# Arithmetical Operations in Quaternary System 

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#### Abstract

The digital circuits became more prominent with incorporating information processing and computing. The modern computers lead to the deterioration in performance of arithmetic operations such as addition, subtraction, multiplication, division, on the aspects of carry propagation delay, large circuit complexity and high power consumption. Arithmetic circuits play a very critical role in both general-purpose and application specific computational circuits. Designing this adder using QSD number representation allows fast addition/subtraction which is capable of carry free addition and borrows free subtraction because the carry propagation chain are eliminated, hence it reduce the propagation time in comparison with radix 2 system. QSD number system based on quaternary system, each digit can be represented by a number from -3 to -3 .


Keywords- quaternary, carry/borrow free addition/ subtraction, number system, qsd, logical system

## 1. INTRODUCTION

Designing this Arithmetic unit using QSD number representation has attracted the interest of many researchers. Additionally, recent advances in technologies for integrated circuits make large scale arithmetic circuits suitable for VLSI implementation. In this paper, we propose a high speed QSD adder which is capable of carry free addition, borrow free subtraction. The QSD addition/subtraction operation employs a fixed number of minterms for any operand size. In QSD number system carry propagation chain are eliminated which reduce the computation time substantially, thus enhancing the speed of the machine. Signed digit number system offers the possibility of carry free addition. QSD Adder / QSD Multiplier circuits are logic circuits designed to perform high-speed arithmetic operations. In QSD number system carry propagation chain are eliminated which reduce the computation time substantially, thus enhancing the speed of the machine. The paper is structured as follows: Section2 presents the Signed digit number. In Section -3 the QSD number system is presented. Section - 4 presents basic concept for performing any operation in QSD, first convert the binary or any other input into quaternary signed digit. Section - 5 presents Adder/Substractor Design. Section - 6 presents simulation results. Then we provide our conclusions in Section - 7.

## 2. SIGNED DIGIT NUMBER

Signed digit representation of number indicates that digits can be prefixed with a - (minus) sign to indicate that they are negative.

$$
\begin{aligned}
(11 \overline{2} 2)_{2} & =1 \times 2^{3}+1 \times 2^{2}-2 \times 2^{1}+1 \times 2^{0} \\
& =8+4-4+2 \\
& =10
\end{aligned}
$$

## 3. QSD NUMBER SYSTEM

QSD numbers are represented using 3-bit 2's complement notation. Each number can be represented by

## $D=\Sigma x i 4^{i}$

Where xi can be any value from the set $\{3,2,1,0,1$, $2,3\}$ for producing an appropriate decimal representation. A QSD negative number is the QSD complement of QSD positive number i.e. $3=-3,2=-$ $2,1=-1$. For digital implementation, large number of digits such as 64,128 , or more can be implemented with constant delay. A high speed and area effective adders and multipliers can be implemented using this technique. For digital implementation, large number of digits such as 64,128 , or more can be implemented with constant delay. A higher radix based signed digit number system, such as quaternary
signed digit (QSD) number system, allows higher information storage density, less complexity, fewer system components and fewer cascaded gates and operations. A high speed and area effective adders and multipliers can be implemented using this technique $[2,1]$.

Examples of n digit QSD number are as follows: $2 \overline{31} 0, \overline{2} 101,3 \overline{2} 11,3101102 \overline{3}$ etc.

For example:

$$
(13 \overline{3} 2)_{Q S D}=1 \times 4^{3}+3 \times 4^{2}-3 \times 4^{1}+2 \times 4^{0}
$$

$=64+48-12+2$
$=(102)_{10}$

The basic quaternary operators are very similar to binary operators and they are obtained from Boolean algebra.

## 4. BASIC CONCEPT

For performing any operation in QSD, first convert the binary or any other input into quaternary signed digit

## 5. ADDER/SUBSTRACTOR DESIGN.

A carry-free addition is desirable as the number of digits is large. The carry-free addition can achieve by exploiting redundancy of QSD number and QSD addition. The redundancy allows multiple representations of any integer quantity i.e., $610=$ $12_{\mathrm{QSD}}=22_{\mathrm{QSD}}$. There are two steps involved in the carry-free addition. The first step generates an intermediate carry and sum from the addend and augends. The second step combines the intermediate sum of the current digit with the carry of the lower significant digit. To prevent carry from further rippling, we define two rules. The first rule states that the magnitude of the intermediate sum must be less than or equal to 2 . The second rule states that the magnitude of the carry must be less than or equal to 1.Consequently, the magnitude of the second step output cannot be greater than 3 which can be represented by a single-digit QSD number; hence no
further carry is required. In step 1, all possible input pairs of the addend and augends are considered [4]. The output ranges from -6 to 6 as shown in Table 1 .

Table 1
The outputs of all possible combinations of a pair of addend (A) and augend (B)

| ${ }^{\text {A }}$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B-3 | -6 | -5 | -4 | -3 | -2 | -1 | 0 |
| -2 | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| -1 | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
| 0 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| 1 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| 2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
| 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |

The range of the output is from -6 to 6 which can be represented in the intermediate carry and sum in QSD format as show in Table 2.Some numbers have multiple representations, but only those that meet the defined rules are chosen. The chosen intermediate carry and sum are listed in the last column of Table 2. Both inputs and outputs can be encoded in 3-bit 2's complement binary number.

The mapping between the inputs, addend and augend, and the outputs, the intermediate carry and sum are shown in binary format in Table 3. Since the intermediate carry is always between -1 and 1 , it requires only a 2-bit binary representation. Finally, five 6 -variable Boolean expressions can be extracted. The intermediate carry and sum circuit is shown in Figure 1.


Figure 1. The intermediate carry and sum generator.

International Journal of Research in Advent Technology, Vol.2, No.2, February 2014
E-ISSN: 2321-9637

Table 2
The intermediate carry and sum between -6 to 6

| SUM | QSD represented <br> Number | QSD coded number |
| :---: | :---: | :---: |
| -6 | $\overline{1} \overline{2}, \overline{2} 2$ | $\overline{1} \overline{2}$ |
| -5 | $\overline{1} \overline{1}, \overline{2} 3$ | $\overline{1}$ |
| -4 | 10 | 10 |
| -3 | $0 \overline{3}, \overline{1} 1$ | 11 |
| -2 | $0 \overline{2}, \overline{1} 2$ | $0 \overline{2}$ |
| -1 | $0 \overline{1}, \overline{1} 3$ | $0 \overline{1}$ |
| 0 | 00 | 00 |
| 1 | $1 \overline{3}, 01$ | 01 |
| 2 | $1 \overline{2}, 02$ | 02 |
| 3 | $1 \overline{1}, 03$ | 11 |
| 4 | 10 | 10 |
| 5 | $2 \overline{3}, 11$ | 11 |
| 6 | 22,12 | 12 |

In step 2, the intermediate carry from the lower significant digit is added to the sum of the current digit to produce the final result. The addition in this step produces no carry because the current digit can always absorb the carry-in from the lower digit. Table-3 shows all possible combinations of the summation between the intermediate carry and the sum.

Table 3
The mapping between the inputs and outputs of the intermediate carry and sum

| INPUT |  |  |  | OUTPUT |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| QSD |  | Binary |  | Decimal <br> Sum | QSD |  | Binary |  |
| Ai | Bi | Ai | Bi |  | Ci | Si | Ci | Si |
| 3 | 3 | 011 | 011 | 6 | 1 | 2 | 01 | 010 |
| 3 | 2 | 011 | 010 | 5 | 1 | 1 | 01 | 001 |
| 2 | 3 | 010 | 011 | 5 | 1 | 1 | 01 | 001 |
| 3 | 1 | 011 | 001 | 4 | 1 | 0 | 01 | 000 |
| 1 | 3 | 001 | 011 | 4 | 1 | 0 | 01 | 000 |
| 2 | 2 | 010 | 010 | 4 | 1 | 0 | 01 | 000 |
| 1 | 2 | 001 | 010 | 3 | 1 | -1 | 01 | 111 |
| 2 | 1 | 010 | 001 | 3 | 1 | -1 | 01 | 111 |
| 3 | 0 | 011 | 000 | 3 | 1 | -1 | 01 | 111 |
| 0 | 3 | 000 | 011 | 3 | 1 | -1 | 01 | 111 |
| 1 | 1 | 001 | 001 | 2 | 0 | 2 | 00 | 010 |
| 0 | 2 | 000 | 010 | 2 | 0 | 2 | 00 | 010 |
| 2 | 0 | 010 | 000 | 2 | 0 | 2 | 00 | 010 |
| 3 | -1 | 011 | 111 | 2 | 0 | 2 | 00 | 010 |
| -1 | 3 | 111 | 011 | 2 | 0 | 2 | 00 | 010 |
| 0 | 1 | 000 | 001 | 1 | 0 | 1 | 00 | 001 |
| 1 | 0 | 001 | 000 | 1 | 0 | 1 | 00 | 001 |
| 2 | -1 | 010 | 111 | 1 | 0 | 1 | 00 | 001 |
| -1 | 2 | 111 | 010 | 1 | 0 | 1 | 00 | 001 |
| 3 | -2 | 011 | 110 | 1 | 0 | 1 | 00 | 001 |
| -2 | 3 | 110 | 011 | 1 | 0 | 1 | 00 | 001 |
| 0 | 0 | 000 | 000 | 0 | 0 | 0 | 00 | 000 |
| 1 | -1 | 001 | 111 | 0 | 0 | 0 | 00 | 000 |
| -1 | 1 | 111 | 001 | 0 | 0 | 0 | 00 | 000 |
| 2 | -2 | 010 | 110 | 0 | 0 | 0 | 00 | 000 |
| -2 | 2 | 11 | 010 | 0 | 0 | 0 | 00 | 000 |
| -3 | 3 | 101 | 011 | 0 | 0 | 0 | 00 | 000 |
| 3 | -3 | 011 | 101 | 0 | 0 | 0 | 00 | 000 |
| 0 | -1 | 000 | 111 | -1 | 0 | -1 | 00 | 111 |
| -1 | 0 | 111 | 000 | -1 | 0 | -1 | 00 | 111 |
| -2 | 1 | 110 | 001 | -1 | 0 | -1 | 00 | 111 |
| 1 | -2 | 001 | 110 | -1 | 0 | -1 | 00 | 111 |
| -3 | 2 | 101 | 010 | -1 | 0 | -1 | 00 | 111 |
| 2 | -3 | 010 | 101 | -1 | 0 | -1 | 00 | 111 |
| -1 | -1 | 111 | 111 | -2 | 0 | -2 | 00 | 110 |
| 0 | -2 | 000 | 110 | -2 | 0 | -2 | 00 | 110 |
| -2 | 0 | 110 | 000 | -2 | 0 | -2 | 00 | 110 |
| -3 | 1 | 101 | 001 | -2 | 0 | -2 | 00 | 110 |
| 1 | -3 | 001 | 101 | -2 | 0 | -2 | 00 | 110 |
| -1 | -2 | 111 | 110 | -3 | -1 | 1 | 11 | 001 |
| -2 | -1 | 110 | 111 | -3 | -1 | 1 | 11 | 001 |
| -3 | 0 | 101 | 000 | -3 | -1 | 1 | 11 | 001 |
| 0 | -3 | 000 | 101 | -3 | -1 | 1 | 11 | 001 |
| -3 | -1 | 101 | 111 | -4 | -1 | 0 | 11 | 000 |
| -1 | -3 | 111 | 101 | -4 | -1 | 0 | 11 | 000 |
| -2 | -2 | 110 | 110 | -4 | -1 | 0 | 11 | 000 |
| -3 | -2 | 101 | 110 | -5 | -1 | -1 | 11 | 111 |
| -2 | -3 | 110 | 101 | -5 | -1 | -1 | 11 | 111 |
| -3 | -3 | 101 | 101 | -6 | -1 | -2 | 11 | 110 |

Table 4
The outputs of all possible combinations of a pair of intermediate carry (A) and sum (B)


The result of addition in this step ranges from -3 to 3 . Since carry is not allowed in this step, the result becomes a single digit QSD output. The inputs, the intermediate carry and sum, are 2-bit and 3-bit binary respectively. The output is a 3-bit binary represented QSD number. The mapping between the 5 - bit input and the 3-bit output is shown in Table 5.


Figure 2. The second step QSD adder
Three 5- variable Boolean expressions can be extracted from Table 4. Figure 2 shows the diagram of the second step adder. The implementation of an $n$-digit QSD adder requires $n$ QSD carry and sum generators and $n-1$ second step adder as shown in Figure 3. The result turns out to be an $n+1$-digit number[4].

Table-5:
The mapping between inputs and outputs of the second step QSD adder

| INPUT |  |  | OUTPUT |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| QSD |  | Binary |  | Decimal | QSD | Binary |
| 1 | 2 | 01 | 010 | 3 | 3 | 111 |
| 1 | 1 | 01 | 001 | 2 | 2 | 010 |
| 0 | 2 | 00 | 010 | 2 | 2 | 010 |
| 0 | 1 | 00 | 001 | 1 | 1 | 001 |
| 1 | 0 | 01 | 000 | 1 | 1 | 001 |
| -1 | 2 | 11 | 010 | 1 | 1 | 001 |
| 0 | 0 | 00 | 000 | 0 | 0 | 000 |
| 1 | -1 | 01 | 111 | 0 | 0 | 000 |
| -1 | 1 | 11 | 001 | 0 | 0 | 000 |
| 0 | -1 | 00 | 111 | -1 | -1 | 111 |
| -1 | 0 | 11 | 000 | -1 | -1 | 111 |
| 1 | -2 | 01 | 110 | -1 | -1 | 111 |
| -1 | -1 | 11 | 111 | -2 | -2 | 110 |
| 0 | -2 | 00 | 110 | -2 | -2 | 110 |
| -1 | -2 | 11 | 110 | -3 | -3 | 001 |



Figure 3. Four digit QSD adder

## 6. SIMULATION RESULTS

The QSD adder written in VHDL, compiled and simulation using ISE simulator. The QSD adder circuit simulated and synthesized. The simulated result for QSD adders as shown in figure-4.


Figure 4: Simulated result QSD adder

# International Journal of Research in Advent Technology, Vol.2, No.2, February 2014 

 E-ISSN: 2321-9637
## 7. CONCLUSION

The proposed QSD adder is better than other binary adders in terms of number of gates and higher number of bits addition within constant time. Efficient design for adder block to perform addition or multiplication will increase operation speed. QSD number uses less space than BSD to store number; higher number of gates can be tolerated for further improvement of QSD adder.

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