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Analysis of Stress & Vibrations With Shape Optimization In Isotropic And Orthotopic Plate

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ABSTRACT

A rectangular isotropic or orthotropic plate with central circular hole under transverse static loading, have found widespread applications in various fields of engineering such as aerospace, marine, automobile and mechanical. For design of plate with hole, accurate knowledge of deflection, stresses & stress concentration are required. Stress concentration arises from any abrupt change in geometry of plate under loading. As a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue cracking and plastic deformation frequently occur at point of stress concentration.

A finite element study is made for reduction of stress concentration factor (SCF) in square simply supported and fixed isotropic & orthotropic plate with central circular hole subjected to transverse static loading. The finite element formulation is carried out by the analysis section of the package ANSYS. The reduction in stress concentration for normal stress in X,Y direction, shear stress in XY plane, VON-MISES (equivalent) stress are achieved by introducing four auxiliary oval cavities or four auxiliary holes on either side of main hole. The deflection in transverse direction are also calculated and compared for different cases. The optimized design of plate will not only reduce the stress concentration, but also reduce the weight of components. With such reduction in maximum stress levels, the improvement in fatigue life of a component can be significant.

INTRODUCTION

Materials can be classified isotropic and anisotropic. But the design requirement involves incorporation of cutouts of various geometry like circular and square, which results in stress concentration in thereby reducing the mechanical strength of the structure.

- ISOTROPIC MATERIALS.

In the study of mechanical properties of materials, "isotropic" means having identical values of a property in all directions none of the properties depend the orientation. Glass and metals are examples of isotropic materials. Isotropic materials are useful since they are easier to shape, and their behavior is easier to predict.

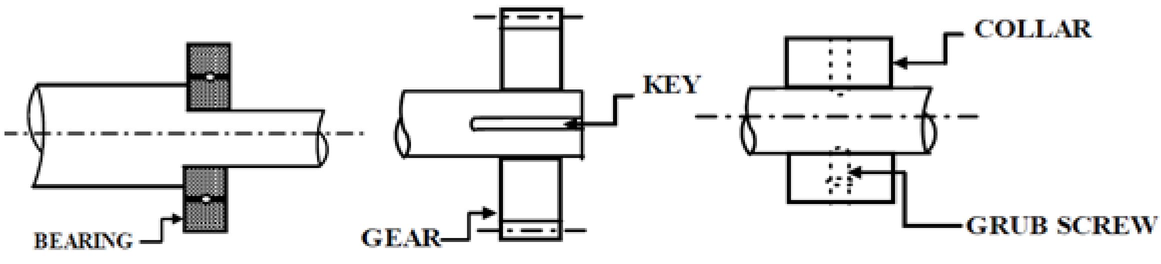
ORTHOTROPIC MATERIALS.

A material is a orthotropic if it's mechanical properties are unique and independent in directions of three mutually perpendicular axis.

Examples of orthotropic material are wood, crystals, rolled metals.

- STRESS CONCENTRATION

In developing a machine it is impossible to avoid changes in cross-section, holes, Notches, shoulders etc. Some examples are-



Any such discontinuity in a member affects the stress distribution in the Neighbourhoods and the discontinuity acts as a stress raiser.

Consider a plate with a centrally located hole and the plate is subjected to uniform tensile load at the ends. Stress distribution at a section A-A passing through the hole and another section BB away from the hole are shown in below figure. Stress distribution away from the hole is uniform but at AA there is a sharp rise in stress in the vicinity of the hole. Stress concentration factor k_t is defined as $k_t = \sigma_3 / \sigma_{av}$, where σ_{av} at section AA is simply $P/ t(w - 2b)$ and $\sigma_1 = P/ tw$. This is the theoretical or geometric stress concentration factor and the factor is not affected by the material properties.

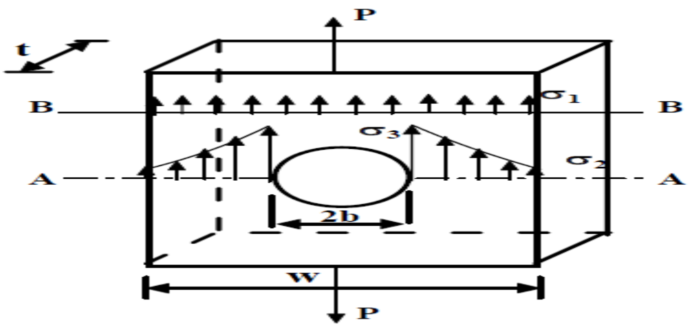


Fig:- Stress concentration due to center hole in a plate subjected to uni-axial loading.

It is possible to predict the stress concentration factors for certain geometric shapes using theory of elasticity approach. For example, for an elliptical hole in an infinite plate, subjected to a uniform tensile stress σ_1 in below figure stress distribution around the discontinuity is disturbed and at points remote from the discontinuity the effect is insignificant. According to such an analysis,

$$\sigma_3 = \sigma_1\left(1 + \frac{2b}{a}\right)$$

If a=b the hole reduces to a circular one and therefore $\sigma_3 = 3\sigma_1$ which gives k_t =3. If, however 'b' is large compared to 'a' then the stress at the edge of transverse crack is very large and consequently k is also very large. If 'b' is small compared to a then the stress at the edge of a longitudinal crack does not rise and k_t =1.

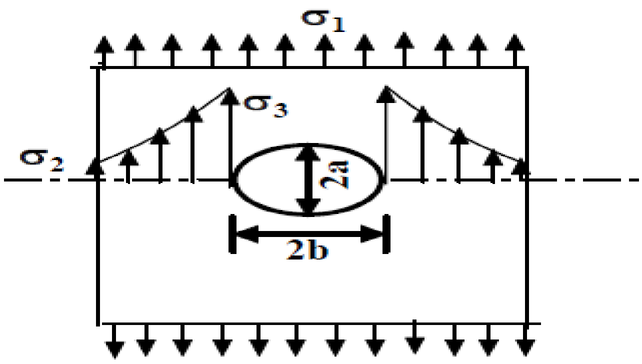


Fig:- Stress concentration due to center elliptic hole in a plate subjected to uni-axial loading.

- **VIBRATION**

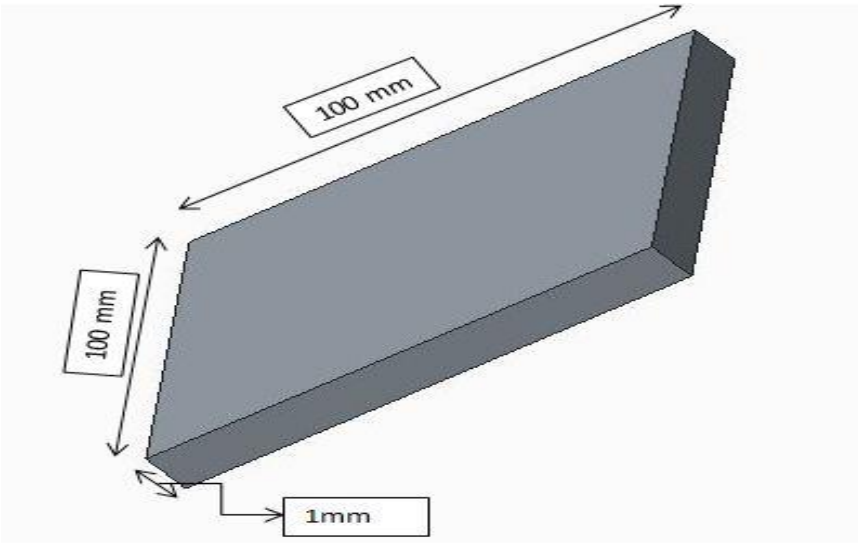
Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. The oscillations may be periodic such as the motion of a pendulum or random such as the movement of a tire on a gravel road.

The equations of motion for free un-damped vibration of an elastic system undergoing large displacements can be expressed in the following matrix form.

$$[K]\{\delta\} + [M]\{\ddot{\delta}\} = \{0\}$$

[K] and [M] are overall stiffness matrix & mass matrix and {δ} is displacement vector.

- **PROBLEM STATEMENT**
  - Stress analysis of isotropic and orthotropic material in fixed and simply support.
  - Optimization of hole shape to reduce stress concentration in isotropic and orthotropic material with fixed and simply support.
  - Three mode vibration analysis of isotropic and orthotropic material in fixed and simply support.



MODEL –(Solid plate)

MATERIAL PROPRTIES –

- ISOTROPIC MATERIAL.

Material	Modulus of Elasticity(E_X)	Poisson ratio	Density
Aluminium	70.9 GPa	.33	2660Kg/m^3

- ORTHOTROPIC MATERIAL.

Material	Boron / epoxy
E_X	208GPa
E_Y	18.9GPa
E_Z	18.9GPa
G_{XY}	5.7GPa
G_{YZ}	5.7GPa
G_{ZX}	5.7GPa
Poisson ratio	.23
Density	2000Kg/m^3

MESHING & ANALYSIS IN ANSYS –

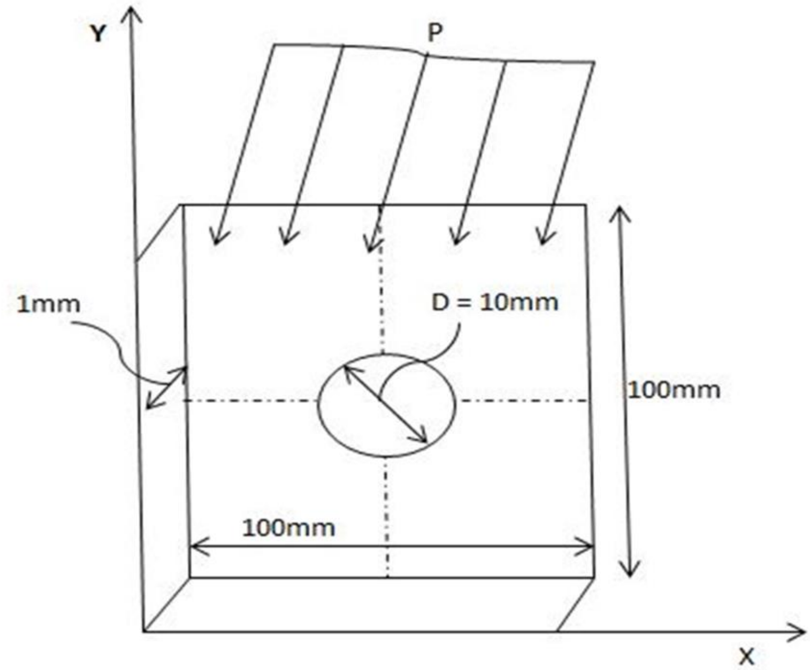
- Element – 3D 4 node 181.
- Meshing – Mapped.
- Type of analysis – Two dimensional (2D).
- Load – 1N/m^2 (transverse direction).

NOTATIONS –

- D – Diameter of central hole (D=10mm).
- A – Width of plate (A=10cm).
- L – Length of rectangle in oval cavity.
- t – Thickness of plate.

- ϕ - Width of rectangle or diameter of semicircles in oval cavity.

BASIC MODEL OF ANALYSIS



Stress Analysis of solid Isotropic Plate

FEM analysis was done using Ansys on a square plate of dimension 0.1 meter using mapped meshing with element length of 0.001 meter and transverse load of $1N/m^2$. The following support conditions are used for analysis:-

1) Simply supported.
2) Fixed support.

The results so obtained from analysis are as shown through following tables:

SIMPLY SUPPORTED

σ_x	σ_y	τ_{xy}	σ_{eqv}	U_z
2949.4	2949.4	1879.7	3256.3	0.6161×10^{-7}

FIXED SUPPORT

σ_x	σ_y	τ_{xy}	σ_{eqv}	U_z
2896.8	2896.8	490.02	2557.0	0.1912×10^{-7}

Stress Analysis of solid orthotropic plate

FEM analysis was done using Ansys on a square orthotropic plate of dimension 0.1 meter using mapped meshing with element length of 0.001 meter and transverse load of$1N/m^2$. The following support conditions are used for analysis:-

1) Simply supported.
2) Fixed support.

The results so obtained from analysis are as shown through following tables:

SIMPLY SUPPORTED

σ_x	σ_y	τ_{xy}	σ_{eqv}	U_z
7527.3	763.04	526.50	7220.9	0.7411×10^{-7}

FIXED SUPPORT

σ_x	σ_y	τ_{xy}	σ_{eqv}	U_z
5011.9	937.3	100.14	4960.5	0.1591×10^{-7}

Stress Analysis of Isotropic plate with hole

FEM analysis was done using Ansys on a square isotropic plate of dimension 0.1 meter with a hole of 0.01 meter at center using mapped meshing with element length of 0.001 meter and transverse load of $1N/m^2$.
The following support conditions are used for analysis:-

1) Simply supported.
2) Fixed support.

The results so obtained from analysis are as shown through following tables:

SIMPLY SUPPORTED

	σ_X	σ_Y	τ_{XY}	σ_{EQ}	U_Z
	5121.1	5121.1	2200.3	4898.3	0.6589×10^{-7}
SCF	1.7363	1.7363	1.17056	1.5042	

FIXED SUPPORT

	σ_X	σ_Y	τ_{XY}	σ_{EQ}	U_Z
	2950.3	2949.8	995.37	2604.0	0.2039×10^{-7}
SCF	1.01776	1.01759	2.0312	1.01838	

Where, SCF is calculated as ratio of maximum stress in plate with hole to maximum stress in plate without hole.

Stress Analysis of Orthotropic plate with hole

FEM analysis was done using Ansys on a square orthotropic plate of dimension 0.1 meter with a hole of 0.01 meter at center using mapped meshing with element length of 0.001 meter and transverse load of $1N/m^2$.
The following support conditions are used for analysis:-

1) Simply supported
2) Fixed support.

The results so obtained from analysis are as shown through following tables:

SIMPLY SUPPORTED

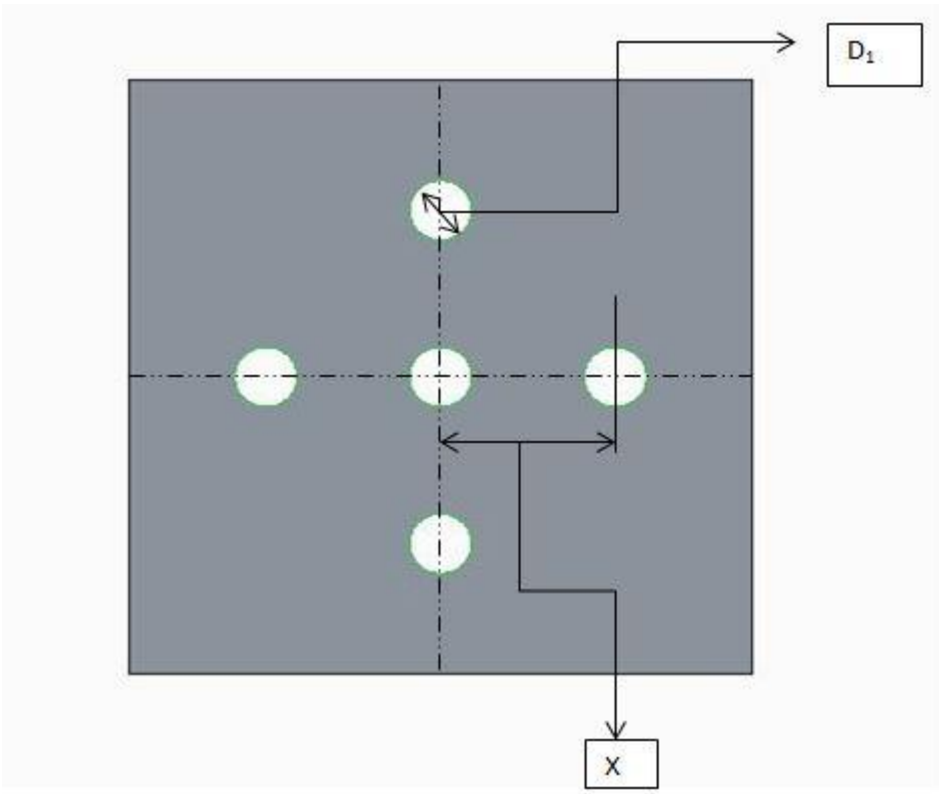
	σ_X	σ_Y	τ_{XY}	σ_{EQ}	U_Z
	6494.8	6494.5	8970.7	15933	0.1941×10-7
SCF	.8628	8.51134	17.038	2.20651	

FIXED SUPPORT

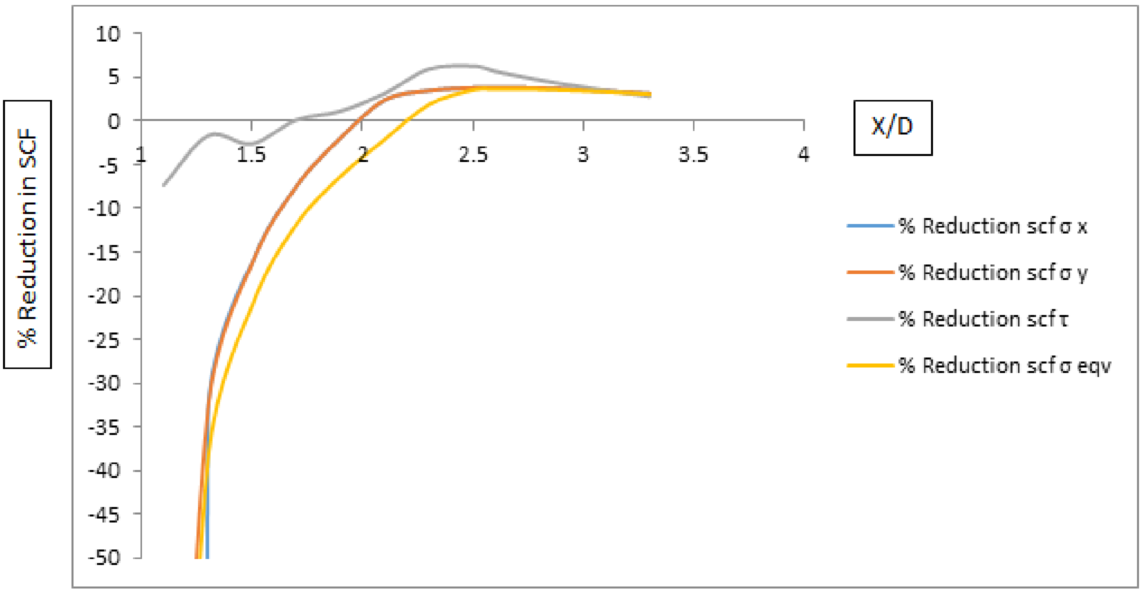
	σ_X	σ_Y	τ_{XY}	σ_{EQ}	U_Z
	3254.4	3253.4	1052.1	3220.8	0.1593×10^{-7}
SCF	.64933	3.4709	10.5062	.64928	

OPTIMIZATION

Optimizing the distance of circular cavity from center of plate-

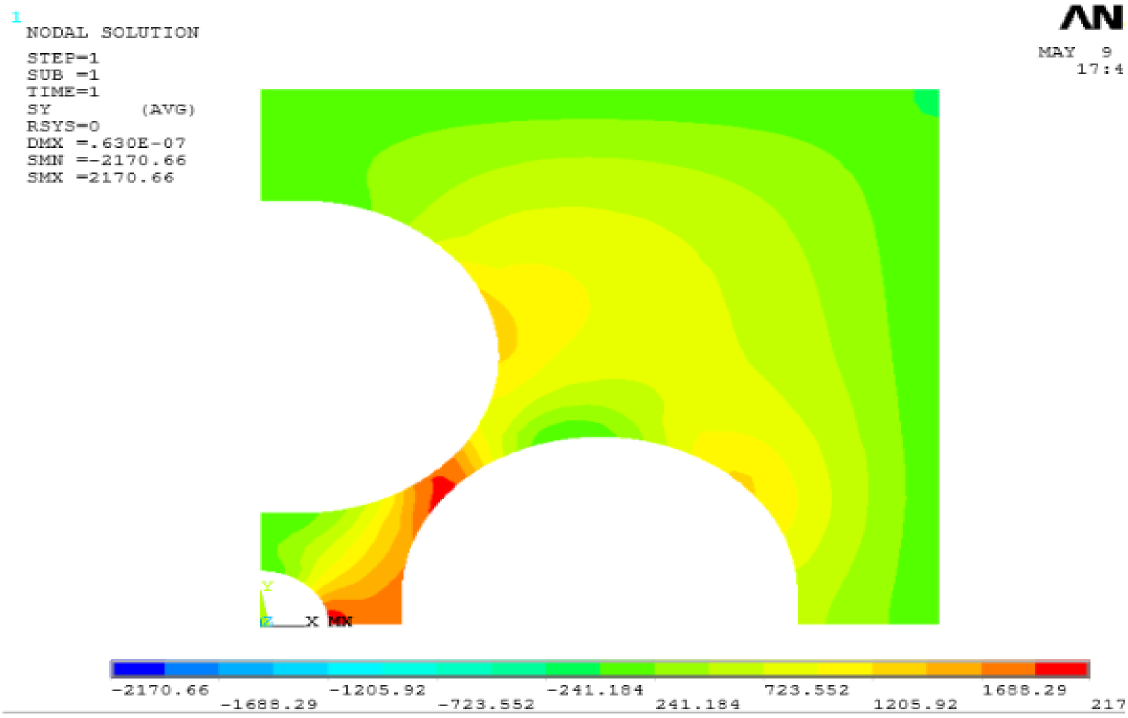


The analysis for reduction in stress concentration at the edges of main hole has been done for different center distance of main hole and auxiliary hole in model 2. It was very necessary to analyze, the influence of center distance of main hole and auxiliary holes on stress concentration for different stresses. Following figure shows the effect of X/D on % reduction in SCF (for σ_{x}, σ_{y}, τ_{xy}, σ_{eqv}) and % increase in U_z in model 'A' for $D_1/D=1$. This analysis has been done to find out the critical value of X/D, so that maximum reduction in SCF could be achieved. The X/D ration is varied from 1.1 to 3.3.



It is clear from the above graph that maximum reduction in SCF for σ_{x}, σ_{y}, and σ_{eqv} is achieved at X/D = 2.6, while maximum reduction in SCF for τ_{xy} is achieved at X/D=1.5. Since the effect of normal stress is more important than shear stress in this case i.e. X/D is selected as 2.5

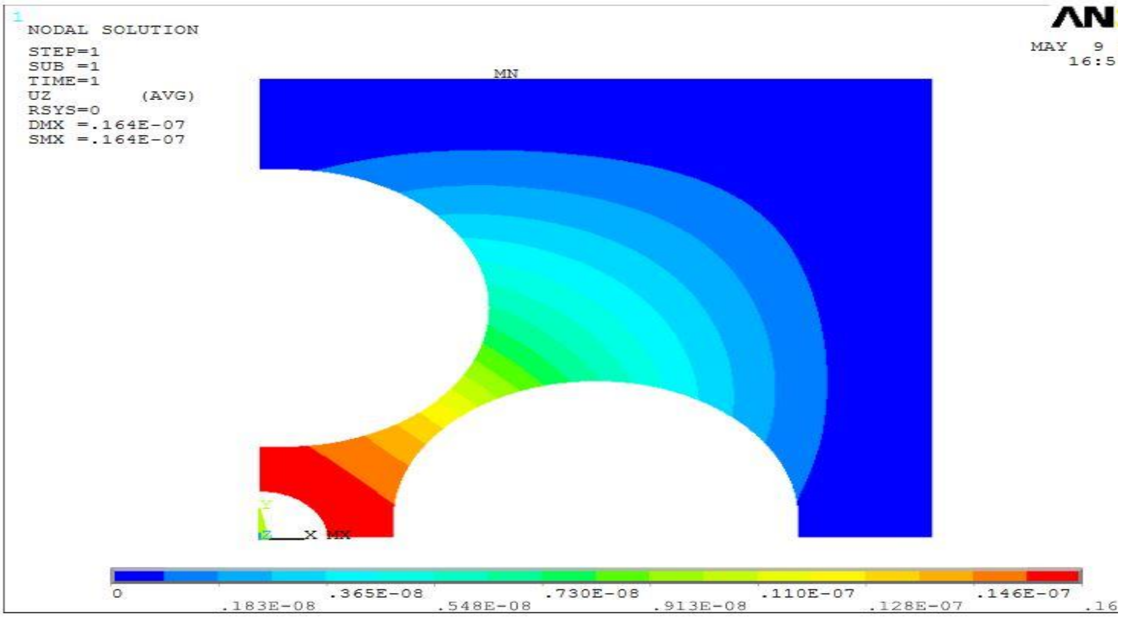
- **OPTIMIZATION OF ISOTROPIC SIMPLY SUPPORTED PLATE**



The L/D ratio is varied from 2.5 to 0.15 and for each L/D ratio, % reduction in SCF is achieved for different values of φ/D. Following observations can be made from above table. For L/D=2.50, increasing φ/D from 1.0 to 1.40 results in % reduction of SCF for σ_{x} and σ_{y} continuously increased from 24.5 to 44.37 and 24.07 to 45.82 % respectively. Increasing φ/D from 1.0 to 1.4 results in % reduction of SCF for τ_{xy} and σ_{eqv} continuously decreased from -5.01 to -16.44 and 6.44 to -3.00 % respectively.
For L/D = 2.50, maximum % reduction in SCF for σ_{x} and σ_{y} can be achieved as 44.37 and 45.82 % respectively. For L/D = 0.6 and φ/D = 2.90, maximum % reduction in SCF for σ_{x} and σ_{y} can be achieved as 57.19 and 57.61 % respectively, but, this is also accompanied by an increase in U_z of 4.58 % .
***Inference*:** In general, the maximum stress concentration for all stresses and maximum deflection in transverse direction is always occurred on hole boundary in a simply supported isotropic square plate with central circular hole under transverse static loading. The reduction in SCF depends on size and place of auxiliary holes and cavities in model 3. Three parameters X/D, L/D and φ/D in model 3 influence the reduction in SCF for all stresses. Maximum reduction in SCF for σ_{x} and σ_{y} is achieved as 57.19 and 57.61 % respectively. It has been seen that, when maximum reduction in SCF for σ_{x} and σ_{y} is achieved in model 3 for all sets of L/D and φ/D , this is also accompanied by an increase always in SCF for τ_{xy} and in U_z and sometimes in SCF for σ_{eqv}. Since the normal stresses in X and Y directions is always greater than shear stress and the values of U_z for all cases is very small, so slight increase in shear stress and in U_z can be neglected. For an isotropic plate, such reductions in maximum SCF improve fatigue life of component. It must be remembered that only three parameters X/D, L/D and φ/D in model 3 affect the reduction in SCF, and these should be selected as per requirement and convenience of designer.

- **OPTIMIZATION OF ISOTROPIC FIXED PLATE**

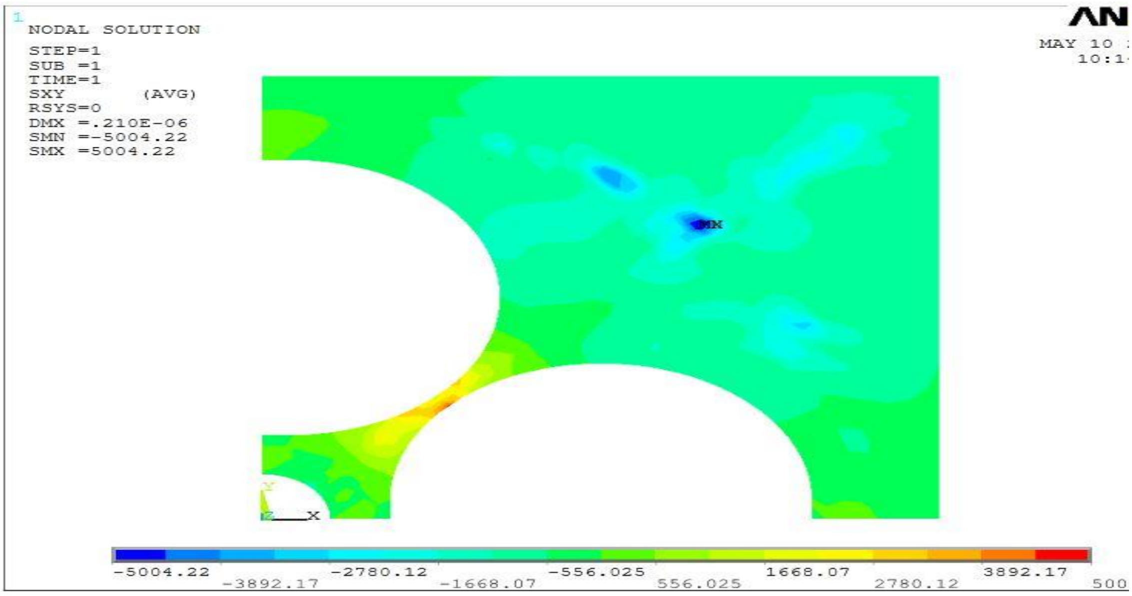
The percentage reduction in SCF (for σ_{x}, σ_{y}, σ_{eqv} and τ_{xy}) and percentage increase in U_z with different sets of L and ϕ in model 3 for X/D = 2.5. were analysed in ansys



Inference:In general, the maximum stress concentration for all stresses and maximum deflection in transverse direction is always occurred on hole boundary in a simply supported isotropic square plate with central circular hole under transverse static loading. The reduction in SCF depends on size and place of auxiliary holes and cavities in model 3. Three parameters X/D, L/D and φ/D in model 3 influence the reduction in SCF for all stresses. Maximum reduction in SCF for σ_{x} and σ_{y} is achieved as 51.38 and 51.45 % respectively. It has been seen that, when maximum reduction in SCF for σ_{x} and σ_{y} is achieved in model 3 for all sets of L/D and φ/D , this is also accompanied by an increase always in SCF for τ_{xy} and in U_z and sometimes in SCF for σ_{eqv}.

- **OPTIMIZATION OF ORTHOTROPIC SIMPLY SUPPORTED PLATE**

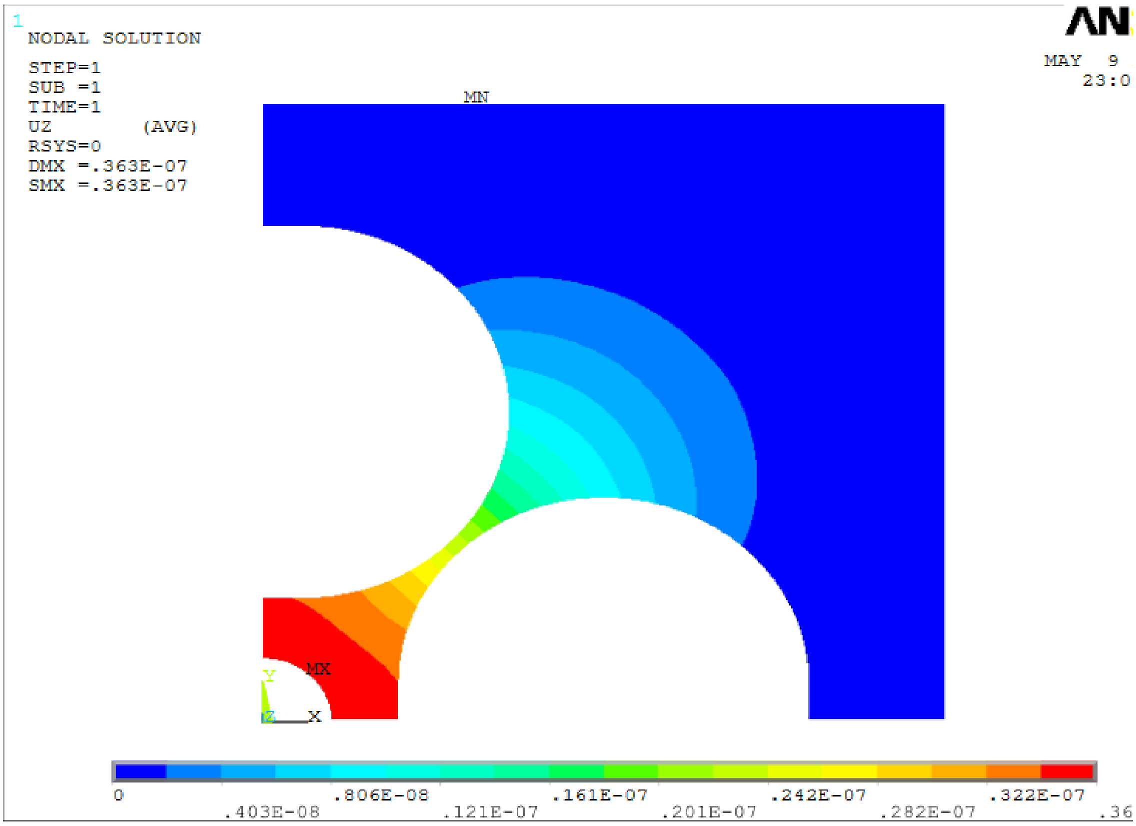
The percentage reduction in SCF (for σ_{x}, σ_{y}, σ_{eqv} and τ_{xy}) and percentage increase in U_z with different sets of L and ϕ in model 3 for X/D = 2.5.were analysed in ANSYS



Inference:In general, the maximum stress concentration for all stresses and maximum deflection in transverse direction is always occurred on hole boundary in a simply supported isotropic square plate with central circular hole under transverse static loading. The reduction in SCF depends on size and place of auxiliary holes and cavities in model 3. Three parameters X/D, L/D and φ/D in model 3 influence the reduction in SCF for all stresses. Maximum reduction in SCF for σ_x and σ_y is achieved as 30.65 and 29.29 % respectively. It has been seen that, when maximum reduction in SCF for σ_x and σ_y is achieved in model 3 for all sets of L/D and φ/D , this is also accompanied by an increase always in SCF for τ_{xy} and in U_z and sometimes in SCF for σ_{eqv}.

- **OPTIMIZATION OF ORTHOTROPIC FIXED PLATE**

The percentage reduction in SCF (for σ_{x}, σ_{y}, σ_{eqv} and τ_{xy}) and percentage increase in U_z with different sets of L and ϕ in model 3 for X/D = 2.5 were analysed in ANSYS



Inference:In general, the maximum stress concentration for all stresses and maximum deflection in transverse direction is always occurred on hole boundary in a simply supported isotropic square plate with central circular hole under transverse static loading. The reduction in SCF depends on size and place of auxiliary holes and cavities in model 3. Three parameters X/D, L/D and φ/D in model 3 influence the reduction in SCF for all stresses. Maximum reduction in SCF for σ_x and σ_y is achieved as 32.00 and 32.65 % respectively. It has been seen that, when maximum reduction in SCF for σ_x and σ_y is achieved in model 3 for all sets of L/D and φ/D , this is also accompanied by an increase always in SCF for τ_{xy} and in U_z and sometimes in SCF for σ_{eqv}.

Vibration analysis of plate with hole

FEM analysis was done using Ansys on a square plate of dimension 0.1 meter using mapped meshing with under self weight (ρ = 2660 Kg/m^3).The vibrational analysis was done by varying D/A ratio and using constant central hole diameter equals to 0.005 meter. element length of 0.001 meter

The following support conditions are used for analysis:-

1) Simply supported.
2) Fixed support.

ISOTROPIC PLATE
VIBRATIONAL ANALYSIS USING
D/A = 0.1, R= 0.005
Element length = 0.001

FIXED SUPPORT

MODE	FREQUENCY
1)	896.67
2)	3267.2
3)	3280.8

SIMPLY SUPPORTED

MODE	FREQUENCY
1)	488.33
2)	2446.0
3)	2454.6

Similarly, the same analysis was done for orthotropic square plate with central hole under same D/A ratio and same two support conditions.

ORTHOTROPIC PLATE
VIBRATIONAL ANALYSIS USING
D/A = 0.1, R= 0.005
Element length = 0.001

FIXED SUPPORT

MODE	FREQUENCY
1)	1229.7
2)	3573.7
3)	4191.3

SIMPLY SUPPORTED

MODE	FREQUENCY
1)	346.54
2)	2281.5
3)	3061.2

Now same analysis with D/A=0.2

VIBRATIONAL ANALYSIS
D/A = 0.2, R= 0.01
Element length = 0.001

ISOTROPIC PLATE

FIXED SUPPORT

MODE	FREQUENCY
1)	910.49
2)	3198.3
3)	3460.1

SIMPLY SUPPORTED

MODE.	FREQUENCY
1)	480.23
2)	2408.8
3)	2539.8

ORTHOTROPIC PLATE

FIXED SUPPORT

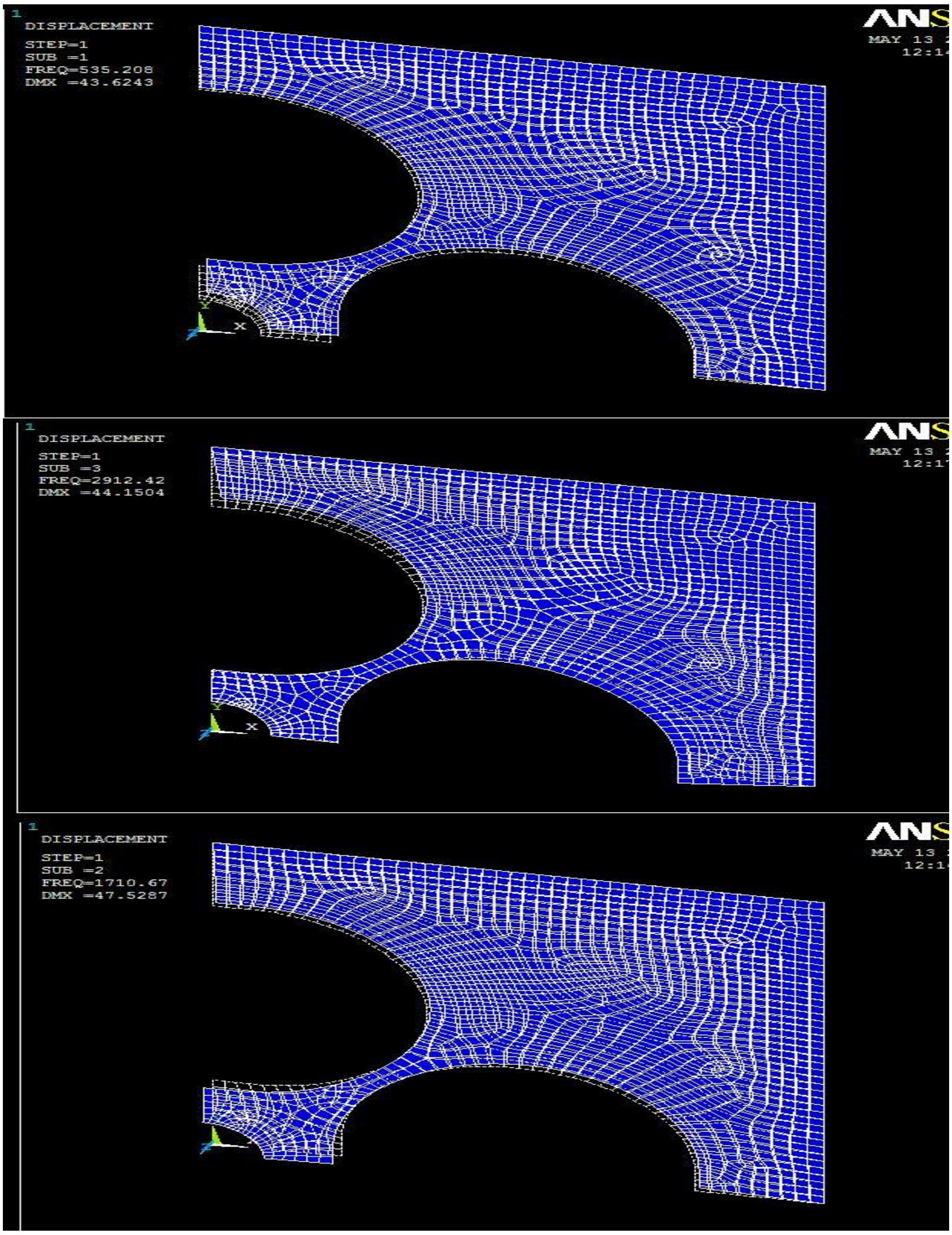
MODE	FREQUENCY
1)	1322.4
2)	3113
3)	4356.2

SIMPLY SUPPORTED

MODE	FREQUENCY
1)	339.95
2)	2034.4
3)	3174.3

Effects of shape optimization on vibration of plate

While optimizing the shape of plate following readings of frequency (3 modes) are analyzed for plate. Keeping X/D = 2.5 , and varying the parameters L/D and φ/D in model 3 under self weight.
Further the following observation were carried out through ANSYS for
X/D=2.5 mm and varying other parameters L/D, φ/D for isotropic plate with simply supported.
X/D=2.5 mm and varying other parameters L/D, φ/D for isotropic plate with Fixed supported
X/D=2.5 mm and varying other parameters L/D, φ/D for orthotropic plate with simply supported.



RESULTS & CONCLUSION

- Comparison between isotropic solid plate with hole and optimized plate.

Fixed support.

Condition	Solid plate with hole	Optimized plate
% reduction in SCF of σ_X	-	51.38
% reduction in SCF of σ_y	-	51.45
1^{st} mode frequency	896.67	970.83
2^{nd} mode frequency	3267.2	2746.3
3^{rd} mode frequency	3280.8	5644.7

Simply support.

Condition	Solid plate with hole	Optimized plate

% reduction in SCF of σ_x	-	57.19
% reduction in SCF of σ_y	-	57.61
1^{st} mode frequency	488.30	535.21
2^{nd} mode frequency	2446.0	1710.7
3^{rd} mode frequency	2454.6	2912.2

- Comparison between orthotropic solid plate with hole and optimized plate

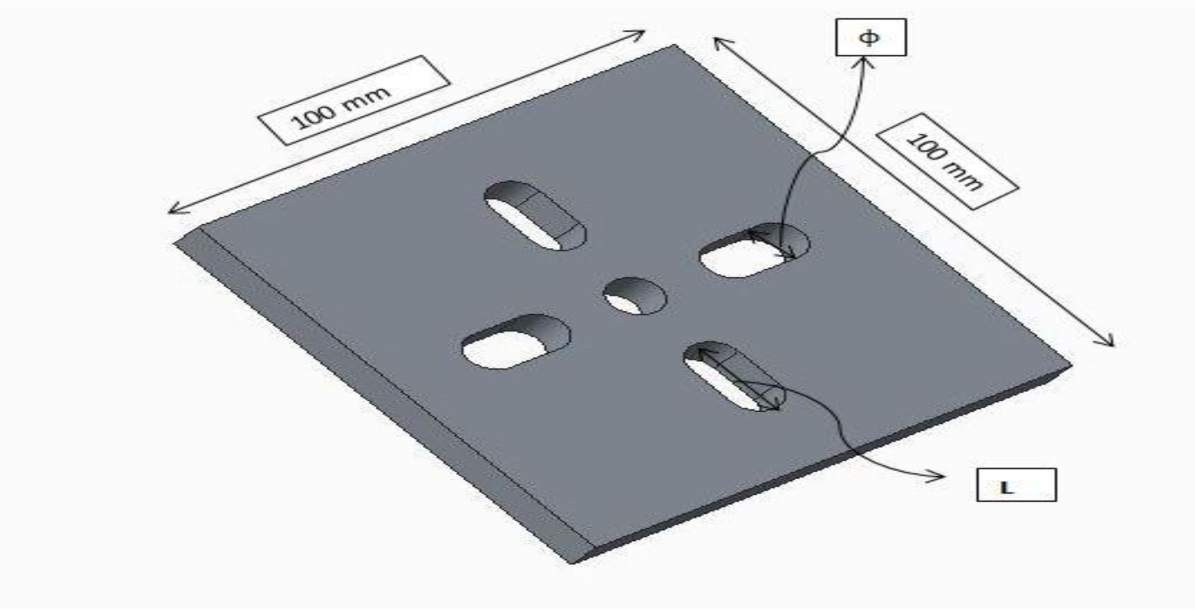
Fixed support.

Condition	Solid plate with hole	Optimized plate
% reduction in SCF of σ_x	-	32.00
% reduction in SCF of σ_y	-	32.65
1^{st} mode frequency	1229.7	696.94
2^{nd} mode frequency	3573.7	2609.2
3^{rd} mode frequency	4191.3	5675.2

Simply support.

Condition	Solid plate with hole	Optimized plate
% reduction in SCF of σ_X	-	30.65
% reduction in SCF of σ_y	-	29.29
1^{st} mode frequency	346.54	348.25
2^{nd} mode frequency	2281.5	1131.0
3^{rd} mode frequency	3061.2	2116.1

CONCLUSION



In general, the maximum stress concentration for all stresses and maximum deflection in transverse direction is always occurred on hole boundary in a simply supported isotropic square plate with central circular hole under transverse static loading. The reduction in SCF depends on size and place of auxiliary holes and cavities in model 3. Three parameters X/D, L/D and φ/D in model 3 influence the reduction in SCF for all stresses.

It has been seen that-

- For isotropic simply supported at L/D = 0.6 and φ/D = 2.9, minimum stresses are found.
- For isotropic Fixed support at L/D = 0.4 and φ/D = 3.0, minimum stresses are found.
- For orthotropic simply supported at L/D = 0.4 and φ/D = 3.1, minimum stresses are found..
- For orthotropic Fixed support at L/D = 0.6 and φ/D = 3.0, minimum stresses are found..

Hence, above optimized shapes can be used as a replacement of plate with hole for minimum value of stresses.

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