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An Approach to Solve Fuzzy Time Cost Trade off Problems

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Abstract: This paper deals with mathematical structure and procedure for solving a fuzzy time cost trade-off problems using linear programming problem in an uncertain environment. The decomposition method for solving the fuzzy linear programming problem has been used. In this method a multiple number of linear programming problems can be aggregated into a single linear programming problem which gives the optimum solution using LINGO solver package. The method is examined through mathematical illustrations.
Keywords: Fuzzy theory, fuzzy numbers, fuzzy time cost trade-off problems, fuzzy linear programming problem, decomposition techniques, aggregation of m-LPPs.

1. INTRODUCTION

Project Management is one of the most important fields in business and industry. The tradeoff between the project cost and the project completion time and the uncertainty of the environment are considerable issues for all real life project decision makers. An important aspect of project management is to schedule the time accurately.

In the literature, there are several approaches proposed over the past years for finding the optimum duration with minimum cost. James E. Kelley[6] first introduced the critical path planning and scheduling and followed by that Ghazanfari et al.[5] presented the new optimal model for time cost trade off problem in a fuzzy environment using goal programming problem.

P. Pandian et al. [8] proposed a new method called decomposition method to solve integer linear programming problems by using triangular fuzzy variables and also a new approach to fuzzy network crashing in a project network whose activity times are uncertain finding an optimal duration without converting the fuzzy activity time to classical number was proposed by Shakeela sathish et al. [9] , Evangeline Jebaseeli et al.[4] formulated a new solution for time cost trade off problems in which both times and costs are fuzzy variables in the same era.

Evangeline Jebaseeli et al.[3] proposed an algorithm to solve fully fuzzy time cost trade off models through multi objective linear programming technique. Aggregated techniques of m-LPPs was proposed by Antony raj et al[1].

In this paper we proposed a solving procedure for fully fuzzy time cost trade off problem using decomposition and aggregated techniques in fuzzy linear programming problem to obtain the optimum solution of the project with numerical illustration.

2. PRELIMINARIES

Definition 2.1

The characteristic function μ_A of a crisp set A⊆X assign a value 0 or 1 to each member in X. The function can be generalised to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}: X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set

$\tilde{A} = \{A, \mu_A(x): x\epsilon X\}$

defined $\mu_{\tilde{A}}(x)$ for each x∈X is a fuzzy set.

Definition 2.2

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: R \to [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x)$ = 0 for all $(-\infty, a] \cup [c, \infty)$.
3. $\mu_{\tilde{A}}(x)$is strictly increasing on [a,b] and strictly decreasing on [b,c].
4. $\mu_{\tilde{A}}(x) = 1$ for all x∈b where a$\leq x \leq c$.

Definition 2.3

Triangular fuzzy number is a fuzzy number represented with three points as follows:

$\tilde{A} = (a_1, a_2, a_3)$ this representation in interpreted as membership functions:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & if x < a_1 \; and x > a_3 \\ \frac{x - a_1}{a_2 - a_1} & if a_1 \le x \le a_2 \\ \frac{a_3 - x}{a_3 - a_2} & if a_2 \le x \le a_3 \end{cases}$$

We use F(R) to denote the set of all triangular fuzzy numbers.

Definition 2.4

Let (a_1, a_2, a_3) and (b_1, b_2, b_3) be two triangular fuzzy numbers. Then

$(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1,\ a_2 + b_2, a_3 + b_3)$.

$(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 - b_3,\ a_2 - b_2, a_3 - b_1)$.

$k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$, $for\ k \ge 0$.

$k(a_1, a_2, a_3) = (ka_3, ka_2, ka_1)$, $for\ k < 0$.

$$\frac{(a_1, a_2, a_3)}{(b_1, b_2, b_3)} = (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1})$$

Definition 2.5

Let F(R) denotes the set of all triangular fuzzy numbers. Let us define a ranking function $\mathcal{R}$: F(R)→R which maps all triangular fuzzy numbers into R. If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number, then Graded Mean Integration Representation(GMIR) method to defuzzify the number is given by $\mathcal{R}(\tilde{A}) = \frac{a+2b+c}{4}$

Definition 2.6

A fuzzy project network is an acyclic digraph, where the vertices represent events and the directed edges represents activities, to be performed in a project. We denote this fuzzy project network by $\tilde{N} = < V, A, \tilde{D} >$. Let $V = \{v_1, v_2, \dots, v_n\}$ be the set of all vertices (events), where v_1 and v_n are the tail and head events of the project. Let$A \subset V \times V$ be the set of all directed edges, $A = \{a_{ij} = (v_i, v_j)/ v_i, v_j \in V\}$, that represents the activities to be performed in the project. A critical path is a longest path from the initial event $v_1$to terminal event v_n of the project, and an activity a_{ij} on a critical path is called a critical activity.

Definition 2.7

Linear programming is one of the most frequently applied operations research technique. We assume that all parameters and variables are real numbers. But in real world environment, do not have precise information. So, the fuzzy numbers and fuzzy variables should be used Linear programming problem. The standard form fully fuzzy linear programming problems with m fuzzy equality constraints and n fuzzy variables as follows:

Maximize (or Minimize) $(\tilde{C}^T \otimes \tilde{X})$ (1)

Subject to $\tilde{A} \otimes \tilde{X} = \tilde{b}$

$\tilde{X}$ is a non-negative fuzzy number.

Where $\tilde{C}^T = [\tilde{c}_j]_{1xn,}$

$\tilde{X}=[\tilde{x}_j]_{nx1}$, $\tilde{A}=[\tilde{a}_{ij}]_{mxn}$, $\tilde{b} = [\tilde{b}_i]_{mx1}$ and $\tilde{c}_j, \tilde{x}_j, \tilde{a}_{ij}, \tilde{b}_i \in F(R)$

$where\ i = 1,2 \dots, m\ and\ j = 1,2, \dots, n.$

2.8 Complete Fuzzy Mathematical Model

A project can be represented by an activity-on-activity arc network G=(V,A) where V={1,2,...,n} is the set of nodes and A is the set of arcs representing the activities. In the network, node 1 and n represent the start and end of the project respectively. The **complete fuzzy Mathematical model** for fully fuzzy time cost trade-off problems is presented as follows:

Min $\tilde{Z} = \sum_i \sum_j \tilde{C}_{ij}$

Subject to

$$\tilde{T}_1 = 0$$
$$\tilde{T}_j - \tilde{T}_i - \tilde{x}_{ij} \ge 0 \qquad (2)$$
$$\tilde{T}_n \le \tilde{T}$$
$$\tilde{c}_{ij} = \tilde{s}_{ij} * (N\tilde{T}_{ij} - \tilde{x}_{ij})$$
$$C\tilde{T}_{ij} \le \tilde{x}_{ij} \le N\tilde{T}_{ij} \forall (i, j) \in P$$
$$\tilde{C}_{ij} = \sum_i \sum_j \tilde{c}_{ij} + \tilde{I} * (\tilde{T}_n - \tilde{T}_1) + \sum_n \tilde{K}_n$$

Theorem 1

A triangular fuzzy number $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ is an optimal solution of the problem (P) iff $\tilde{x}_1, \tilde{x}_2\ and\ \tilde{x}_3$ are optimal solutions of the following crisp linear programming problems (P2) , (P1) and (P3) respectively where

(P) Maximize $\tilde{Z}$= C$\tilde{x}$
Subject to A$\tilde{x} \le \breve{b}, \tilde{x} \ge 0$

(P2) Maximize Z_2= Cx_2
Subject to $Ax_2 \le b_2$, $x_2 \ge 0$

(P1) Maximize Z_1= Cx_1
Subject to $Ax_1 \le b_1$,
$x_1 \ge 0$, $x_1 \le x_2$

(P3) Maximize Z_3= Cx_3
Subject to $Ax_3 \le b_3$,
$x_3 \ge 0, x_3 \ge x_2$.

3. AGGREGATION OF M-LPPS [1]

3.1 Notations

$i: i^{th}\ problem\ (i = 1,2, \dots, m).$

$j: j^{th}\ problem\ (j = 1,2 \dots, n_i).$

$x_{ij} :\ j^{th}\ variable\ of\ the\ i^{th}\ problem.$

$c_{ij}: Cost\ coefficient\ of\ the\ j^{th}\ variable$

$of\ the\ i^{th}\ problem.$
n_i: $Number\ of\ variables\ in\ the\ i^{th}\ problem.$
k_i: $Number\ of\ constraints\ in\ the\ i^{th}\ problem.$
b_{ik_i}: $Right\ hand\ side\ value\ of\ the\ k_i{}^{th}\ constraint\ of\ the\ i^{th}\ problem.$

3.2 General LPP structure of the ith- problem (i=1,2,...m) can be given as:

$$Max\ Z_i = c_{i1}x_{i1} + c_{i2}x_{i2} + \cdots + c_{in_i}x_{in_i}$$

Subject to the constraints:

$$a_{i11}x_{i1} + a_{i12}x_{i2} + \cdots + a_{i1n_i}x_{in_i}\ \{\le, =, \ge\} b_{i1}$$
$$a_{i21}x_{i1} + a_{i22}x_{i2} + \cdots + a_{i2n_i}x_{in_i}\ \{\le, =, \ge\} b_{i2}$$

....

....

$$a_{ik_i1}x_{i1} + a_{ik_i2}x_{i2} + \cdots + a_{ik_in_i}x_{in_i}\ \{\le, =, \ge\} b_{ik_i}$$
$$x_{ij} \ge 0, \{i = 1, \ldots. m, j = 1,2, \ldots, n_i\}$$

3.3 Aggregated structure of m-LPPs together

$$Max\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n_i} c_{ij}x_{ij}$$

Subject to the constraints:

$$\begin{cases} \boldsymbol{a_{111}x_{11} + a_{112}x_{12} + \cdots + a_{11n_1}x_{1n_1}}\ \{\le, =, \ge\}\ \boldsymbol{b_{11}} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots. \\ \boldsymbol{a_{1k_11}x_{11} + a_{1k_12}x_{12} + \cdots + a_{1k_1n_1}x_{1n_1}}\ \{\le, =, \ge\}\ \boldsymbol{b_{1k_1}} \end{cases}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots..$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\begin{cases} \boldsymbol{a_{m11}x_{11} + a_{m12}x_{12} + \cdots + a_{m1n_1}x_{1n_1}}\ \{\le, =, \ge\}\ \boldsymbol{b_{m1}} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots. \\ \boldsymbol{a_{mk_m1}x_{11} + a_{1k_m2}x_{12} + \cdots + a_{1k_mn_1}x_{1n_1}}\ \{\le, =, \ge\}\ \boldsymbol{b_{mk}} \end{cases}$$
$$\boldsymbol{x_{ij} \ge 0,\ \{i = 1, \ldots. m, j = 1, 2, \ldots, n_i\}}$$

4. SOLVING PROCEDURE

Step 1 Determine the cost slope and the direct cost of the triangular fuzzy time cost trade off problem.
Step 2 Convert the triangular fuzzy time cost trade off problem into a fuzzy linear programming problem using the fully fuzzy mathematical model (2).
Step 3 Fuzzy linear programming problem is decomposed into crisp linear programming problems.
Step 4 Decomposed crisp linear programming problems are aggregated into single linear programming problem.
Step 5 Attain the optimum solution of fully fuzzy mathematical model using LINGO solver package.
Step 6 The optimum solution of the crash duration and crash cost of each activity can be easily found in the corresponding variables.

5. NUMERICAL ILLUSTRATION

List of activities for creating a one way traffic system to connect four places in a city so as to avoid a traffic jam is given below with the relevant details. Table 1 represents the description of the project. In this project, time parameter and costs of the project are considered in triangular fuzzy number form. Indirect cost of the project per day is (100,100,100). In this project, the project manager decides to complete within (63, 64, 65) days. Activities information is given in table 2.

Table 1 Project description

Activity	Description
1→2 (A)	Vayallur to Bishop Heber college
2→3 (B)	Bishop Heber College to Trichy fort
2→4 (C)	Bishop Heber College to Pallakarai
3→4 (D)	Trichy fort to Pallakarai

Table 2 Fuzzy data of the project

Activity	**Crash time**	**Normal time**	**Crash cost**	**Normal cost**
1→2	**(20,21,22)**	**(23,24,25)**	**(400, 500, 600)**	**(700,800,900)**
2→3	**(18,18,18)**	**(20,20,20)**	**(500,500,500)**	**(600,600,600)**
2→4	**(19,20,21)**	**(22,22,22)**	**(600,700,800)**	**(900,900,900)**
3→4	**(16,16,16)**	**(20,20,20)**	**(600,600,600)**	**(800,800,800)**

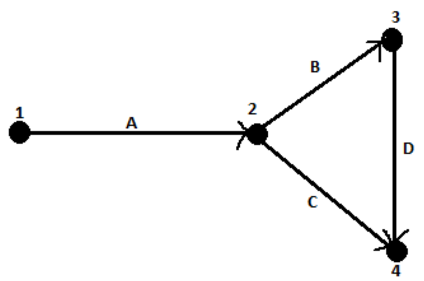

Figure 1

Table 3 Crash slope of the project

Activity	ΔT	ΔC	Crash slope (ΔC/ ΔT)
1-2(A)	(1,3,5)	(100,300,500)	(20,100,500)
2-3(B)	(2,2,2)	(100,100,100)	(50,50,50)
2-4(C)	(1,2,3)	(100,200,300)	(33.3,100,300)
3-4(D)	(4,4,4)	(200,200,200)	(50,50,50)

Direct cost of the given project is (2100, 2300, 2500) and the total duration is (63, 64, 65).
By using problem (2)

Min $\tilde{Z} = [\sum_i \sum_j \tilde{C}_{ij} + \tilde{I} * (\tilde{T}_n - \tilde{T}_1) + \sum_n \tilde{K}_n]$

Subject to the constraints

$\tilde{T} = 0$
$\tilde{T}_2 - \tilde{T}_1 - \tilde{x}_{12} \ge 0$
$\tilde{T}_3 - \tilde{T}_2 - \tilde{x}_{23} \ge 0$
$\tilde{T}_4 - \tilde{T}_2 - \tilde{x}_{24} \ge 0$

$\tilde{T}_4 - \tilde{T}_3 - \tilde{x}_{34} \geq 0$
$\tilde{T}_4 \geq (62,62,62)$
$\tilde{c}_{12} = \tilde{s}_{12} * (N\tilde{T}_{12} - \tilde{x}_{12})$
$\tilde{c}_{23} = \tilde{s}_{23} * (N\tilde{T}_{23} - \tilde{x}_{23})$
$\tilde{c}_{24} = \tilde{s}_{24} * (N\tilde{T}_{24} - \tilde{x}_{24})$
$\tilde{c}_{34} = \tilde{s}_{34} * (N\tilde{T}_{34} - \tilde{x}_{34})$
$c\tilde{T}_{12} \leq \tilde{x}_1 \leq N\tilde{T}_{12}$
$c\tilde{T}_{23} \leq \tilde{x}_2 \leq N\tilde{T}_{23}$
$c\tilde{T}_{24} \leq \tilde{x}_2 \leq N\tilde{T}_{24}$
$c\tilde{T}_{34} \leq \tilde{x}_3 \leq N\tilde{T}_{34}$

All the triangular fuzzy variables are decomposed in to 3 crisp variables and then aggregated.

Table 4 crash cost for each activity

Activity	Project duration	Crashing Cost
1-2(A)	(22.75,22.75,22.75)	(45,125,125)
2-3(B)	(19.25,19.25,19.25)	(37.5,37.5,37.5)
2-4(C)	(22,22,22)	(0,0,0)
3-4(D)	(20,20,20)	(0,0,0)

The minimum values of the fuzzy total cost and planned duration of the project have been determined using LINGO solver package. The optimal crashing cost of each activity is given in table 4. The optimal fuzzy total cost of the project is (9500, 9500, 9500) at (63, 64, 65) days

6. CONCLUSION

In this paper we have presented a new solving procedure for fuzzy time cost trade off problem using fuzzy linear programming problem with simple numerical illustration. This method is easier and time consuming compared with the existing methods. As in earlier existing methods we get a crisp solution for the fuzzy variables. Using this new approach we need not to defuzzify the triangular fuzzy variables, we were able to get a fuzzy solution for the fuzzy variables.

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