

An Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging

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Abstract- In this paper we developed a general inventory model for deteriorating items with constant deterioration rate under the consideration of time dependent demand rate and partial backlogging. Shortages are allowed and completely back-logged. The purpose of our study is to minimize the total variable inventory cost and a numerical example is also given to demonstrate the developed model. A sensitivity analysis is also given for the developed model.

Keywords- Inventory, time dependent demand, deteriorating items, shortages and partial backlogging.

1. INTRODUCTION-

In the recent years there is a state of interest of studying time dependent demand rate. It is observed that the demand rate of newly launched products such as electronics items, mobile phones and fashionable garments increases with time and later it becomes constant. Deterioration of items cannot be avoided in business scenarios. In most of the cases the demand for items increases with time and the items that are stored for future use always lose part of their value with passage of time. In inventory this phenomenon is known as deterioration of items. The rate of deterioration is very small in some items like hardware, glassware, toys and steel. The items such as medicine, vegetables, gasoline alcohol, radioactive chemicals and food grains deteriorate rapidly over time so the effect of deterioration of physical goods cannot be ignored in many inventory systems. The deterioration of goods is a realistic phenomenon in many inventory systems and controlling of deteriorating items becomes a measure problem in any inventory system. Due to deterioration the problem of shortages occurs in any inventory system and shortage is a fraction that is not available to satisfy the demand of the customers in a given period of time. Dye [2002] developed an inventory model with partial backlogging and stock dependent demand. Chakrabarty et al. [1998] extended the Philip's model [1974]. Skouri and Papachristos [2003] determine an optimal time of an EOQ model for deteriorating items with time dependent partial backlogging. Manjusri Basu and Sudipta Sinha [2007] extended the Yan and Cheng model [1998] for time dependent backlogging rate. Rau et al. [2004] considered an inventory model for determining an economic ordering policy of deteriorating items in a supply chain management

system. Teng and Chang [2005] determined an economic production quantity in an inventory model for deteriorating items. Dave and Patel [1983] developed an inventory model together with an instantaneous replenishment policy for deteriorating items with time proportional demand and no shortage. Roy and Chaudhury [1983] considered an order level inventory model with finite rate of replenishment and allowing shortages. Mishra [1975], Dev and Chaudhuri [1986] assumed time dependent deterioration rate in their models. In this regard an extended summary was given by Raafat[1991]. Berrotoni [1962] discussed the difficulties of fitting empirical data to mathematical distributions. Covert and Philip [1973] developed an inventory model for deteriorating items by considering two parameters weibull distribution. Mandal and Phaujdar [1989] developed a production inventory model for deteriorating items with stock dependent demand and uniform rate of production. In this direction some work also done by Padmanabhan and Vrat [1995]. Ray and Chaudhuri [1997], Mondal and Moiti [1999], Biermans and Thomas [1997], Buzacoh [1975], Chandra and Bahner [1988], Jesse et al. [1983], Mishra [1979] developed their models and show the effect of inflation in inventory models by taking a constant rate of inflation. Liao et al [2000] discuss the effect of permissible delay in payment for an inventory model of deteriorating items under inflation. Bhahmbhatt [1982] developed an EOQ model under price dependent inflation rate. Ray and Chaudhuri [1997] considered an EOQ model with shortages under the effect of inflation and time discount. Goyal [1985] developed an EOQ model under the conditions of permissible delay in payment. Chung et al [2002] and Hung [2003] considered an optimal replenishment policy for

EOQ model under permissible delay in payments. Aggarwal and Jaggi [1995] extended the EOQ model with constant rate of deterioration. Hwang and Shinn [1997] determined the lot size policy for the items with exponential demand and permissible delay in payment. In the presence of trade credit policy, Chung and Hung [2005] developed an EOQ model. Vinod kumar Mishra and Lal sahab singh [2010] developed an inventory model for deteriorating items with time dependent demand and partial backlogging. Further Vinod kumar Mishra [2013] developed an inventory model involving controllable deterioration rate to extend the traditional EOQ model. Mandal [2013] developed an inventory model for random deteriorating items with time dependent demand and partial backlogging. It has been observed that the unsatisfied demand is completely back-logged and during the shortage period either all the customers wait for the arrival of next order (completely backlogged) or all the customers leave the system (completely lost). The length of waiting time for the replenishment is the main factor for determining whether the backlogging is accepted or not.

In the present paper we developed an inventory model for deteriorating items with constant deterioration rate and time dependent demand and partial backlogging. Shortages are allowed and completely backlogged for the next replenishment cycle. The purpose of our study is to minimize the total variable inventory cost. A numerical example is considered to illustrate the developed model.

2. ASSUMPTIONS AND NOTATIONS

We consider the following assumptions and notations

1. The time dependent demand is $R(t) = (a+bt)$, $a, b > 0$ where a is constant and b is the fraction of demand varying with time.
2. The replenishment rate is infinite.
3. The planning horizon is infinite.
4. The deterioration rate θ is constant.
5. Shortages are allowed and completely backlogged. The backlogging rate is defined by $R(t) = \frac{-(a + bt)}{1 + \delta(T - t)}$ where backlogging parameter δ is a positive constant.
6. $I(t)$ is the inventory level at any time t .
7. S , is the inventory level at time $t=0$.

8. S_0 , is the inventory level at time $t = T_0$.
9. A is the ordering cost per order.
10. C is the purchase cost per unit.
11. h is the holding cost per unit per cycle.
12. π_B is the backordered cost per unit time.
13. π_L is the lost sales cost per unit.
14. T is the cycle length.
15. T_1 is the time at which shortages start.
16. $TC(T_0, T_1, T_2)$ is the average total inventory cost per unit time

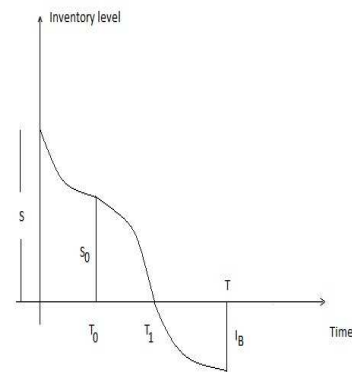


Figure I

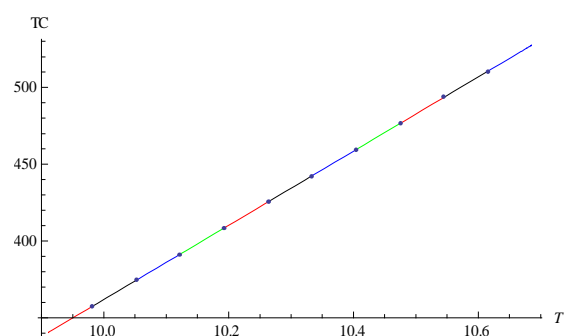


Figure II (with respect to parameter δ)

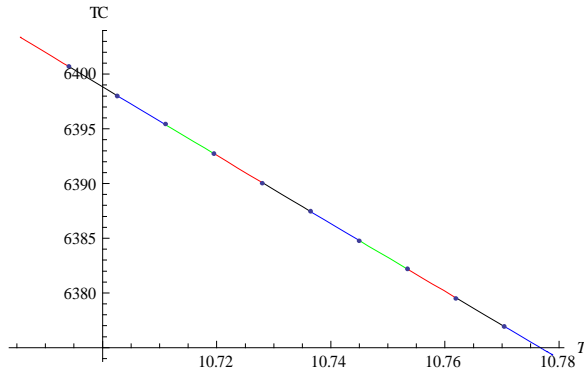


Figure III (with respect to parameter θ)

3. MATHEMATICAL FORMULATION-

The instantaneous inventory level at any time t in the interval $[0, T]$ is given by the differential equations

$$\frac{dI_1}{dt} + \theta I_1 = -(a + bt), \quad 0 \leq t \leq T_0. \quad \dots\dots\dots (1)$$

With the Boundary Condition $I_1(0) = S$

$$\frac{dI_2}{dt} + \theta I_2 = -(a + bt), \quad T_0 \leq t \leq T_1. \quad \dots\dots\dots (2)$$

With the Boundary Condition $I_2(T_1) = 0$

$$\frac{dI_3}{dt} = -\frac{(a + bt)}{1 + \delta(T - t)}, \quad T_1 \leq t \leq T. \quad \dots\dots\dots (3)$$

With the Boundary Condition $I_3(T_1) = 0$

The solution of equation (1) is

$$I_1 = S - (a + \theta S)t + \frac{(a\theta + S\theta^2 - b)t^2}{2} \quad \dots\dots\dots (4)$$

The solution of equation (2) is

$$I_2 = a(T_1 - t) + \frac{(b + a\theta)T_1^2}{2} + \frac{(a\theta - b)t^2}{2} - at\theta T_1 \quad \dots\dots\dots (5)$$

The solution of equation (3) is

$$I_3 = a(T_1 - t) + \frac{b(T_1^2 - t^2)}{2} + a\delta(T_1^2 - t^2 - 2TT_1 + 2Tt) \quad \dots\dots\dots (6)$$

The ordering cost per unit time is

$$O_c = \frac{A}{T} \quad \dots\dots\dots (7)$$

The holding cost per unit time is

$$\begin{aligned} H_c &= \frac{h}{T} \left[\int_0^{T_0} I_1(t) dt + \int_{T_0}^{T_1} I_2(t) dt \right] \\ &= \frac{h}{T} \left[\int_0^{T_0} \left\{ S - (a + \theta S)t + \frac{(a\theta + S\theta^2 - b)t^2}{2} \right\} dt \right. \\ &\quad \left. + \int_{T_0}^{T_1} \left\{ a(T_1 - t) + \frac{(b + a\theta)T_1^2}{2} + \frac{(a\theta - b)t^2}{2} - at\theta T_1 \right\} dt \right] \\ H_c &= \frac{h}{T} \left[ST_0 - (a + S\theta)T_0^2 + \frac{(a\theta + S\theta^2 - b)T_0^3}{6} \right. \\ &\quad \left. + \frac{a(T_1 - T_0)^2}{2} - \frac{(b + a\theta)T_0T_1^2}{2} - \frac{(a\theta - b)T_0^3}{6} \right. \\ &\quad \left. + \frac{(b + 2a\theta)T_1^3}{3} - \frac{a\theta(T_1^3 - T_1T_0^2)}{2} \right] \end{aligned} \quad \dots\dots\dots (8)$$

The shortages cost per unit time is

$$\begin{aligned} S_c &= -\frac{\pi_B}{T} \int_{T_1}^T I_3(t) dt \\ &= -\frac{\pi_B}{T} \int_{T_1}^T \left\{ a(T_1 - t) + \frac{b(T_1^2 - t^2)}{2} \right. \\ &\quad \left. + a\delta(T_1^2 - t^2 - 2TT_1 + 2Tt) \right\} dt \\ S_c &= \frac{-\pi_B}{T} \left[a(2TT_1 - T^2 - T_1^2) \right. \\ &\quad \left. + \frac{b(TT_1^2 - \frac{2T_1^3}{3} - \frac{T_1^3}{3})}{2} \right. \\ &\quad \left. + a\delta(2TT_1^2 - \frac{2T_1^3}{3} + \frac{2T^3}{3} - 2T_1T^2) \right] \quad \dots\dots\dots (9) \end{aligned}$$

The lost sales cost per cycle is

$$\begin{aligned} L_s &= \pi_L \int_{T_1}^T \left\{ (a + bt) - \frac{(a + bt)}{1 + \delta(T - t)} \right\} dt \\ &= \pi_L \left[\frac{a\delta(T - T_1)^2}{2} + \frac{bT\delta(T - T_1)^2}{2} \right] \quad \dots\dots\dots (10) \end{aligned}$$

The purchase cost per cycle is

$$P_c = (\text{Purchase cost per unit})(\text{Order quantity in one cycle}) \quad \dots\dots\dots (11)$$

$$\text{Since } I_{\max} = S$$

The maximum backordered inventory is obtained by putting $t = T$ in equation (6) so

$$I_B = -\{a(T_1 - T) + \frac{b(T_1^2 - T^2)}{2} + a\delta(T_1 - T)^2\}$$

Therefore $Q = I_{\max} + I_B$

$$Q = S - \{a(T_1 - T) + \frac{b(T_1^2 - T^2)}{2} + a\delta(T_1 - T)^2\}$$

The purchase cost per cycle is

$$P_C = CQ$$

$$P_C = C[S - a(T_1 - T) - \frac{b(T_1^2 - T^2)}{2} - a\delta(T_1 - T)^2]$$

..... (12)

Therefore the total cost per unit time is

$$TC(T_0, T_1, T) = \frac{1}{T}[O_C + H_C + S_C + L_S + P_C]$$

$$\begin{aligned} &= \frac{1}{T}[A + h\{ST_0 - (a + S\theta)T_0^2 \\ &\quad + \frac{(a\theta + S\theta^2 - b)T_0^3}{6} + \frac{a(T_1 - T_0)^2}{2} \\ &\quad - \frac{(b + a\theta)T_0T_1^2}{2} + \frac{(b - a\theta)T_0^3}{6} \\ &\quad + \frac{(b + 2a\theta)T_1^3}{3} - \frac{a\theta(T_1^3 - T_1T_0^2)}{2}\} - \pi_B\{-a(T - T_1)^2 \\ &\quad + \frac{b(TT_1^2 - 2T_1^3 - T^2)}{3} + a\delta(2TT_1^2 - \frac{2T_1^3}{3} + \frac{2T^3}{3} \\ &\quad - 2T_1T^2)\} + \pi_L(\frac{a\delta}{2} + \frac{bT\delta}{2})(T - T_1)^2 + C\{S \\ &\quad + a(T - T_1) - \frac{b(T_1^2 - T^2)}{2} - a\delta(T - T_1)^2\}] \end{aligned}$$

..... (13)

The necessary condition for $TC(T_0, T_1, T)$ to be minimum is that

$$\frac{\partial TC}{\partial T_0} = 0, \quad \frac{\partial TC}{\partial T_1} = 0, \quad \frac{\partial TC}{\partial T} = 0, \quad \text{and the}$$

determinant of the principal minors H_1, H_2, H_3, \dots of Hessian matrix

(H-matrix) of $TC(T_0, T_1, T)$ are positive definite.

$$H = \begin{vmatrix} \frac{\partial^2 TC}{\partial T_0^2} & \frac{\partial^2 TC}{\partial T_0 \partial T_1} & \frac{\partial^2 TC}{\partial T_0 \partial T} \\ \frac{\partial^2 TC}{\partial T_1 \partial T_0} & \frac{\partial^2 TC}{\partial T_1^2} & \frac{\partial^2 TC}{\partial T_1 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial T_0} & \frac{\partial^2 TC}{\partial T \partial T_1} & \frac{\partial^2 TC}{\partial T^2} \end{vmatrix}$$

of $TC(T_0, T_1, T)$ are positive definite.

$$\begin{aligned} \frac{\partial TC}{\partial T_0} &= \frac{1}{T}[h\{S - 2(a + S\theta)T_0 + \frac{(a\theta + S\theta^2 - b)T_0^2}{2} \\ &\quad + a(T_1 - T) - \frac{(b + a\theta)T_1^2}{2} + \frac{(b - a\theta)T_0^2}{2} + a\theta T_0T_1\}] \end{aligned}$$

..... (14)

$$\begin{aligned} \frac{\partial TC}{\partial T_1} &= \frac{1}{T}[h\{a(T_1 - T_0) - (b + a\theta)T_0T_1 \\ &\quad + (b + \frac{a\theta}{2})T_1^2 + \frac{a\theta T_0^2}{2}\} - \pi_B\{2a(T - T_1) \\ &\quad + (b + 4a\delta)TT_1 - (b + 2a\theta)T_1^2 - 2a\delta T^2\} \\ &\quad - \pi_L\{\delta(a + bT)(T - T_1)\} - C\{a + (b \\ &\quad + 2a\delta)T_1 - 2a\delta T\}] \end{aligned}$$

..... (15)

$$\begin{aligned} \frac{\partial TC}{\partial T} &= \frac{1}{T}[-\pi_B\{-2a(T - T_1) + \frac{b(T_1^2 - T^2)}{2} \\ &\quad + 2a\delta(T - T_1)^2\} + \pi_L\{\delta(a + bT)(T - T_1) \\ &\quad + \frac{b\delta(T - T_1)^2}{2}\} + C\{a + bT - 2a\delta(T - T_1)\}] \\ &\quad - \frac{1}{T^2}[A + h\{ST_0 - (a + S\theta)T_0^2 + \frac{(a\theta + S\theta^2 - b)T_0^3}{6} \\ &\quad + \frac{a(T_1 - T_0)^2}{2} - \frac{(b + a\theta)T_0T_1^2}{2} + \frac{(b - a\theta)T_0^3}{6} + \frac{(b - a\theta)T_0^3}{6} \end{aligned}$$

$$\begin{aligned}
 & + \frac{(b+2a\theta)T_1^3}{3} - \frac{a\theta(T_1^3 - T_1T_0^2)}{2} \} - \pi_B \{ -a(T-T_1)^2 \\
 & + \frac{b(TT_1^2 - 2T_1^3 - T^2)}{2} + a\delta(2TT_1^2 - \frac{2T_1^3}{3} + \frac{2T^3}{3} - 2T_1T^2) \} \\
 & + \pi_L (\frac{a\delta}{2} + \frac{bT\delta}{2})(T-T_1)^2 + C\{S + a(T-T_1) \\
 & - \frac{b(T_1^2 - T^2)}{2} - a\delta(T-T_1)^2 \}] \\
 & \dots\dots\dots (16)
 \end{aligned}$$

Solving the equations (14), (15) and (16) for T_0, T, T_1 and for which $TC(T_0, T_1, T)$ is minimum.

4. NUMERICAL EXAMPLE

Let us consider an inventory system with the given data in appropriate units as follows:

$A=50, S=500, \pi_L = 10, \pi_B = 5, h=0.5, \delta = 3, \theta = 0.001, a=2, b=8, C=6$

Table I

θ	T_0	T_1	T	TC
0.001	7.68152	10.8158	10.6856	6418.65
0.004	6.55108	10.6785	10.5629	6241.78
0.008	7.01796	10.2886	12.1451	5361.85
0.015	5.72252	10.0241	9.91096	5460.89

Table II

δ	T_0	T_1	T	TC
3	7.68152	10.8158	10.6856	6418.65
6	7.73318	10.8115	10.7409	12784.10
9	7.75234	10.8089	10.7615	19145.40
15	7.76846	10.8086	10.7789	63756.90

Table III

h	T_0	T_1	T	TC
0.5	7.68152	10.8158	10.6856	6418.65
1.0	6.33218	10.9265	10.8855	6514.18
2.0	5.67915	10.9797	11.1128	6499.11
4.0	5.43949	10.9992	11.4601	6375.46

5. SENSITIVITY ANALYSIS

From the Table I we see that as we increase the deterioration parameter θ then the time periods T, T_0, T_1 decreases and the total cost also decreases. From the Table II we see that as we

increase the backloging parameter δ then the time periods T, T_0 increases and the time period T_1 decreases and the total cost increases.

From the Table III we see that as we increase the holding parameter h then the time periods T_0 decreases and the time periods T, T_1 increases and the total cost increases. Thus the parameter δ is more sensitive than the parameters θ and h.

6. CONCLUSION

In the present paper we developed an inventory model for deteriorating items with time dependent demand and partial back-logging under constant deterioration rate. Shortages are allowed and completely backlogged for the next replenishment cycle. From the sensitivity analysis it is observed that the parameter δ is more sensitive than the parameters h and θ . We make a decision to determine the optimum cycle time for minimizing the total average inventory cost. In future study this model can be generalized by considering time dependent deterioration and holding costs and also probabilistic model

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