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Fuzzy Decision Making applying Z numbers with AHP & BWM

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Abstract-Since MCDM has been widely applied in various decision making process, a fuzzy model using z number has been proposed using AHP and modified by Best worst method. To illustrate the proposed method a real-life example has been discussed.
Index Terms-AHP, Decision making, Z-number, BWM

1. INTRODUCTION

In the real world, most of the decisions taken by the humans are unpredictable which is uncertain. Thus in (1970) Fuzzy Decision making model was first proposed by Zadeh[2] and later in (2011), he proposed a notion namely Z-number, which is an order pair of fuzzy numbers. As there exists numerous methods in solving decision making problems, Analytical Hierarchical Process (AHP) is one amongst to rank the best alternative. In (2015) Mardani et.al[9] discussed the MCDM techniques with the fuzzy set.

In this paper an extension has been proposed for the FAHP with z numbers [7] by means of Best worst method which was proposed by Jafer Rezaei [8]in (2014) to evaluate the best criterion for the decision makers and it has been discussed with an example.

2. PRELIMINARIES[23]

2.1 Fuzzy Set

A fuzzy set A is defined on a universe X may be given as A= $\{(x, \mu_{\ddot{A}}(x))) / x \in X\}$

Where $\mu_{\ddot{A}} : X \to [0,1]$ is the membership function A. The membership value $\mu_A(x)$ describes the degree of belongingness of $x \in X$ in A

2.2.*Z- number*

A Z- number is an ordered pair of fuzzy numbers denoted as Z = (A,B), the first component A which is a real valued uncertain variable X plays the role of fuzzy number restriction, R(x), where A is a fuzzy set, The second component B, is referred to as a measure of reliability for the first component.

2.3. Fuzzy Analytic Hierarchy Process (FAHP)

Fuzzy Analytic Hierarchy Process was developed for solving hierarchical problems in handling the uncertainty and vagueness involved in the pairwise comparison process [18]. Thus, the traditional AHP uses an exact value to the decision makers opinion in a comparison of alternatives (Wang & Chen 2007) [20].

2.4 Best – worst Method (BWM)

It is a multi-criteria decision making (MCDM) method. This method is used to evaluate a set of alternatives with respect to a set of decision criteria. The BWM is based on a systematic pairwise comparison of the decision criteria. Once the decision criteria has been identified by the decision maker (DM), two criteria are selected by the DM: the best criterion and the worst criterion. The best criterion is the one which has the most important role in making the decision, while the worst criterion has the opposite role. The DM then gives his/her preferences of the best criterion over all the other criteria and also his/her preferences of all the criteria over the worst criterion using a number from a predefined scale. These two sets of pairwise comparisons are used as input for an optimization problem, the optimal results of which are the weights of the criteria. The salient feature of the BWM is that it uses a structured way to generate the pairwise comparisons which leads to reliable results.
In order to execute a pairwise comparison for n criteria using a scale, the matrix would be

$$A = \begin{bmatrix} a_{11} & a_{12} \dots\dots\dots & a_{1n} \\ a_{21} & a_{22} \dots\dots\dots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & \dots\dots. & a_{nn} \end{bmatrix}$$

where a_{ij} shows the relative preference of criterion i to criterion j. $a_{ij} = 1$ shows that i and j are of same importance**.**

3. PROPOSED METHODOLOGY

As Fuzzy Analytic hierarchy process is a good decision making tool, it still has an ambiguity in language description and does not respond well to the evaluators, decision makers and so on. Z number appears to be a good solution to overcome this issue. Combining this and converting it to a crisp value by

means of AHP and BWM method gives the closest approximation to the decision maker. It is also more helpful for us to verify the consistency of the solution made by the decision maker. Methodology are as follows,

Step-1: Construction of the detailed hierarchy of the problem:

The hierarchy is constructed based on all criteria, sub criteria and alternatives to the practical research problem

Step-2: Constructing a pairwise comparison matrix:

Once the hierarchy was established and a series of equations were asked to direct pairwise comparisons, each expert performed a pairwise comparison. Assuming expert gives his or her opinion as follows:

Consider $\ddot{A}(a_1, b_1, c_1)$ and $\ddot{B}(a_2, b_2, c_2)$ as an Z number, where $\ddot{A}$ and $\ddot{B}$ are triangular fuzzy numbers such as $[(a_1, b_1, c_1), (a_2, b_2, c_2)]$

Converting $\ddot{B}$ to a crisp number, secondly, we add weight of $\ddot{B}$to the $\ddot{A}$ according to the equation $\ddot{z} = (\ddot{A}, \alpha) = [(a_1, b_1, c_1), \alpha]$, $\ddot{z}(\ddot{A}, \alpha)$ to a fuzzy number $\ddot{Z} = (\ddot{A}, \ddot{B})$ as $\ddot{Z} = (\sqrt{\alpha}a, \sqrt{\alpha}b, \sqrt{\alpha}c)$ again construct a pairwise comparison matrix according to the expert's opinion as triangular fuzzy number and applying into FAHP.

Step-3: Convert the above pairwise comparison matrix of a triangular fuzzy number to a normal pairwise crisp value using the ranking formula such as

$$R(\ddot{Z}) = \frac{c + 4a + b}{6}$$

Step-4: The decision making has been evaluated using the normal AHP and the priorities have been determined for the several n criteria

Step-5:Determine the best (e.g. most desirable, most important) and the worst (e.g. least desirable, least important) criteria.

Step-6: Determine the preference of the best criterion over all the other criteria, using a number between 1 and 9, The resulting best-to-others(BO) vector would be

$$A_B = (a_{B1}, a_{B2}, \dots\dots\dots a_{Bn})$$

Where a_{Bj} indicates the preference of the best criterion B over criterion j. it is clear that $a_{BB} = 1$

Step-7:Determine the preference of all the criteria over the worst criterion, using a number between 1 and 9. The resulting others-to-worst(OW) vector would be:

$$A_W = (a_{1W}, a_{2W}, \dots\dots\dots a_{nW})^T$$

Where a_{jw} indicates the preference of the criterion j over the worst criterion W. It is clear that $a_{WW} = 1$.

Step-8: Find the optimal weights. The aim is to determine the optimal weights of the criteria, such that the maximum absolute differences $\left|\frac{W_B}{W_j} - a_{Bj}\right|$and $\left|\frac{W_j}{W_B} - a_{jW}\right|$for all j is minimized, which is translated to the following minimax model:

$$\min max\left\{\left|\frac{W_B}{W_j} - a_{Bj}\right|, \left|\frac{W_j}{W_B} - a_{jW}\right|\right\}$$

s.t

$$\sum_j w_j = 1$$

$w_j \geq 0$, for all j.

Step-9: Thus, based on this BWM method, the result obtained in the AHP has been verified which helps the decision makers to obtain the closest approximation solution.

Step-10: Based on the weighted priorities of the several criteria, the best alternative is determined using original AHP.

4. NUMERICAL EXAMPLE

Here we give an example of selection of the best safety measure to control the dengue fever to illustrate the procedure of the proposed approach. Some of the measures taken by the government to improve this global problem are assigned as several criteria such as C_1: Domestic breeding checkers to take up mosquito control steps, C_2: Household survey of containers and awareness to store water inside homes to protect houses getting infested with larvae, C_3: People were asked not to let the water get stagnated, C_4: Fogging, C_5: Medical Camps being held; mosquito breeding curtailed. Here the alternatives are assigned as the most affected areas such as Kerala, Karnataka, Tamil Nadu, West Bengal andNew Delhi.

Based on the proposed methodology, the linguistic criteria evaluation of the decision makers and using the concept of z number, the reliability of the evaluation has been done. Hence the weighted z number has been converted to a crisp number using step2-4 and the fuzzy pairwise comparison matrix and the weighted crisp value priority vector using z number are tabulated as follows.

4.1TABLE
FUZZY PAIRWISE COMPARISON MATRIX

	C_1	C_2	C_3	C_4	C_5
C_1	(1,1,1)	(1/5,1/3,1)	(1/4,1/2,1)	(1/7,1/3,1)	(1/3,1,1)
C_2	(1,3,5)	(1,1,1)	(2,4,6)	(1,1,2)	(5,7,9)
C_3	(1,2,4)	(1/6,1/4,2)	(1,1,1)	(1/7,1/5,1/3)	(1/9,1/7,1/5)
C_4	(1,3,7)	(1/2,1,1)	(3,5,7)	(1,1,1)	(5,8,9)
C_5	(1,1,3)	(1/9,1/7,1/5)	(5,7,9)	(1/9,1/8,1/5)	(1,1,1)

4.2 TABLE
WEIGHTED CRISP VALUE USING Z NUMBER

	C_1	C_2	C_3	C_4	C_5	Priority
C_1	1.000	0.325	0.358	0.267	0.419	0.090
C_2	1.563	1.000	2.505	1.097	5.308	0.344
C_3	1.423	0.212	1.000	0.168	0.117	0.090
C_4	1.877	0.548	3.437	1.000	5.315	0.319
C_5	1.265	0.120	5.373	0.120	1.000	0.157
SUM	7.128	2.204	12.673	2.652	12.159	

Next step is to choose the best criterion and the worst criterion as in step 5. To check the consistency measure, let us choose $\mathbf{C_2}$(row 2) as the bestand$\mathbf{C_3}$ (Column 3)as worst from table 4.2. Hence by step-6 and 7

$a_{Bj} = (1.563,1.00,2.505,1.097,5.308)$
and
$a_{jW} = (0.358,2.505,1.00,3.437,5.373)^T$
Thus, the optimal weights for the best and the worst are as follows

4.3 TABLE
BWM ANALYSIS

a_{Bj}	a_{jW}	$\frac{W_B}{W_j}$	$\frac{W_j}{W_B}$	$\left\|\frac{W_B}{W_j} - a_{Bj}\right\|$	$\left\|\frac{W_j}{W_B} - a_{jW}\right\|$	$max\left\{\left\|\frac{W_B}{W_j} - a_{Bj}\right\|, \left\|\frac{W_j}{W_B} - a_{jW}\right\|\right\}$
1.563	0.358	0.136	35.399	1.427	35.041	35.041
1	2.505	0.087	5.059	0.913	2.554	2.554
2.505	1	0.2183	12.673	2.2867	11.673	11.673
1.097	3.437	0.0956	3.687	2.59	0.25	2.559
5.308	5.373	0.4626	2.3586	4.8454	3.0144	4.845

From the above table, it has been found that the minimum value is 2.554. It corresponds to the criteria C_2. Hence we chooseC_2as the best criteria.

5. CONCLUSION:

In this paper, an additional concept of best worth method has been introduced as an extension of FAHP with z numbers. Eventhough the government, is taking various steps to control dengue, we find that C_2 which is household survey and educating is the best method when compared to other methods. So, more attention must be given for educating the public and keep proper sanitation in their individual houses.

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