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CFD Analysis on Convergent Nozzle

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Abstract: Nozzle is a mechanical device used to increase the velocity by the use of pressure energy and enthalpy of the fluid. The nozzles are used in subsonic and supersonic velocity applications and it is important to know the behavior of the flow in nozzles for the effective design. In this analysis convergent nozzle is considered and mainly focused on the effect of back pressure on the performance of the nozzle. When inlet and back pressures are equal flow is not possible and with the decrease of back pressure mass flow rate and Mach number increases. At the end results compared with theoretical values.

Key Words: Nozzle, subsonic velocity, supersonic velocity, flow rate, Mach number etc.

1. INTRODUCTION

Nozzle

Nozzle is a mechanical device of varying cross sectional area and used to

1. Increase the velocity with the expense of pressure in incompressible fluids.
2. Increase the velocity with the expense of pressure, enthalpy in compressible fluids.

1 Types Of Nozzles

Depending on the shape, nozzles can be divided into two types.

1. Convergent nozzle
2. Convergent – Divergent nozzle

1.1.1 Convergent Nozzle

In this type of nozzle, cross sectional area is continuously decreasing. Generally this type of nozzle used where subsonic speeds are required. Maximum Mach number obtained by the use of convergent nozzle is 1.0.

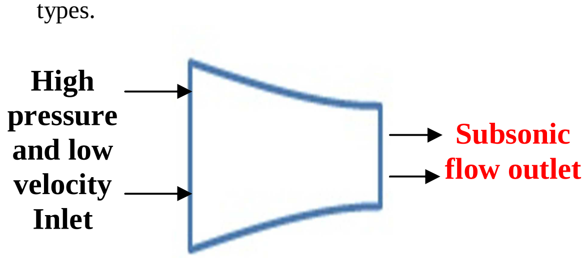


Fig 1.1: Convergent Nozzle

Fig 1.2: Convergent – Divergent Nozzle

1.1.2 Convergent – Divergent Nozzle

This nozzle is extension of convergent nozzle. Divergent portion is added at the end of convergent section to get the supersonic speeds.

1.2 CFD

CFD is a tool which is used to convert the partial differential equations into algebraic equations. With the advancement of computers solving complex flow problems made easy. Results obtained by the analysis are more accurate and also reduces time for solving. Analyzing problems by using experiments leads to high cost and time taken is more with the use of analytical approaches. To overcome these difficulties usage of software increased in the recent years. It also reduces complexity for designing the models and avoids the rework. In this analysis different techniques are used for different analyses to determine the nozzle performance effected by various parameters.

2. ANALYSIS OF NOZZLE FLOW

2.1 Theoretical Analysis

2.1.1 Governing equations of fluid flow:

1. Conservation of mass
2. Conservation of momentum
3. Conservation of energy

Conservation of mass

Governing equation of conservation of mass is

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

For steady state, 2-dimensional analysis equation can be reduced as follows

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$$

Conservation of momentum

Governing equations of Conservation of momentum are as follows

X- Direction

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}+\frac{\partial(\rho uw)}{\partial z}=-\frac{\partial p}{\partial x}+\frac{1}{R_e}\left[\frac{\partial(\tau_{xx})}{\partial x}+\frac{\partial(\tau_{yx})}{\partial y}+\frac{\partial(\tau_{zx})}{\partial z}\right]$$

For steady state, 2-dimensional analysis equation can be reduced as follows

$$\frac{\partial(\rho u^2)}{\partial x}+\frac{\partial(\rho uv)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{R_e}\left[\frac{\partial(\tau_{xx})}{\partial x}+\frac{\partial(\tau_{yx})}{\partial y}\right]$$

Y- Direction

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho v^2)}{\partial y}+\frac{\partial(\rho vw)}{\partial z}=-\frac{\partial p}{\partial y}+\frac{1}{R_e}\left[\frac{\partial(\tau_{xy})}{\partial x}+\frac{\partial(\tau_{yy})}{\partial y}+\frac{\partial(\tau_{zy})}{\partial z}\right]$$

For steady state, 2-dimensional analysis equation can be reduced as follows

$$\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho v^2)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{R_e}\left[\frac{\partial(\tau_{xy})}{\partial x}+\frac{\partial(\tau_{yy})}{\partial y}\right]$$

Z- Direction

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial(\rho uw)}{\partial x}+\frac{\partial(\rho vw)}{\partial y}+\frac{\partial(\rho w^2)}{\partial z}=-\frac{\partial p}{\partial z}+\frac{1}{R_e}\left[\frac{\partial(\tau_{xz})}{\partial x}+\frac{\partial(\tau_{yz})}{\partial y}+\frac{\partial(\tau_{zz})}{\partial z}\right]$$

For steady state, 2- Dimensional analysis Z- Direction term is eliminated

Conservation of energy

Governing equations of conservation of energy is

$$\frac{\partial[\rho\left(e+\frac{V^2}{2}\right)]}{\partial t}+\frac{\partial[\rho u\left(e+\frac{V^2}{2}\right)]}{\partial x}+\frac{\partial[\rho v\left(e+\frac{V^2}{2}\right)]}{\partial y}+\frac{\partial[\rho w\left(e+\frac{V^2}{2}\right)]}{\partial z}=K\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right]+\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}+\frac{\partial p}{\partial z}\right)+\mu(u\frac{\partial^2 u}{\partial x^2}+\frac{\partial}{\partial x}\left(v\frac{\partial v}{\partial x}+w\frac{\partial w}{\partial x}\right)+v\frac{\partial^2 v}{\partial y^2}+\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}+w\frac{\partial w}{\partial y}\right)+w\frac{\partial^2 w}{\partial z^2}+\frac{\partial}{\partial z}\left(v\frac{\partial v}{\partial z}+u\frac{\partial u}{\partial z}\right)+2\mu[(\frac{\partial u}{\partial x})^2+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+(\frac{\partial v}{\partial y})^2+\frac{\partial v}{\partial z}\frac{\partial w}{\partial y}+(\frac{\partial w}{\partial z})^2+\frac{\partial w}{\partial x}\frac{\partial u}{\partial z}$$

For steady state, 2-dimensional analysis equation can be reduced as follows

$$\frac{\partial[\rho u\left(e+\frac{V^2}{2}\right)]}{\partial x}+\frac{\partial[\rho v\left(e+\frac{V^2}{2}\right)]}{\partial y}=K\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right]+\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right)+\mu[(u\frac{\partial^2 u}{\partial x^2}+\frac{\partial}{\partial x}\left(v\frac{\partial v}{\partial x}\right)+v\frac{\partial^2 v}{\partial y^2}+\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial y}\right)]+2\mu[(\frac{\partial u}{\partial x})^2+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+(\frac{\partial v}{\partial y})^2]$$

2.1.2 Continuity equation

Continuity equation based on the principle of conservation of mass.

$m = \rho AV =$ constant

Applying logarithm and differentiating equation for steady, one dimensional flow the continuity equation i s

$$\frac{d\rho}{\rho}+\frac{dV}{V}+\frac{dA}{A}=0$$

2.1.3 Energy equation

According to first law of thermodynamics energy equation for steady, one dimensional flow is

$$h_2 + \frac{V_2^{\ 2}}{2000} + Z_2 + W = h_1 + \frac{V_1^{\ 2}}{2000} + Z_1 + Q$$

For nozzle work done is zero, no heat transfer across nozzle wall (adiabatic) and potential head at inlet and outlet are same (Z_1=Z_2).

Then the energy equation is reduced to

$$h_2 + \frac{V_2^{\ 2}}{2000} = h_1 + \frac{V_1^{\ 2}}{2000}$$

For a nozzle $V_1 <<<< V_2$

$$V_2 = \sqrt{2000(h_1 - h_2)}$$

For flow process h = U + Pυ
Where U = internal energy Pυ = Flow work

2.1.4 Mass flow rate

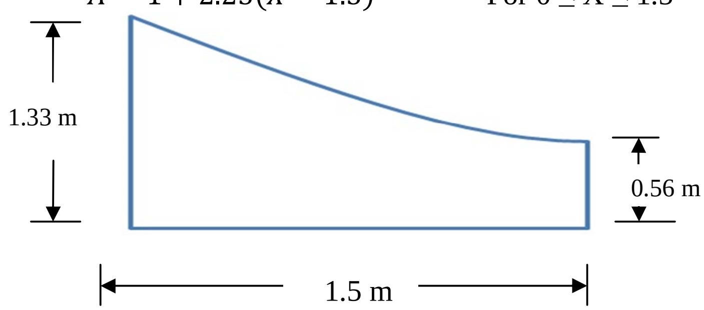
Mass flow rate along the nozzle is given by the following equation

$$\frac{m}{A} = \sqrt{\frac{\Upsilon}{R}}\frac{P_0 M}{\sqrt{T_0}}\left(\frac{T_0}{T}\right)^{\frac{-(\Upsilon+1)}{2(\Upsilon-1)}}$$

3. MODELS FOR CFD ANLYSIS

A two dimensional Convergent nozzle is modeled by using the area equation

$$A = 1 + 2.23(X - 1.5)^2 \qquad \text{For } 0 \le X \le 1.5$$



Effect of back pressure

Back pressure plays a very important role in the analysis of convergent nozzle. Variation of the back pressure causes changes in the Pressure, Mach number, density and temperatures in the nozzle. This analysis is required to know the influence of back pressure on convergent nozzle.

Parameters used for the analysis are,

Mesh size	640 x 15	Residue	$1e^{-3}$
Pressure inlet	100 kPa, 300 K	Pressure outlet	90 kPa to 10 kPa
Material	Air (Ideal gas)	Fluid	Inviscid

4. RESULTS AND DISCUSSION

Mass flow rate in the nozzle increases with the decrease of back pressure and attains maximum at critical pressure. Further decrease of pressure will not affect the mass flow rate. Variation of mass flow rate with back pressure resulting from theory and CFD are shown in Figure 5.1. Exit pressure is equal to back pressure up to the critical condition and remains constant below the critical pressure as shown in Figure 5.2.

Figure 5.3 represents the Mach number variation through the nozzle. Mach number at the exit of nozzle increases with decrease of back pressure up to critical pressure. At critical pressure Mach number reached to one at the exit of the nozzle. Further decrease of back pressure has not affected the variation of Mach number.

Pressure, density and temperatures decreased with the decrease of back pressure up to the critical pressure and a further decrease of pressure not affected the properties as shown in Figures 5.4, 5.5 & 5.6.

Figures 5.7 & 5.8 clearly show the Mach number and pressures contours along the nozzle at different back pressures. From these contours we can clearly observe the Mach number and pressure variation with back pressure.

The exit values for different back pressures are shown in table 5.1. A comparison of theoretical and CFD results at critical condition is made in table 5.2 and from this table it can be clearly observed that the theoretical and CFD values are nearly equal with insignificant error.

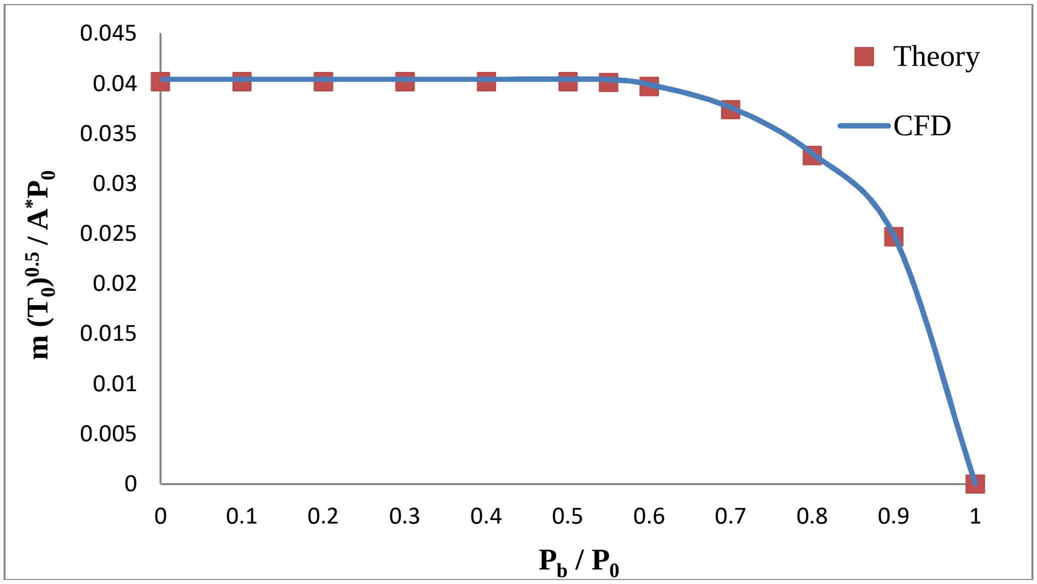


Fig 5.1: Mass flow rate variation with back pressures ratio

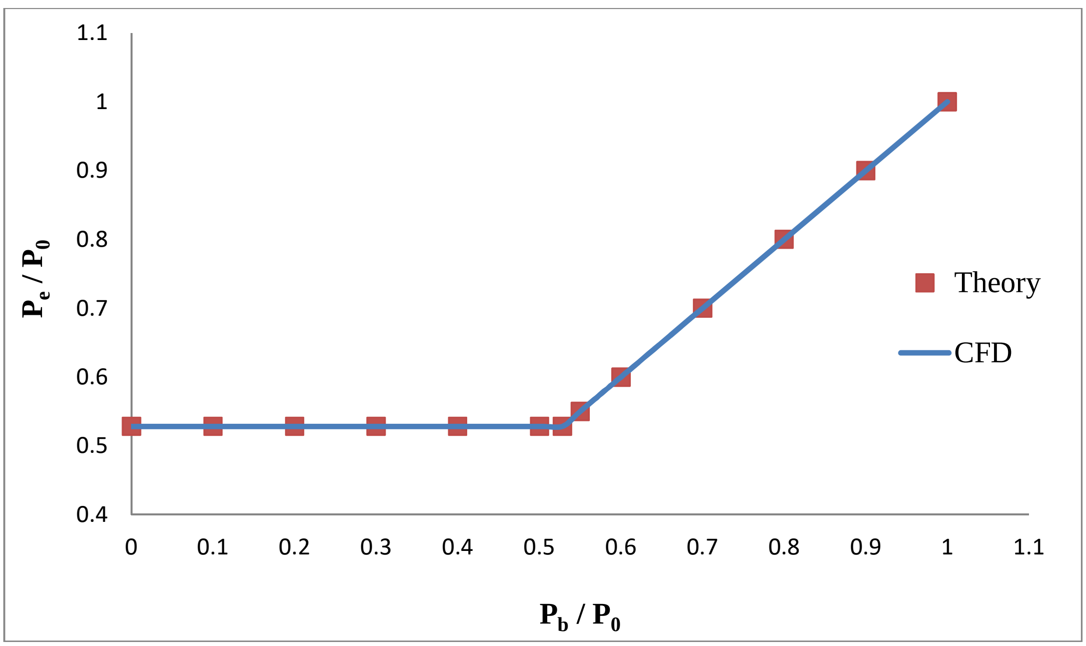


Fig 5.2: Exit pressure ratio Variation with back pressure ratio

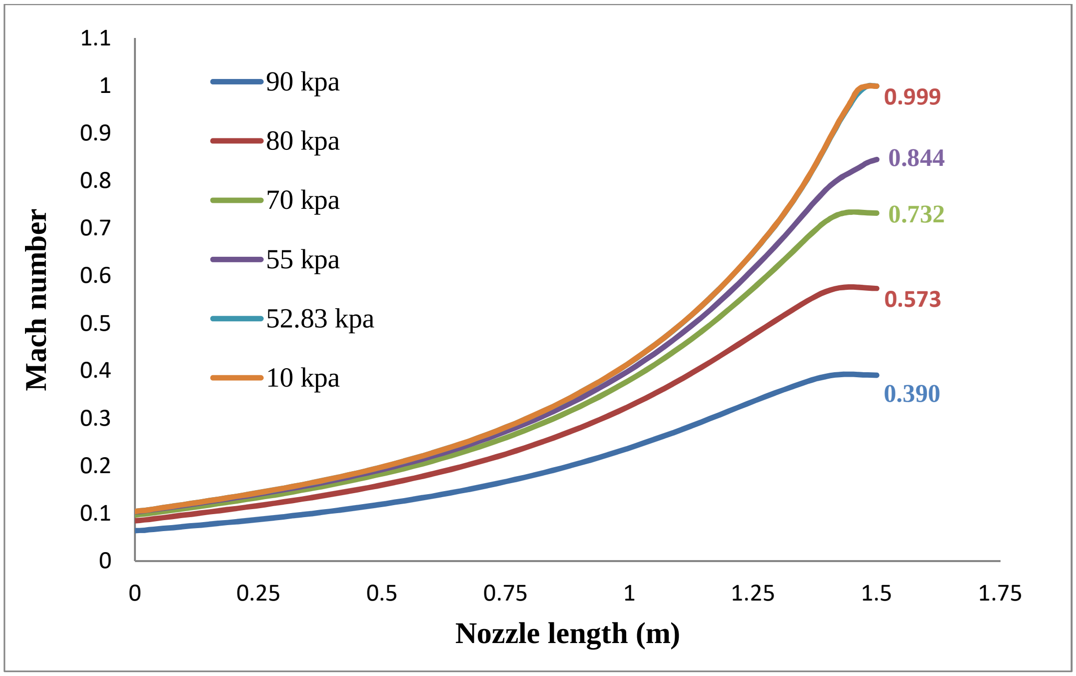


Fig 5.3: Mach number variation with back pressure

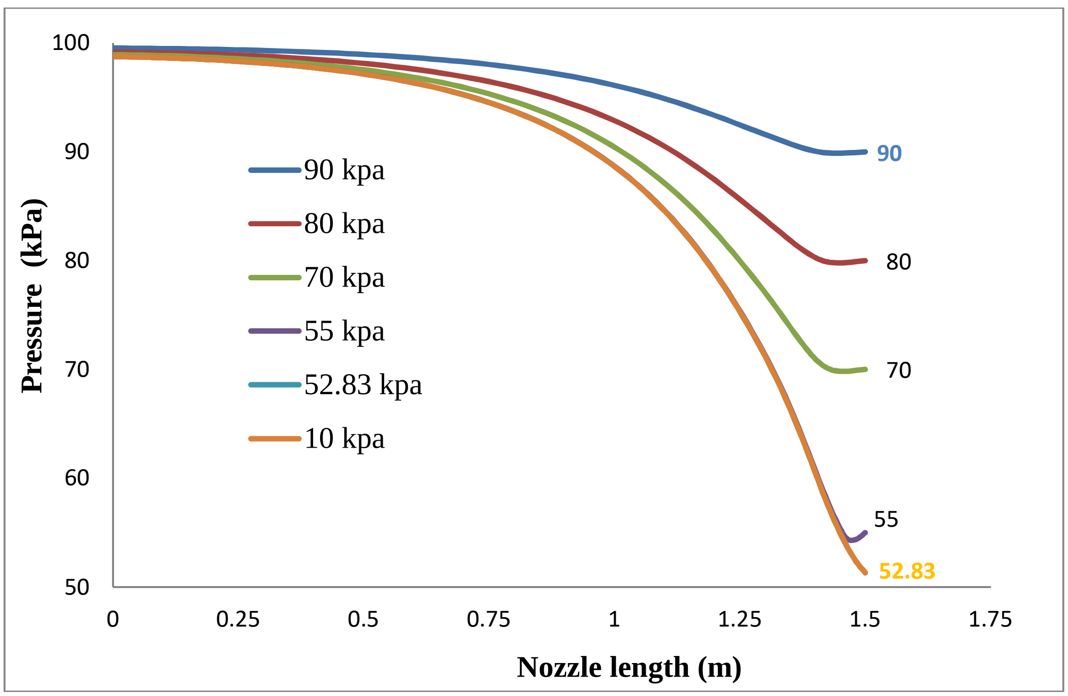


Fig 5.4: Pressure variation with back pressure

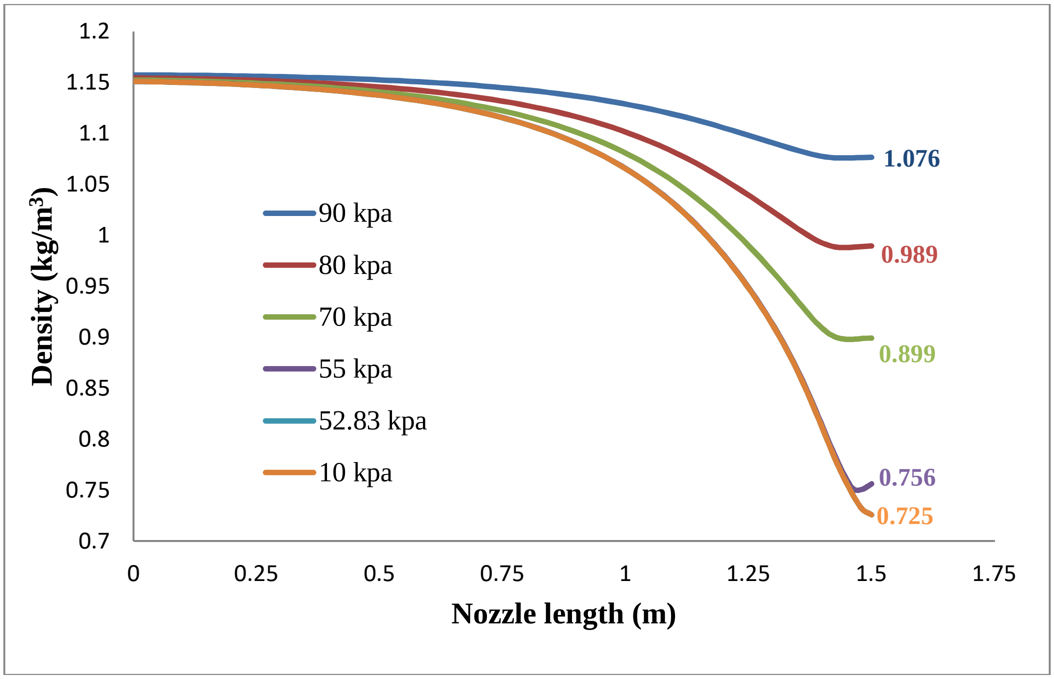


Fig 5.5: Density variation with back pressure

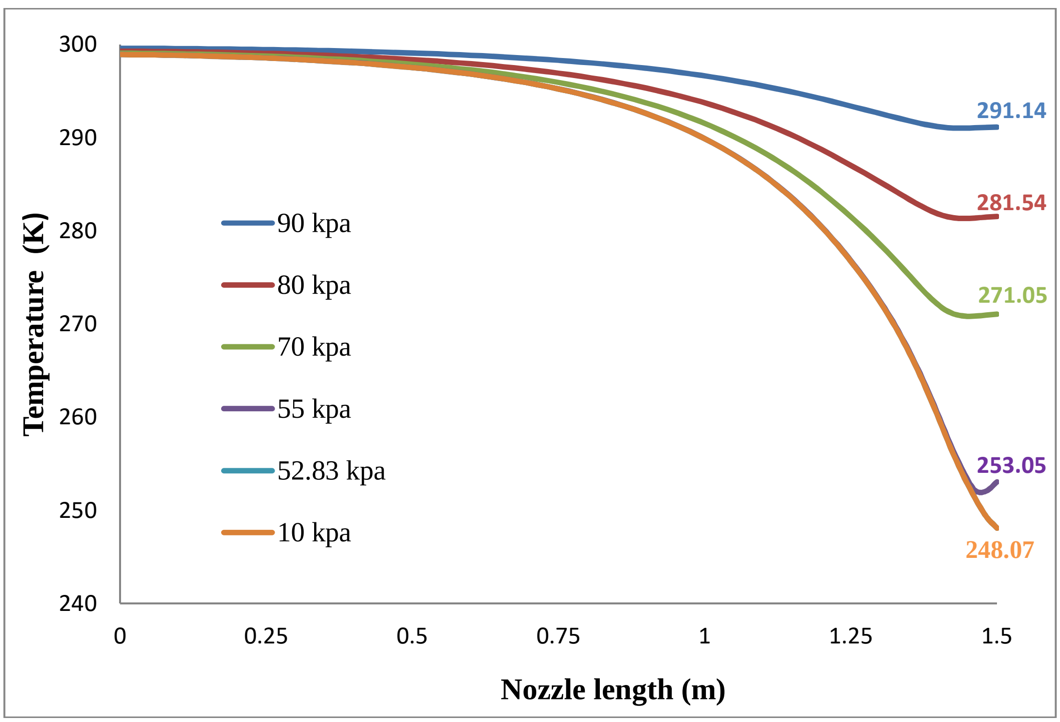


Fig 5.6: Temperature variation with back pressure

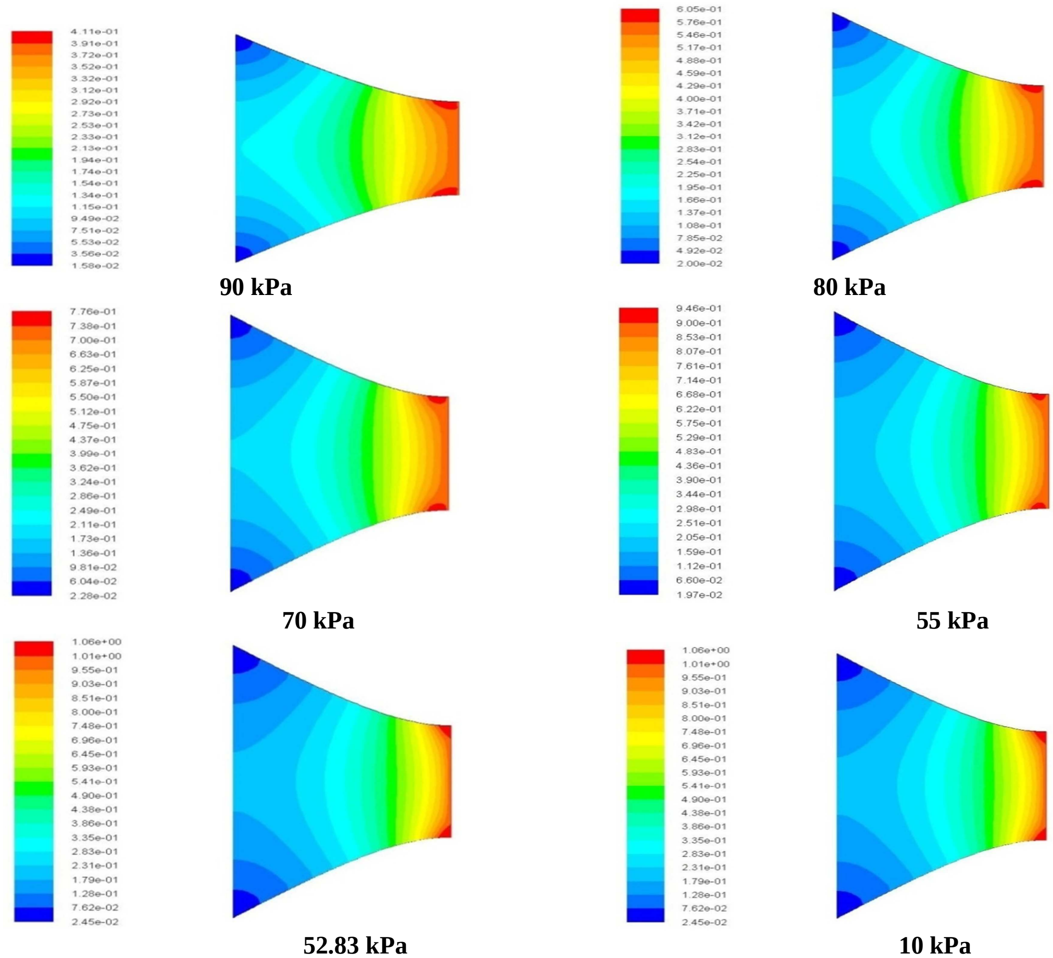


Fig 5.7: Mach number contours with back pressure

Table 5.1: Exit values of Convergent nozzle

Back Pressure (kPa)	Exit Mach number	Exit Pressure (Pa)	Exit Density (Kg/m^3)	Exit Temperature (K)	Mass flow rate (kg/s)
90	0.390	90000	1.076	291.14	143.597
80	0.573	80000	0.989	281.54	190.97
70	0.732	70000	0.899	271.05	217.23
55	0.844	55000	0.756	253.05	233.25
52.83	0.999	52830	0.725	248.07	233.44

10	0.999	52830	0.725	248.07	233.44

Table 5.2: Comparison of CFD and Theoretical values at sonic condition

Variable	CFD	Theory	% Error
Mach number	0.999	1.00	**0.075**
Pressure (Pa)	52830	52830	**0**
Density (kg/m^3)	0.725	0.734	**0.084**
Temperature (K)	248.074	250	**0.0872**
Mass flow rate (kg/s)	233.44	233.33	**0.0417**

5. CONCLUSIONS

1. For the increase of back pressure Mach number and Mass flow rate increases up to some value and constant although we decrease the back pressure.
2. The numerical method adopted (CFD) predicted the performance of subsonic nozzle exactly with theory (less than 1% error) and hence CFD tool can be easily extended for any improvements or modifications in design.
3. Values obtained by analysis is nearly matches with theoretical values.

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