Weighted Goal Programming Multiple Non-Linear Regression Model with Two-way Interaction Effect

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Abstract- Weighted Goal programming has been proven a valuable mathematical programming form in a number of venues. Weighted Goal Programming is the most widely used approach in the field of multiple criteria decision making that enables the decision maker to incorporate numerous variations of constraints and goals. Weighted Goal programming model serves a valuable purpose of cross checking answers from other methodologies. Different computer programmes are used to solve these GP models. Likewise, multiple regression models can also be used to more accurately combine multiple criteria measures that can be used in GP model parameters. Those parameters can include the relative weighting and the goal constraint parameters. This paper gives a brief review of the Weighted Goal Programming Multiple Non-Linear Regression Model with Interaction Effect.
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Keywords: Weighted Goal Programming, Multiple Regression, Least Square Method, Interaction effect

1. INTRODUCTION

Regression analysis is used to understand the statistical dependence of one varia-ble on other variables. Linear regression is the oldest and most widely used predictive model in decision making in managerial sciences, environmental science, and all the areas wherever it is required to describe possible relationships between two or more variables.

This technique can show what proportion of variance between variables is due to the dependent variable, and what proportion is due to the independent variables. The earliest form of regression was the method of least squares, which was published by Legendre [2] and by Gauss [5]. The linear regression can be classified into two types, simple linear regression and multiple linear regressions (MLR). The simple linear regression describes the relationship between two variables and MLR analysis describes the relationship between several independent variables and a single dependent variable. A number of methods for the estimation of the reg

ression parameters are available in the literature. These include methods of minimizing the sum of absolute residuals, minimizing the maximum of absolute residuals and minimizing the sum of squares of residuals [13], where the last method of minimizing the sum of squares of residuals popularly known as least square methods is commonly used. Alp *et. al.* [17] explained that linear goal pro-gramming can be proposed as an alternative of the Least Square Method. For this he took an example of vertical network adjustment. Hassonpour et. al. [14] pro-posed a linear programming model based on goal programming to calculate regression coefficient. Saha [20] used the binary logistic regression model to analyze the school examination result (scores) of 1002 student.

An interaction occurs when the magnitude of the effect of one independent varia-ble on a dependent variable varies as a function of a second independent variable. This is also known as a moderation effect, although some have more strict criteria for moderation effects than for interactions. Now days interaction effects through regression models is a widely interested area of investigation as there has been a great deal of confusion about the analysis of moderated relationships involving continuous variables. Alken and West [1] have analyzed such interaction effects, further this method was applied into several models by the researchers, for exam-ple, Curran et. al. [6] applied into hierarchical linear growth models.

Multiple Objective optimization techniques provides more realistic solutions for most of the problems as it deals with multiple objective whereas single objective optimization techniques provides solutions to the problems that deals with single objective. Goal programming (GP)is a type of multiple objective optimization technique that converts a multi objective optimization model into a single objec-tive optimization model. GP model has been proven a valuable tool in support of decision making. The first publication using GP was the form of a constrained re-gression model was used by Charnes *et. al.* [4]. There have been many books de-voted to this topic over past years (Ijiri [10]; Lee [11]; Spronk [12]; Ignizio [9] and others). This tool often represents a substantial improvement in the modeling and analysis of multi-objective problems (Charnes and Cooper [4]; Eiselt et al [7]; Ignizio

[8]). By minimizing deviation the GP model can generate decision variable values that are the same as the beta values in some types of multiple regression models. Tamiz et. Al. [15] presents the review of current literature on the branch of multi criteria decision modeling known as Goal Programming (GP). Machiel Kruger [16] proposed a goal programming approach to efficiently managing a bank's balance sheet while maximizing returns and at the same time taking into account the conflicting goals such as minimizing risk, subject to regulatory and managerial constraints.

2. MULTIPLE REGRESSION

Multiple regression is a technique that allows additional factors enter in the regression analysis separately, so that the effect of each can be estimated.

In other words a linear regression model that contains more than one predictor variable is called a multiple regression model.

Let $X_{i0} = 1$ for $i = 1,2,\dots,n$. Let $X_{i1}, X_{i2}, \dots, X_{im}$ be m independent variables. Then a linear relationship can be modeled as:

$$Y_i = \sum_{j=0}^{m} b_j X_{ij} + e_i$$

Where $b_0, b_1, \dots, b_m$ are the parameters to be estimated and e_i is the error components which are assumed to be normally and independently distributed with zero mean and constant variance.

The linear absolute residuals method requires us to estimate the values of the unknown parameters $b_0, b_1, \dots, b_m$ so as to

$$\text{Minimize } \sum_{i=1}^{n} |Y_i - \hat{Y}_i|$$

Where

$$\hat{Y}_i = \sum_{j=0}^{m} \hat{b}_j X_{ij}$$

$$j = 0,1,\dots,m$$

Where estimated values of the unknown parameters are represent by $\hat{b}_j$.

The least squares principle requires us to choose $b_0, b_1, \dots, b_m$ which

$$\text{Minimize} \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

2.1 Interaction effect

Gupta et. al. [19] solved a multi objective investment management planning problem using fuzzy min sum weighted fuzzy goal programming technique. Application of a multi-objective programming model like goal programming model is an important tool for studying various aspects of management systems (Sen and Nandi [20]). As an extension to the findings of Sharma et. al. [21], the paper is focused on study of the Weighted Goal Programming Multiple Non-Linear Regression Model with two way Interaction Effect.

Interaction effect represents the combined effect of variables on the criterion or dependent measure. In other words an "interaction effect" has traditionally implied a separate effect of an independent variable on the dependent variable. The "product term" actually represents a portion of the effect of the independent variables on the dependent variable.

An interactive model can be represented as:

$$Y = a_0 + a_1(X_1) + a_2(X_2) + a_3(X_1X_2)$$

In equation a_3 represents the interaction effect between X_1 and X_2 independent variables, which is referred to here as the "product term". In a traditional linear regression model (without a product term), the slope of Y on X_1 has a constant value across all values of X_2.

3. REGRESSION FORMULATION

The regression equation used to analyze and interpret a two- way interaction effect, represent

$$a_0 + \sum_{j=1}^{n} a_j X_{ij} + \sum_{k=n+1}^{2n} a_k X_{ik} + \sum_{l=2n+1}^{2n+\binom{n}{2}} a_l X_{il} + e_i = Y_{ir}$$

$i = 1,2,\dots,m$
$j = 1, 2, \dots, n$
$k = n+1,\dots,2n$ $\quad l = 2n+1,\dots, 2n + \binom{n}{2}$

Where $a_0, a_1, a_2, a_3, a_4, a_5, \dots\dots a_j \dots\dots\dots, a_k \dots\dots\dots a_l$ are the parameters to be estimated, these are contribution of decision variables $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \dots\dots X_{ij} \dots\dots X_{ik} \dots\dots X_{il}$ respectively, except a_0. Here Y_{iG} taken as dependent variable and $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \dots\dots X_{ij}$ are first, second and nth independent variable in linear form. Square of $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \dots\dots X_{ij}$ (nonlinear form) taken in linear form as X_{ik}, also product term of $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \dots\dots X_{ij}$ (representing interaction effect) taken in linear form as X_{il} respectively.

The linear absolute residual method requires us to estimate the values of unknown parameters so as to

$$\text{Minimize} \sum_{i=1}^{m} |Y_i - Y_{ir}|$$

4. WEIGHTED GOAL PROGRAMMING FORMULATION

Let Y_{iG} be the ith goal, d_i^+ be positive deviation from the ith goal and d_i^- be the negative deviation from the ith goal . w_i^+ and w_i^+ represent weights, those reflect the decision maker's preferences regarding the relative importance of each goal.

Then the problem of Minimize $\sum_{i=1}^{m}|Y_i - Y_{iG}|$ may be reformulated as:

$$\text{Minimize} \sum_{i=1}^{m}(d_i^+ + d_i^-)$$

Subject to:

$$a_0 + \sum_{j=1}^{n} a_j X_{ij} + \sum_{k=n+1}^{2n} a_k X_{ik} + \sum_{l=2n+1}^{2n+\binom{n}{2}} a_l X_{il} + d_i^- - d_i^+ = Y_{iG}$$

Non-negativity constraint,

$X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \ldots\ldots X_{ij} \ldots\ldots X_{ik} \ldots\ldots X_{il} \geq 0,$

$d_i^+ \geq 0\ ,\ d_i^- \geq 0$

Complementary constraints

$d_i^+ \times d_i^- = 0$

$i = 1,2, \ldots, m$

$j = 1, 2, \ldots,n$

$k = n+1,\ldots\ldots,2n$

$l = 2n+1,\ldots\ldots, 2n + \binom{n}{2}$

Where $a_0, a_1, a_2, a_3, a_4, a_5, \ldots\ldots a_j \ldots\ldots\ldots, a_k \ldots\ldots\ldots a_l$ are the parameters to be estimated, these are contribution of decision variables $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \ldots\ldots X_{ij} \ldots\ldots X_{ik} \ldots\ldots X_{il}$ respectively, except a_0. Here Y_{iG} taken as dependent variable and $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \ldots\ldots X_{ij}$ are first, second and nth independent variable in linear form. Square of $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \ldots\ldots X_{ij}$ (nonlinear form) taken in linear form as X_{ik} ,also product term of $X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}, \ldots\ldots X_{ij}$ (representing interaction effect) taken in linear form as X_{il} respectively, to formulate the multiple nonlinear regression problem with two way interaction effect into linear goal programming model.

5. CONCLUSION

Over the last 30 years, GP Multiple Regression problems have been deployed extensively. This paper has briefly reviewed many of the highlights. There is a huge capacity for future developments and applications of GP.

The weighted goal programming technique provides the users with a better degree of estimates of the Multiple Non-Linear Regression parameters with two - way Interaction Effect. Solution of various problems related to estimation in science and technology obtained through presented weighted goal programming technique using various software packages like Microsoft Office Excel, LINGO, LINDO and TORA.

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