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = R(v)$
1.	1	v
2.	t	1
3.	t^2	$\frac{2!}{v}$
4.	$t^n, n \in N$	$\frac{n!}{v^{n-1}}$

5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^{n-1}}$
6.	e^{at}	$\frac{v^2}{v-a}$
7.	$sinat$	$\frac{av^2}{(v^2+a^2)}$
8.	$cosat$	$\frac{v^3}{(v^2+a^2)}$
9.	$sinhat$	$\frac{av^2}{(v^2-a^2)}$
10.	$coshat$	$\frac{v^3}{(v^2-a^2)}$

4. SOME IMPORTANT PROPERTIES OF ERROR AND COMPLEMENTARY ERROR FUNCTIONS:

4.1 The sum of error and complementary error functions is unity:

$erf(x) + erfc(x) = 1$

Proof: we have $\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2} dt = 1$$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt + \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt = 1$$

$$\Rightarrow erf(x) + erfc(x) = 1$$

4.2 Error function is an odd function:

$erf(-x) = -erf(x)$

4.3 The value of error function at $x = 0$ is 0:

$erf(0) = 0.$

4.4 The value of complementary error function at $x = 0$ is 1:

$erfc(0) = 1.$

4.5 The domain of error and complementary error functions is $(-\infty, \infty)$.

4.6 $erf(x) \to 1 \text{ as } x \to \infty$.

4.7 $erfc(x) \to 0 \text{ as } x \to \infty$.

4.8 The value of error functions $erf(x)$ for different values of x [60]:

Table: 2

S.N.	x	$erf(x)$
1.	0.00	0.00000
2.	0.02	0.02256
3.	0.04	0.04511
4.	0.06	0.06762
5.	0.08	0.09008
6.	0.10	0.11246
7.	0.12	0.13476
8.	0.14	0.15695
9.	0.16	0.17901
10.	0.18	0.20094
11.	0.20	0.22270

5. MOHAND TRANSFORM OF ERROR FUNCTION:

By equation (1), we have

$$erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-x^2} dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} \left[1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots \ldots\right] dx$$

$$= \frac{2}{\sqrt{\pi}} \left[x - \frac{x^3}{3.1!} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \cdots \ldots\right]_0^{\sqrt{t}}$$

$$= \frac{2}{\sqrt{\pi}} \left[t^{1/2} - \frac{t^{3/2}}{3.1!} + \frac{t^{5/2}}{5.2!} - \frac{t^{7/2}}{7.3!} + ..\right] \quad (5)$$

Applying Mohand transform both sides on equation(5), we get

$$M\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}} M\left\{\left[t^{1/2} - \frac{t^{3/2}}{3.1!} + \frac{t^{5/2}}{5.2!} - \frac{t^{7/2}}{7.3!} + ..\right]\right\} \quad (6)$$

Applying the linearity property of Mohand transform on equation (6), we get

$$M\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}} \left[\Gamma(3/2) v^{1/2} - \frac{\Gamma(5/2)}{v^{1/2}.3.1!} + \frac{\Gamma(7/2)}{v^{3/2}.5.2!} - \frac{\Gamma(9/2)}{v^{5/2}.7.3!} + \cdots\right]$$

$$= \frac{2}{\sqrt{\pi}} \Gamma(3/2) v^{1/2} \left[1 - \frac{1}{2}\left(\frac{1}{v}\right) + \frac{1.3}{2.4}\left(\frac{1}{v}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{1}{v}\right)^3 + \cdots \ldots \ldots\right]$$

$$= v^{1/2} \left(1 + \frac{1}{v}\right)^{-1/2} = \frac{v}{\sqrt{(1+v)}} \quad (7)$$

6. MOHAND TRANSFORM OF COMPLEMENTARY ERROR FUNCTION:

We have, $erf(\sqrt{t}) + erfc(\sqrt{t}) = 1$

$$\Rightarrow erfc(\sqrt{t}) = 1 - erf(\sqrt{t}) \quad (8)$$

Applying Mohand transform both sides on equation(8), we have

$$M\{erfc(\sqrt{t})\} = M\{1 - erf(\sqrt{t})\} \quad (9)$$

Applying the linearity property of Mohand transform on equation(9), we get

$$M\{erfc(\sqrt{t})\} = M\{1\} - M\{erf(\sqrt{t})\}$$

$$\Rightarrow M\{erfc(\sqrt{t})\} = v - \frac{v}{\sqrt{(1+v)}}$$

$$\Rightarrow M\{erfc(\sqrt{t})\} = v\left[\frac{\sqrt{(1+v)}-1}{\sqrt{(1+v)}}\right] \quad (10)$$

7. APPLICATIONS:

In this section, some applications are given in order to explain the advantage of Mohand transform of error function for evaluating the improper integral, which contain error function.

7.1 Evaluate the improper integral

$$I = \int_0^\infty e^{-t}\, erf(\sqrt{t})\, dt.$$

We have $M\{erf(\sqrt{t})\} = v^2 \int_0^\infty erf(\sqrt{t})\, e^{-vt} dt$

$$\Rightarrow M\{erf(\sqrt{t})\} = \frac{v}{\sqrt{(1+v)}} \qquad (11)$$

Taking $v \to 1$ in above equation, we have

$$I = \int_0^\infty e^{-t}\, erf(\sqrt{t})\, dt = \frac{1}{\sqrt{2}}$$

7.2 Evaluate the improper integral

$$I = \int_0^\infty te^{-3t}\, erf(\sqrt{t})\, dt.$$

We have $M\{erf(\sqrt{t})\} = \frac{v}{\sqrt{(1+v)}}$

$$\Rightarrow M\{t\, erf(\sqrt{t})\} = \left[\frac{2}{v} - \frac{d}{dv}\right]\frac{v}{\sqrt{(1+v)}}$$

$$= \frac{2}{\sqrt{(1+v)}} - \frac{2+v}{2(1+v)^{\frac{3}{2}}} \qquad (12)$$

By the definition of Mohand transform, we have

$$M\{t\, erf(\sqrt{t})\} = v^2 \int_0^\infty t\, erf(\sqrt{t})\, e^{-vt} dt \qquad (13)$$

Now by equations (12) and (13), we get

$$v^2 \int_0^\infty t\, erf(\sqrt{t})\, e^{-vt} dt = \frac{2}{\sqrt{(1+v)}} - \frac{2+v}{2(1+v)^{\frac{3}{2}}}$$

Taking $v \to 3$ in above equation, we have

$$9\int_0^\infty te^{-3t}\, erf(\sqrt{t})\, dt = 1 - \frac{5}{16} = \frac{11}{16}$$

$$I = \int_0^\infty te^{-3t}\, erf(\sqrt{t})\, dt = \frac{11}{144}$$

7.3 Evaluate the improper integral

$$I = \int_0^\infty e^{-(v-2)t}\, erf(\sqrt{t})\, dt.$$

We have $M\{erf(\sqrt{t})\} = \frac{v}{\sqrt{(1+v)}}$

Now by shifting theorem of Mohand transform, we have

$$M\{e^{2t}\, erf(\sqrt{t})\} = \frac{v^2}{(v-2)^2}\left[\frac{v-2}{\sqrt{(v-1)}}\right]$$

$$\Rightarrow M\{e^{2t}\, erf(\sqrt{t})\} = \frac{v^2}{(v-2)\sqrt{(v-1)}} \qquad (14)$$

By the definition of Mohand transform, we have

$$M\{e^{2t}\, erf(\sqrt{t})\} = v^2 \int_0^\infty e^{2t}\, erf(\sqrt{t})\, e^{-vt} dt$$

$$\Rightarrow M\{e^{2t}\, erf(\sqrt{t})\} = v^2 \int_0^\infty e^{-(v-2)t}\, erf(\sqrt{t})\, dt \qquad (15)$$

Now by equations (14) and (15), we get

$$v^2 \int_0^\infty e^{-(v-2)t}\, erf(\sqrt{t})\, dt = \frac{v^2}{(v-2)\sqrt{(v-1)}}$$

$$\Rightarrow I = \int_0^\infty e^{-(v-2)t}\, erf(\sqrt{t})\, dt = \frac{1}{(v-2)\sqrt{(v-1)}}.$$

7.4 Evaluate the improper integral

$$I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{u})du\right\} dt.$$

We have $M\{erf(\sqrt{t})\} = \frac{v}{\sqrt{(1+v)}}$

Now by the property of Mohand transform of integral of a function, we have

$$M\left\{\int_0^t erf(\sqrt{u})du\right\} = \frac{1}{v}\left[\frac{v}{\sqrt{(1+v)}}\right]$$

$$\Rightarrow M\left\{\int_0^t erf(\sqrt{u})du\right\} = \frac{1}{\sqrt{(1+v)}} \qquad (16)$$

By the definition of Mohand transform, we have

$$M\left\{\int_0^t erf(\sqrt{u})du\right\} = v^2 \int_0^\infty e^{-vt}\left\{\int_0^t erf(\sqrt{u})du\right\} dt \qquad (17)$$

Now by equations (16) and (17), we get

$$v^2 \int_0^\infty e^{-vt}\left\{\int_0^t erf(\sqrt{u})du\right\} dt = \frac{1}{\sqrt{(1+v)}}$$

Taking $v \to 1$ in above equation, we have

$$I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{u})du\right\} dt = \frac{1}{\sqrt{2}}.$$

7.5 Evaluate the improper integral

$$I = \int_0^\infty e^{-2t}\left[\frac{d}{dt} erf(2\sqrt{t})\right] dt.$$

We have $M\{erf(\sqrt{t})\} = \frac{v}{\sqrt{(1+v)}}$

Now by change of scale property of Mohand transform, we have

$$M\{erf(2\sqrt{t})\} = 4\left[\frac{v/4}{\sqrt{(1+v/4)}}\right]$$

$$\Rightarrow M\{erf(2\sqrt{t})\} = \frac{2v}{\sqrt{(4+v)}}$$

Now using the property of Mohand transform of derivative of a function, we have

$$M\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} = v.\frac{2v}{\sqrt{(4+v)}} - v^2.0$$

$$\Rightarrow M\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} = \frac{2v^2}{\sqrt{(4+v)}} \qquad (18)$$

By the definition of Mohand transform, we have

$$M\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} = v^2 \int_0^\infty e^{-vt}\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} dt \qquad (19)$$

Now by equations (18) and (19), we get

$$v^2 \int_0^\infty e^{-vt}\left\{\frac{d}{dt} erf(2\sqrt{t})\right\} dt = \frac{2v^2}{\sqrt{(4+v)}}$$

Taking $v \to 2$ in above equation, we have

$$4\int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf\left(2\sqrt{t}\right)\right\}dt = \frac{8}{\sqrt{(6)}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf\left(2\sqrt{t}\right)\right\}dt = \frac{2}{\sqrt{(6)}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf\left(2\sqrt{t}\right)\right\}dt = \sqrt{\frac{2}{3}}.$$

7.6 Evaluate the improper integral

$$I = \int_0^\infty e^{-5t}\left[erf\sqrt{t} * erf\sqrt{t}\right]dt.$$

By convolution theorem of Mohand transform, we have

$$M\{erf\sqrt{t} * erf\sqrt{t}\} = \frac{1}{v^2}M\{erf(\sqrt{t})\}M\{erf(\sqrt{t})\}$$

$$= \frac{1}{v^2}\left[\frac{v}{\sqrt{(1+v)}}\right]\left[\frac{v}{\sqrt{(1+v)}}\right] = \frac{1}{1+v} \quad (20)$$

Now by the definition of Mohand transform, we have

$$M\{erf\sqrt{t} * erf\sqrt{t}\} = v^2\int_0^\infty e^{-vt}\{erf\sqrt{t} * erf\sqrt{t}\}dt \quad (21)$$

Now by equations (20) and (21), we get

$$v^2\int_0^\infty e^{-vt}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{1+v} \quad (22)$$

Taking $v \to 5$ in above equation, we have

$$25\int_0^\infty e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{6}$$

$$\Rightarrow I = \int_0^\infty e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{150}.$$

8. CONCLUSIONS:

In this article, we have successfully discussed the Mohand transform of error function. The given numerical applications in application section show the advantage of Mohand transform of error function for evaluating the improper integral, which contain error function. Results of numerical applications show Mohand transform give the exact solution without any tedious calculation work.

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