International Journal of Research in Advent Technology, Special Issue, August 2018
E-ISSN: 2321-9637
International Conference on "Topical Transcends in Science, Technology and Management" (ICTTSTM-2018)
Available online at www.ijrat.org

Mixed Convection Flow and Heat Transfer of Micropolar Fluid in a Vertical Channel with Source Or Sink And Thermal Radiation Effect

PATIL MALLIKARJUN B
Department of Studies & Research in Mathematics
Tumkur University
Tumkur, India
patilmallikarjun@tumkuruniversity.in

KAVITHA L
Department of Mathematics
REVA University,
Bengaluru, India
lkavitha29@gmail.com

ABSTRACT

The analytical solutions of fully developed mixed convection flow of a micropolar fluid with heat source or heat sink and thermal radiation effect in a vertical channel has been presented by perturbation series method. The two boundaries of the channel are kept either at equal or at different temperatures for the left-right walls of the channel and these are isothermal-isothermal, isoflux-isothermal and isothermal-isoflux cases. The effect of material parameter is to decrease velocity and microrotation velocity. Heat source promotes the flow and heat sink suppresses the flow. The radiation effect also enhances the velocity and microrotationvelocity.

Keywords—mixedconvection, micropolarfluids**,**perturbation series method, micro rotation velocity**,** Thermal radiation.

I. INTRODUCTION

Micro polar fluids consists of randomly oriented molecules. Each volume element of the micro polar fluid exhibits translation as well as rotation. The analysis of physical problems in these Non Newtonian fluids has gained popularity and has revealed several interesting phenomena, which are not found in Newtonian fluids. The theory of micropolar fluids, which takes into account the inertial characteristics of the substructure particles, which are allowed to undergo rotation, has been proposed by Eringen[1] . This theory can be used to explain the flow of colloidal fluids, liquid crystals, animal blood, etc. He extended the micropolar fluid theory and developed the theory of thermomicropolar fluids [2]. The problem of laminar flow of a viscous incompressible electrically conducting micro polar fluid over a semi-infinite vertical moving porous plate in the presence of transverse magnetic field has been analyzed by Youn J Kim and J C Lee [3] where it is shown that there exists completely oscillating behavior of the velocity distribution in the boundary layer . The study of combined effect of the internal heat generation and magnetic field on free convection with mass transfer flow of a viscous electrically conducting micropolar fluid occupying semi-infinite region of the space has been carried out by M F El-Amin [4]. Mixed convection flow and heat transfer of micropolar fluid in a vertical channel with heat generation or heat absorption has been studied by Patil Mallikarjun B [5] and further extended his work with symmetric and asymmetric wall heating conditions [6]. Mixed convection of micropolar fluid in a vertical channel with boundary conditions of third kind has been discussed by J C Umavathi and Jaweriya Sultana [7] and further flow through vertical double passage channel has been analyzed by J C Umavathi [8].Fully developed free convection flow of micropolar and viscous fluids in a vertical channel is completely derived by J Prathap Kumar et al [9]. Natural convection flow of micropolar fluid along a vertical and a semi infinite plate embedded in a porous medium has been considered by et al I A Hassanien [10] in which it is studied that unlike the Newtonian fluids, micropolar fluids enhances skin friction when embedded in a saturated porous medium.

With reference to the above mentioned applications it is the aim of this paper to study the mixed convection flow and heat transfer of micropolar fluid in a vertical channel with heat source or sink and thermal radiation effect. This will be done for two different thermal boundary conditions which are isoflux-isothermal and isothermal- isoflux cases.

II MATHEMATICAL FORMULATION

Consider a steady, laminar, micropolar fully developed flow with heat source or heat sink in a parallel-plate vertical channel. The origin of the axes is such that the channel walls are at positions $Y = -L/2$ and $Y = L/2$. The evaluation of the gravitational body force is by using Oberbeck-Boussinissq approximation. The equation of state is

$$\rho = \rho_0 \left(1 - \beta \left(T - T_0\right)\right) \tag{1}$$

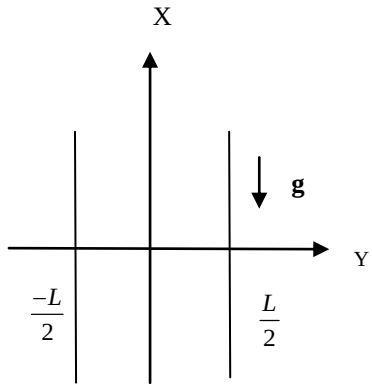


Fig1: Drawing of the parallel plates and rectangular coordinates

For the fully developed flow $\partial U / \partial X = 0$, since the velocity field U is solenoidal, one obtains $\partial V / \partial Y = 0.$ As a consequence, the velocity component V is constant in any channel section and is equal to zero at the channel walls, so that V must be vanishing at any position. The Y-momentum balance equation can be expressed as $\partial P / \partial Y = 0$ where $P = p + \rho_0 \, g \, X$ is the difference between the pressure and the hydrostatic pressure. Therefore P depends only on X and the Y momentum balance equation is given by

$$\rho_0 g \beta (T - T_0) - \frac{dP}{dX} + (\mu + k)\frac{d^2 u}{dy^2} + k\frac{d\overline{N}}{dY} = 0 \tag{2}$$

$$\gamma \frac{d^2 \overline{N}}{dY^2} - 2k\overline{N} - k\frac{dU}{dY} = 0 \tag{3}$$

The walls of the channel are isothermal. The boundary temperatures are T_1 at $Y = -L/2$ and T_2 at $Y = L/2$ with $T_2 \geq T_1$. These conditions are compatible with the momentum equation (2) only when dP/dX is independent of X. Hence, there exists a constant A such that

$$\frac{dP}{dX} = A \tag{4}$$

The energy balance equation including the heat source or heat sink coefficient becomes,

$$K\frac{d^2 T}{dY^2} \pm Q(T - T_0) - \frac{1}{\rho_0 C_P}\frac{dq_R}{dY} = 0 \tag{5}$$

The corresponding boundary conditions are

$$U = \overline{N} = 0 \text{ at } Y = \pm\frac{L}{2}$$

$$T = T_1 \text{ at } Y = -\frac{L}{2}$$

$$T = T_2 \text{ at } Y = \frac{L}{2} \tag{6}$$

The non-dimensional quantities are,

$$u = \frac{U}{U_0}; \qquad y = \frac{Y}{D}; \;\; j = D^2 \;;\; k = \mu K_1 \;; \overline{N} = \frac{U_0}{D} N; \quad U_0 = -\frac{AD^2}{48\mu}$$

$$\mathrm{Re} = \frac{U_0 D}{\nu}; \;\; \mathrm{Pr} = \frac{\nu}{\alpha} \;\; ; \gamma = (\mu + k/2) j \;;\; \phi = \frac{QD^2}{K}; \quad T_0 = \frac{T_1 + T_2}{2}$$

$$Gr = \frac{g\,\beta\,\Delta T\, D^3}{\nu^2} \;;\; \lambda = \frac{Gr}{\mathrm{Re}}; \;\; R_T = \frac{T_2 - T_1}{\Delta T} \;;\; \theta = \frac{T - T_0}{\Delta T}; \; \frac{dq_R}{dY} = c(T - T_0) \tag{7}$$

The diameters of the channel are given by $D = 2L$ and U_0, T_0 are the corresponding reference velocity and reference temperature. The temperature difference ΔT is given by

$$\Delta T = T_2 - T_1 \text{ if } T_1 < T_2 \quad \text{or} \quad \text{by}$$

$$\Delta \mathrm{T} = \frac{\nu^2}{C_p D^2} \text{ if } T_1 = T_2 \tag{8}$$

Using the non-dimensional quantities from equation (7) in equations (2), (3), (5) and (6) reduces to

$$(1+K)\frac{d^2u}{dy^2} + K\frac{dN}{dy} + \lambda\theta + 48 = 0 \tag{9}$$

$$\left(1 + \frac{K}{2}\right)\frac{d^2N}{dy^2} - 2KN - K\frac{du}{dy} = 0 \tag{10}$$

$$\frac{d^2\theta}{dy^2} = \mp\phi\theta + F^2\theta \tag{11}$$

$$u = N = 0 \qquad \text{at } y = \pm\frac{1}{4} \tag{12}$$

$$\theta = \pm\frac{R_T}{2} \qquad \text{at } y = \pm\frac{1}{4} \tag{13}$$

The solution of temperature field is obtained from equation (11) by using the boundary condition (13) and is given by

$$\theta = \frac{R_T}{2}\frac{Sin\sqrt{\phi - F^2}\,y}{Sin\left(\dfrac{\sqrt{\phi - F^2}}{4}\right)} \tag{14}$$

for the case of heat generation and

$$\theta = \frac{R_T}{2}\frac{Sinh\sqrt{\phi + F^2}\,y}{Sinh\left(\dfrac{\sqrt{\phi + F^2}}{4}\right)} \tag{15}$$

for the case of heat absorption. Integrating the equation (9) with respect to y and by using the equations (14) and (15) we get

$$\frac{du}{dy} = -\frac{K}{(1+K)}N + \frac{\lambda R_T}{2\sqrt{\phi - F^2}(1+K)Sin\left(\frac{\sqrt{\phi - F^2}}{4}\right)}Cos\sqrt{\phi - F^2}\,y - \frac{48}{(1+K)}y - \frac{A}{(1+K)} \tag{16}$$

for the case of heat generation and

$$\frac{du}{dy} = -\frac{K}{(1+K)}N - \frac{\lambda R_T}{2\sqrt{\phi + F^2}(1+K)Sinh\left(\frac{\sqrt{\phi + F^2}}{4}\right)}Cosh\sqrt{\phi + F^2}\,y - \frac{48}{(1+K)}y - \frac{A}{(1+K)} \tag{17}$$

for the case of heat absorption.

Substituting the equations (16) and (17) in equation (10) we get

$$\frac{d^2N}{dy^2} - \tau N = \frac{\lambda K R_T}{\sqrt{\phi - F^2}(1+K)(2+K)Sin\left(\frac{\sqrt{\phi - F^2}}{4}\right)}Cos\sqrt{\phi - F^2}\,y - \frac{48\tau}{(2+K)}y - \frac{A\tau}{(2+K)}$$

for the case of heat generation and (18)

$$\frac{d^2N}{dy^2} - \tau N = \frac{-\lambda K R_T}{\sqrt{\phi + F^2}(1+K)(2+K)Sinh\left(\frac{\sqrt{\phi + F^2}}{4}\right)}Cosh\sqrt{\phi + F^2}\,y - \frac{48\tau}{(2+K)}y - \frac{A\tau}{(2+K)}$$

for the case of heat absorption. (19)

By using the boundary condition given in equation (12), the solutions of above equations (18) and (19) are

$$N = C_3 Cosh\sqrt{\tau}\,y + C_4 Sinh\sqrt{\tau}\,y + l_1 Cos\sqrt{\phi - F^2}\,y + l_2 y + l_3 \tag{20}$$

for the case of heat generation and

$$N = C_3 Cosh\sqrt{\tau}\,y + C_4 Sinh\sqrt{\tau}\,y + l_1 Cosh\sqrt{\phi + F^2}\,y + l_2 y + l_3 \tag{21}$$

for the case of heat absorption.

The solutions of equation (9) along with boundary condition (12) using equations (20) and (21) is

$$u = -\frac{1}{(1+K)}\left(\begin{array}{l}\frac{C_3K}{\sqrt{\tau}}Sinh\sqrt{\tau}y + \frac{C_4K}{\sqrt{\tau}}Cosh\sqrt{\tau}y + \left(\frac{l_1K}{\sqrt{\phi-F^2}} - \frac{\lambda R_T}{2(\phi-F^2)Sin\left(\frac{\sqrt{\phi-F^2}}{4}\right)}\right) \\ Sin\sqrt{\phi-F^2}\,y + (48+l_2K)\frac{y^2}{2} + C_5y + C_6\end{array}\right) \tag{22}$$

for the case of heat generation and

$$u = -\frac{1}{(1+K)}\left(\begin{array}{l}\frac{C_3K}{\sqrt{\tau}}Sinh\sqrt{\tau}y + \frac{C_4K}{\sqrt{\tau}}Cosh\sqrt{\tau}y + \left(\frac{l_1K}{\sqrt{\phi+F^2}} + \frac{\lambda R_T}{2(\phi+F^2)Sinh\left(\frac{\sqrt{\phi+F^2}}{4}\right)}\right) \\ Sinh\sqrt{\phi+F^2}\,y + (48+l_2K)\frac{y^2}{2} + C_5y + C_6\end{array}\right) \tag{23}$$

for the case of heat absorption.

Isoflux-isothermal $(q_1 - T_2)$ walls

The dimensional forms of the thermal boundary conditions for the channel walls are

$$q_1 = -K\frac{dT}{dY} \text{ at } Y = -\frac{L}{2}$$

$$T = T_2 \text{ at } Y = \frac{L}{2} \tag{24}$$

The dimensionless form of equation (24) can be obtained by using the equation (7) with $\Delta T = q_1 D / K$ to give

$$\frac{d\theta}{dy} = -1 \text{ at } y = -\frac{1}{4}$$

$$\theta = \frac{R_{qt}}{2} \qquad \text{at } y = \frac{1}{4} \tag{25}$$

where $R_{qt} = (T_2 - T_0)/\Delta T$ is the thermal ratio parameter for the isoflux-isothermal case. The solutions of temperature, microrotation velocity and velocity fields are obtained from equations (9) to (811) by using the boundary conditions (12) and (25) which are given below.

$$\theta = C_1Cos\sqrt{\phi-F^2}\,y + C_2Sin\sqrt{\phi-F^2}\,y$$

$$N = C_3Cosh\sqrt{\tau}y + C_4Sinh\sqrt{\tau}y + l_1Sin\sqrt{\phi-F^2}\,y + l_2Cos\sqrt{\phi-F^2}\,y + l_3y + l_4$$

$$u=-\frac{1}{(1+K)}\left(\begin{array}{l}\frac{C_3K}{\sqrt{\tau}}Sinh\sqrt{\tau}y+\frac{C_4K}{\sqrt{\tau}}Cosh\sqrt{\tau}y-\left(\frac{l_1K}{\sqrt{\phi-F^2}}+\frac{\lambda C_1}{\phi-F^2}\right)Cos\sqrt{\phi-F^2}y+\\ \left(\frac{l_2K}{\sqrt{\phi-F^2}}-\frac{\lambda C_2}{\phi-F^2}\right)Sin\sqrt{\phi-F^2}y+(48+l_3K)\frac{y^2}{2}+C_5y+C_6\end{array}\right)$$

for the case of heat generation and (26)

$$\theta=C_1Cosh\sqrt{\phi+F^2}y+C_2Sinh\sqrt{\phi+F^2}y$$

$$N=C_3Cosh\sqrt{\tau}y+C_4Sinh\sqrt{\tau}y+l_1Sinh\sqrt{\phi+F^2}y+l_2Cosh\sqrt{\phi+F^2}y+l_3y+l_4$$

$$u=-\frac{1}{(1+K)}\left(\begin{array}{l}\frac{C_3K}{\sqrt{\tau}}Sinh\sqrt{\tau}y+\frac{C_4K}{\sqrt{\tau}}Cosh\sqrt{\tau}y+\left(\frac{l_1K}{\sqrt{\phi+F^2}}+\frac{\lambda C_1}{\phi+F^2}\right)Cosh\sqrt{\phi+F^2}y+\\ \left(\frac{l_2K}{\sqrt{\phi+F^2}}+\frac{\lambda C_2}{\phi+F^2}\right)Sinh\sqrt{\phi+F^2}y+(48+l_3K)\frac{y^2}{2}+C_5y+C_6\end{array}\right)$$

for the case of heat absorption. (27)

Isothermal-isoflux (T_1-q_2) walls

The results are same as we obtained in the case of isoflux-isothermal case with constant $-C_1$ replaced by C_1.

III.RESULTS AND DISCUSSION

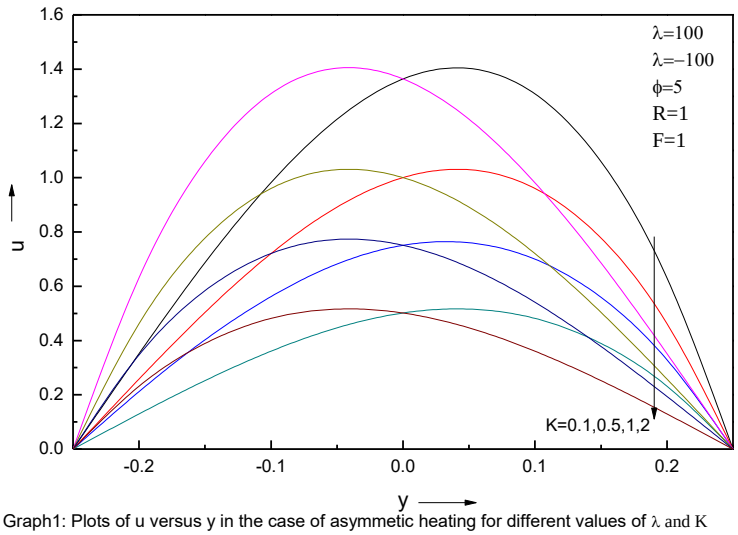
An analytical solution for flow and heat transfer of micropolar fluid in a vertical channel in the presence of source or sink and thermal radiation effect is analyzed. Solutions are given by equations (14), (15) and (20) to (23) and are evaluated numerically for different values of the governing parameters and the results are presented diagrammatically in GRAPHS 1 to 9.

The effect of material parameter K on the velocity profiles is shown in Graphs 1 and 3 for $\lambda=\pm100, \lambda=\pm500$ respectively.. As K increases velocity decreases for all the values of λ. It is interesting to note that for $\lambda=\pm500$ there is a flow reversal closed to the boundary at $y=\pm1/4$. In the reversal flow also, velocity decreases as material parameter K increases. The effect of material parameter K on microrotation velocity *N* is shown in Graphs 2 and 4 for $\lambda=\pm100$, $\lambda=\pm500$. There is flow reversal which is significant for $\lambda=\pm100$ compare to $\lambda=\pm500$ at both the walls $y=\pm1/4$. The magnitude of reversal is large for $\lambda=100$ in the region *–0.25* to *0* where as, the magnitude of *N* is large in the region $y=0$ to *0.25*. The effect of material parameter K is to decrease microrotation velocity in the region *–0.25* to *0* and to increase *N* from *0* to *0.25* for $\lambda=\pm100$. For $\lambda=500$, the effect of K is to increase microrotation velocity for $N>0$ and to decrease for $N<0$ only at $y=-0.25$. The effect of K for $\lambda=-500$ is to increase the microrotation velocity $N<0$ and to increase for $N>0$ only at $y=0.25$.

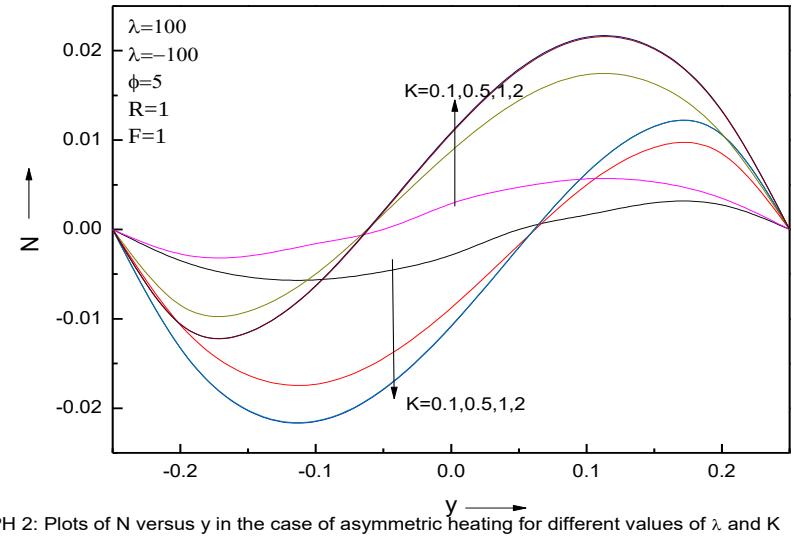
Effect of radiation parameter F on velocity for isoflux-isothermal wall conditions is shown in Graph 5 for K=1 and K=2 (both for micropolar fluids) the effect of ϕ is to increase the velocity for Non- Newtonian micropolar fluid but the magnitude of velocity is large for Newtonian fluid. The effect of F on microrotation velocity increases for $\lambda = 500$ and decreases for $\lambda = -500$ as seen in Graph 6.

The effect of radiation parameter F and K on velocity and microrotation velocity for isoflux-isothermal and isothermal-isoflux wall conditions are shown in Graphs 7 ,8 and 9. For micropolar fluids, effects of F is to increase velocity for $\lambda = 500$ and decrease velocity for $\lambda = -500$ as seen in Graph 7. Where as, there is an opposite effect of F and K for isoflux-isothermal case. That is the effect of F for micropolar fluids is to increase the velocity for $\lambda = -500$ and decrease the velocity for $\lambda = 500$. Thus the effect of F on microrotation velocity for isoflux-isothermal wall conditions is shown in Graph 8. The effect of F is to increase the microrotation velocity N for $\lambda = \pm 500$. It is also noticed that there is a flow separation at $N = 0$. The effect of F is same as that of isoflux-isothermal wall conditions except the profiles are interchanged for $\lambda = \pm 500$.

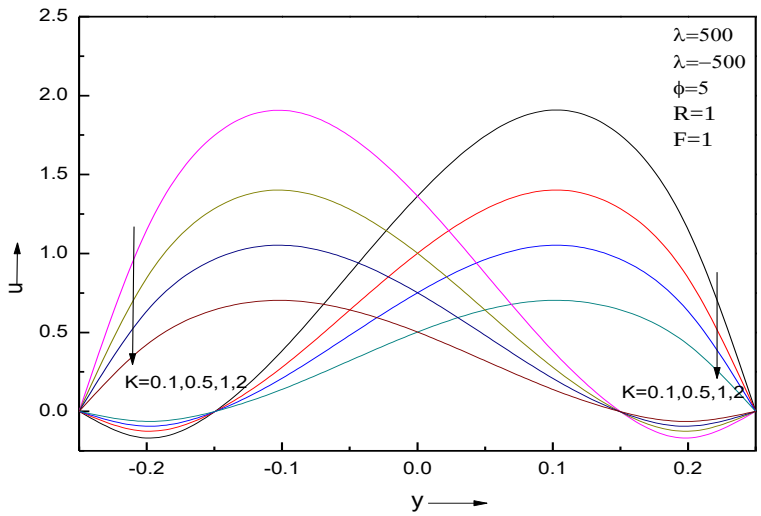
The effect of F is to decrease the velocity for micropolar fluid for $\lambda = 500$ and increases the velocity for $\lambda = -500$ for isoflux-isothermal wall conditions as seen in Graph 9.



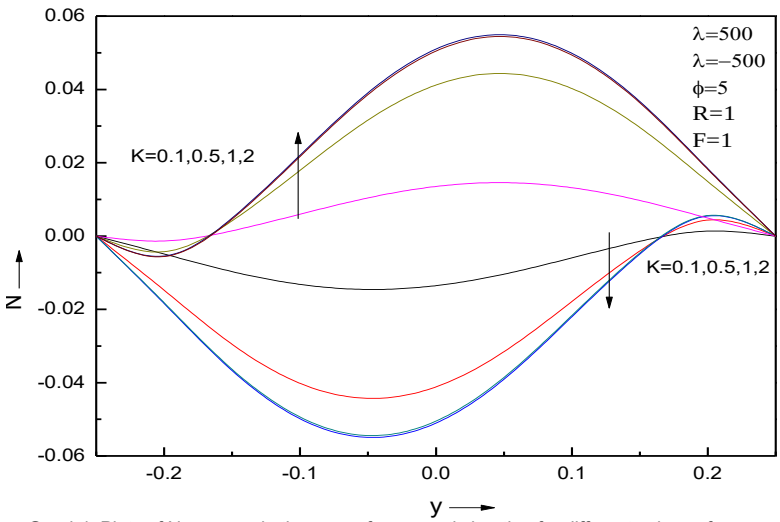
Graph1: Plots of u versus y in the case of asymmetic heating for different values of λ and K



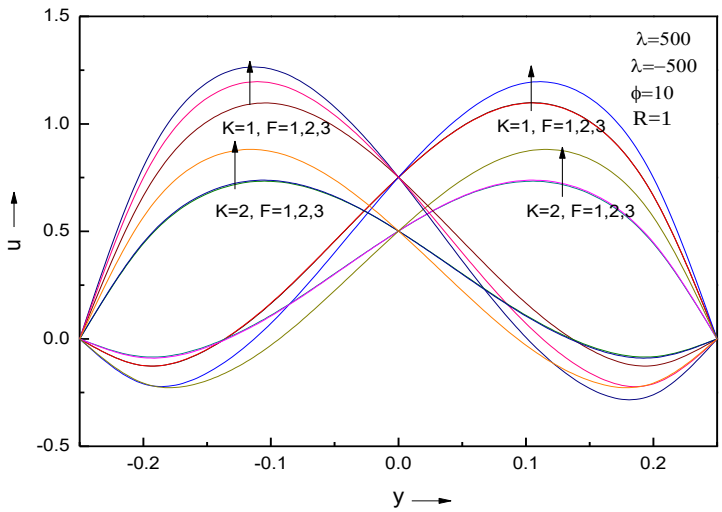
GRAPH 2: Plots of N versus y in the case of asymmetric heating for different values of λ and K



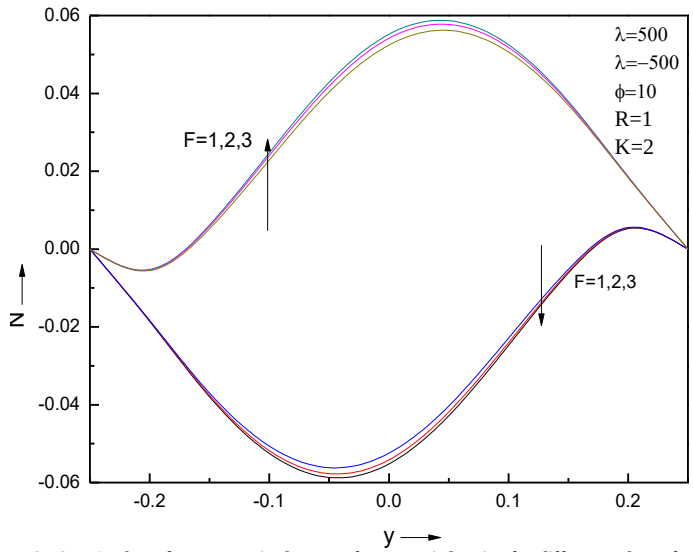
Graph3: Plots of u versus y in the case of asymmetic heating for different values of λ and K



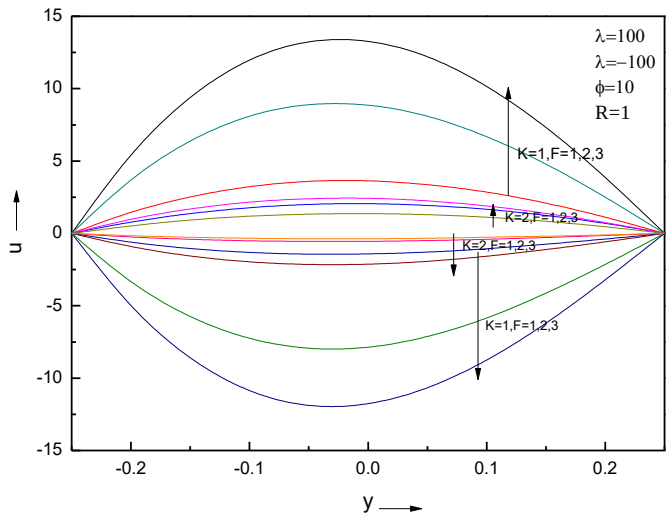
Graph4: Plots of N versus y in the case of asymmetric heating for different values of λ and K



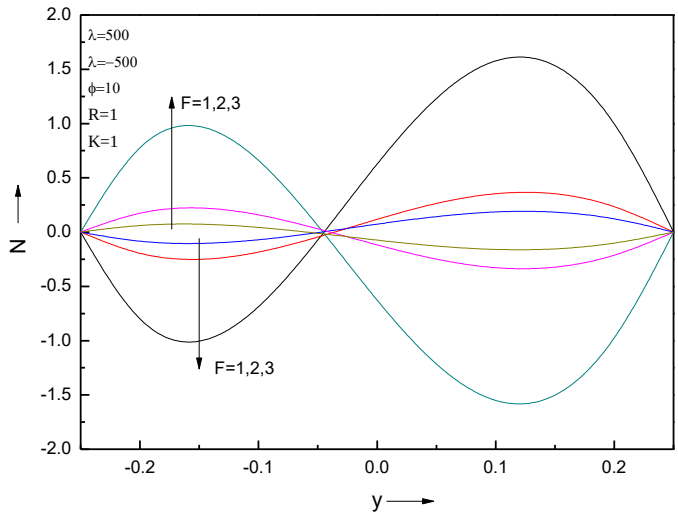
GRAPH5: Plots of u versus y in the case of asymmetric heating for different values of radiaation parameter F and K



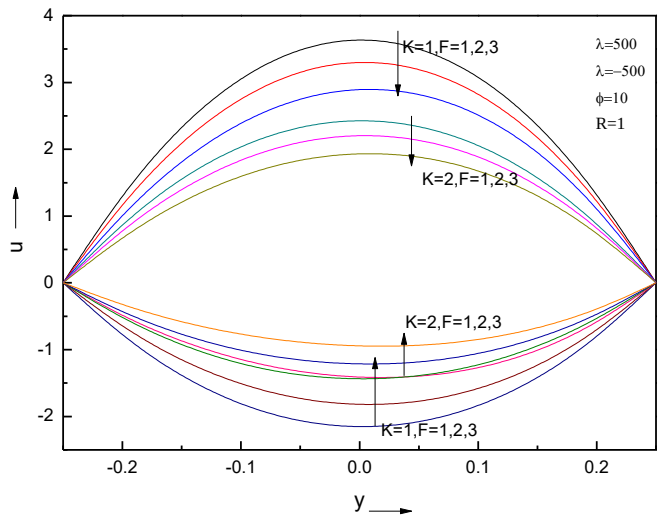
GRAPH6: Plots of N versus y in the case of asymmetic heating for different values of radiation parameter F



GRAPH7: Plots of u versus y for different values of K and radiation parameter F for Isoflux-Isothermal case



GRAPH8: Plots of N versus y for different values of radiation parameter F for the Isoflux-Isothermal case



GRAPH9: Plots of u versus y for different values of Radiation parameter F and K forisoflux-isothermal case

REFERENCES

[1]. A C Eringen, J. Math.Mech,16, 1964,1.
[2]. A C Eringen, J. Math Anal. Appl.38 ,1972,480.
[3]. Youn.J.Kim, J. C. Lee ,"Analytical studies on MHD Oscillatory flow of a micropolar fluid over a vertical porous plate", in Surfaceandcoating Technology, vol.171,pp.187-193,2003.
[4]. M. F.El-Amin," Combined effect of internal heat generation and magnetic field on free convection and mass transfer flow in a micropolar fluid with constant suction" in Journal of Magnetism and Magnetic materials, vol.270,pp.130-135,2004
[5]. Patil Mallikarjun B, "Flow and heat transfer of micropolarfluid in a vertical channel with heat generation or heat absorption" in International journal of Mathematical Education , vol.3,pp.1-13,2013.
[6]. Patil Mallikarjun B, Chandrali Baishya,"Mixed convection flow and heat transfer of micropolar fluid in a vertical channel with symmetric and asymmetric wall heating conditions" in International journal of Engineering research and applications,vol.6,pp.7-18,2016

[7]. J. C. Umavathi, Jaweriya Sultana, "Mixed Convection of a micropolar fluid in a vertical channel with bundary conditions of third kind" in International journal of Engineering Science and Technology, vol.3,pp.213-224,2011.

[8]. J. C. Umavathi, "Mixed Convection of micropolar fluid in a vertical double passage channel" in International journal of Engineering Science and Technology, vol.3,pp.197-209,2011.

[9]. J. Prathap Kumar, J. C. Umavathi, A.J. Chamkha ,I. Pop, "Fully developed free convective flow of micropolar and viscous fluids in a vertical channel" in Applications of Mathematical Modelling, vol.34, pp.1175-1186,2010 .

[10]. A. Hassanien, A.H.Essawy, N. M. Moursy, "Natural convection flow of micropolar fluids from a permeable uniform heat flux surface in porous medium" in Applied Mathematics and Modelling, vol.152,pp.323-335,2004.