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Vertical Vibration Analysis of Suspension Bridges

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Abstract- Suspension bridges are long and flexible structures most vulnerable to dynamic loading. In this regard, continuum models are found to be more appropriate because of its ability to model dynamic problems with sufficient generality. This study compares the responses of linear and non-linear vibrations on continuum suspension bridge models. A continuum model with vertical vibration alone is then taken up for the study and the equation of motion is formulated. The governing differential equations were rendered dimensionless to identify the key parameters that control the dynamic response. The resulting eigenvalue problem was solved using MATLAB to obtain the modal characteristics of the structure. A parametric study was carried out to study the effect of the identified key parameters on the modal characteristics of the structure. Finite element modelling and analysis on an example problem is performed in MIDAS/Civil to find out the effect of hanger spacing on the modal characteristics. The natural frequencies of hangers were determined to study if they participate in the vibration or not.

The study revealed that the effect of hanger flexibility has very less significant role in the controlling the dynamic response of suspension bridges unless higher modes are considered with a relatively stiff girder. The effect of hanger spacing was found to have a very less role in controlling the vibrational characteristics. Hangers are found to have much higher natural frequency than the fundamental modes of vibration of the structure and hence the assumption of massless hangers is hence validated.

Index Terms- Suspension Bridges; Continuum Models; Vibration; Dynamic Response; Parametric Study

1. INTRODUCTION

Suspension bridges belong to the family of cable supported bridges and are distinguished by their ability to span very long distances. In Suspension bridges, bridge deck is supported onto main cables running between pylons by means of vertical or inclined hangers. These are the slenderest among all bridges and dominating type of bridge used for long span. They have been used from very old time and can be traced back to the time when creeper or rope suspended walkways were used by primitive man. An Iron chain bridge across Pan-Po River in China in A.D. 65 is the oldest known metal bridge. A similar record is found in Assam in India. Stiffened suspension bridges with wrought iron chains are originated in Europe in the 16th century and were developed in the 18th century.Today they represent more than 20 of all the longest bridges in the world (Harazaki, 2000). Akashi-Kaykio in Japan (completed in 1998) is the current longest suspension bridge with a central span of 1990.8 m. It is a three span suspension bridge having a total span of 3911.1 m.

As the suspension bridges are very long and flexile structures, the vibration analysis become very significant in predicting the behavior of bridge under the various dynamic loads such as vehicle live load, wind load and earthquake load. Among them, the response of such bridges to aerodynamic load was subjected to a huge amount of research after the infamous collapse of Tacoma Narrows Bridge in 1940 due to a wind of just 65 kmph. The bridge failure is attributed to the interaction between structure and wind, due to which it experienced a combined vertical and torsional motion. Negative damping caused these self-exited oscillations (Bleich,1950). The bridge had only been accounted for static pressure due to wind, the aerodynamic effects has been disregarded. Dynamic analysis become essential for the design of new bridges as well as for the structural health monitoring of existing bridges. It is also essential in designing vibration control systems for suspension bridges. The present study will be dedicated to identify the effect of key parameters to the modal characteristics such as natural frequency and mode shapes of Suspension bridges as they are going to govern the behavior under forced vibration.

2. METHODOLOGY

2.1. *Model considered and Assumptions for Linear Vibrations*

A model with three span (with each span simply supported at ends) suspension bridge with loaded backstays is used for the analysis (Luco, 2010) as given in figure. 1. It consists of a main cable between two pylons of equal height, a stiffening girder, uniformly spaced suspenders and two anchorages. This model is chosen so that it can be easily extended to single suspended span with unloaded backstays.

2.1.1. *Assumptions*

To simplify the complexity of the problem and for a rational mathematical model, several assumptions are made as follows:

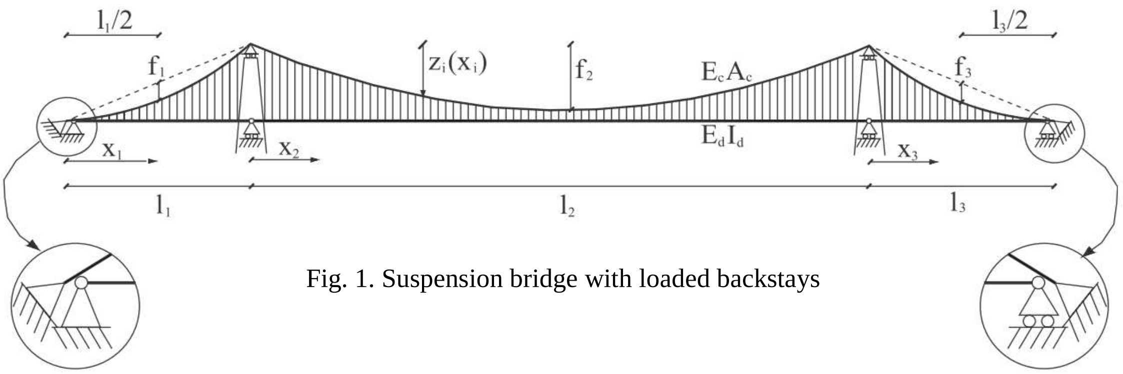

Fig. 1. Suspension bridge with loaded backstays

(i) The main cables are assumed to be uniform, elastic and perfectly flexible and are anchored at both ends.
(ii) The static sag is small so that the mass of cable per unit span can be taken as a constant.
(iii) The horizontal component of cable tension is assumed to be constant throughout the span.
(iv) The stiffening girder is modelled as flexural member with uniform flexural rigidity. Shear deformations, axial deformations and rotary inertia and any camber of the girder are neglected.
(v) The hangers are assumed to be massless, elastic, vertical and continuously and uniformly distributed without carrying any shear.
(vi) The initial dead loads are carried entirely by the cable without causing any stress in the girder. Under such a condition the cable adopts a parabolic profile under initial dead load.
(vii) The displacements from the static configuration is less and the additional horizontal component of cable tension *h(t)* is much smaller compared to static horizontal component of cable tension *H*.
(viii) The initial static tension in suspenders is sufficiently high to prevent slackening due to small vertical vibrations.

The first assumption eliminates the flexural rigidity of cable and allows the cable to carry loads only by tension. The second assumption will enable adding the mass per unit length of deck slab and cable directly, which otherwise would have become a function of distance from support and made the governing differential equations non-linear. This assumption is valid for small sag to span ratios of less than 1/8. The third assumption of constant horizontal tension throughout the span in cable is justifiable since no external load considered on cable having a Horizontal component. The fifth assumption of massless hangers is considered by assuming that the frequency of vibration of hangers is much above the lower frequencies of vibration of the structure so that their mass will not play any role. The sixth assumption is justified by the fact that the cables with hangers will be laid first and the deck will be hung on to the suspenders as precast segments as a usual construction practice. Though the real cable profile is between a catenary and a parabola, it is acceptable to assume the parabolic profile as it leads to greater mathematical simplicity. The second last assumption is supported by the fact that the external loads will be carried both by cable and girder and hence will induce an additional tension in cable which can only be a small fraction of that caused by initial dead load. This will reduce the governing differential equations of motion to linear differential equations. The last assumption is due to the fact that large-scale oscillations of the structure may arise from the slackening of the hangers and will give rise to nonlinear behavior of the bridge. Hence it is assumed that the amplitude of vibration is well within the amplitude initiating slackening.

2.2. ***Equation of Motion***

The equation of motion of the girder for each span l_i *(i=1, 2, 3)* will be of the form,

$$m_c \frac{\partial^2 w_{ic}}{\partial t^2} - H \frac{\partial^2 (w_{ic} + z_i)}{\partial {x_i}^2} - h \frac{\partial^2 z_i}{\partial {x_i}^2} = m_c g + f_{is} + f_{ie}^c$$

$$(\ 0 < x_i < l_i) \qquad \text{Eq. (1.a)}$$

$$m_d \frac{\partial^2 w_{id}}{\partial t^2} + E_d I_d \frac{\partial^4 w_{id}}{\partial {x_i}^4} = m_d g - f_{is} + f_{ie}^d$$

$$(0 < x_i < l_i) \qquad \text{Eq. (1.b)}$$

Where,
m_c and m_d are the m ass of the cable and girder per unit length.
w_{ic} and w_{id} are the vertical displacements of cable and girder from initial static configuration.
H is the dead load horizontal component of cable tension.
h(t) is the horizontal component of incremental cable tension.
z_i (x_i) is the dead load parabolic profile of the each segment of cable.
E_dI_d is the uniform flexural stiffness of the stiffening girder.
$f_{is}(x,t)$ is the distributed force in the suspenders.
$f_{ie}^c(x,t)$ & $f_{ie}^d(x,t)$ are the distributed external force acting on the cable & deck.

The displacements and external forces are considered positive downwards. Tensile force is considered positive.
The boundary conditions of at the ends of each span are,

$$w_{ic}(0,t) = w_{id}(0,t) = w_{id}''(0,t) = 0 \qquad \text{Eq. (2.a)}$$

$$w_{ic}(l_i,t) = w_{id}(l_i,t) = w_{id}''(l_i,t) = 0 \qquad \text{Eq. (2.b)}$$

The equations of motion can now be expressed in the matrix form as,

$$[M]\{\ddot{Y}(\bar{t})\} + [C]\{\dot{Y}(\bar{t})\} + [K]\{Y(\bar{t})\} = \{F(\bar{t})\} \qquad \text{Eq. (3)}$$

The above vetor has a dimension of (6Nx1). For undamped free vibration, the forcing function and damping matrix will become zero and the natural frquencies in terms of normalised natural frequencies can be obtained by solving the eigen value problem.The mass matrix in Eqn. (22) is mass normalised matrix [M] of size (6Nx6N) and is given by,

$$[M] = \begin{bmatrix} [M_1] & [0] & [0] \\ [0] & [M_2] & [0] \\ [0] & [0] & [M_3] \end{bmatrix} \qquad \text{Eq. (4)}$$

Where,

$$[M_i] = \alpha_i \begin{bmatrix} [I] & \vdots & \bar{m}_d[I] \\ \cdots\cdots & \vdots & \cdots\cdots \\ \bar{m}_d[I] & \vdots & \bar{m}_d[I] \end{bmatrix} \qquad \text{Eq. (5)}$$

In which [I] is a unit matix of size (NxN).

The (6Nx6N) normalised stiffness matrix [K] is given by

$$[K] = \begin{bmatrix} [K_{11}] & [K_{12}] & [K_{13}] \\ [K_{21}] & [K_{22}] & [K_{23}] \\ [K_{31}] & [K_{32}] & [K_{33}] \end{bmatrix} \qquad \text{Eq. (6)}$$

Where,

$$[K_{ii}] = \begin{bmatrix} [K_{icc}] & [K_{icr}] \\ [K_{irc}] & [K_{irr}] \end{bmatrix} \quad (i = 1,2,3) \qquad \text{Eq. (7)}$$

$$[K_{12}] = [K_{21}]^T = \begin{bmatrix} [k_{12}] & [0] \\ [0] & [0] \end{bmatrix} \qquad \text{Eq. (7.a)}$$

$$[K_{13}] = [K_{31}]^T = \begin{bmatrix} [k_{13}] & [0] \\ [0] & [0] \end{bmatrix} \qquad \text{Eq. (7.b)}$$

$$[K_{32}] = [K_{32}]^T = \begin{bmatrix} [k_{23}] & [0] \\ [0] & [0] \end{bmatrix} \qquad \text{Eq. (7.c)}$$

In which,

$$[K_{icc}] = \mu^2[k_{inm}^d] + [k_{inm}^H] + \lambda^2\alpha_i^2[k_{nm}^h] \qquad \text{Eq. (8)}$$

$$[K_{icr}] = [K_{irc}] = \mu^2[k_{inm}^d] \qquad \text{Eq. (9)}$$

$$[K_{irr}] = \mu^2[k_{inm}^d] + \chi^2[k_{inm}^s] \qquad \text{Eq. (10)}$$

And,

$$[k_{12}] = \lambda^2\alpha_1\alpha_2[k_{nm}^h] \qquad \text{Eq. (11.a)}$$

$$[k_{13}] = \lambda^2\alpha_1\alpha_3[k_{nm}^h] \qquad \text{Eq. (11.b)}$$

$$[k_{23}] = \lambda^2\alpha_2\alpha_3[k_{nm}^h] \qquad \text{Eq. (11.c)}$$

2.3. *Dimensionless Terms*

2.3.1. *Relative mass distribution,* $\bar{m}_d$

This parameter reflects the relative mass distribution of cable and girder. Relative mass distribution value will approach 1, if the girder is much heavier than the cable. It will have a value of 0.5 if the cable and girder have same self-weight per unit length.

2.3.2. *Dimensionless Cable Stiffness,* λ^2

The second one is the Irvine-Caughey non-dimensional cable parameter (Irvine and Caughey, 1974) which is dependent on the cable stiffness. It is dependent on the total length of the cable and hence is the only one parameter which differentiates between a single span suspension bridge with unloaded backstays and cable fixed at support.

2.3.3 *Dimensionless Girder Stiffness,* μ^2

The third parameter, μ^2 reflects the relative bending stiffness of the girder. It was introduced by Steinman. This parameter will have higher value if the flexural rigidity of the stiffening girder is more but will be lesser for longer spans.

2.3.4. *Dimensionless Hanger Stiffness,* χ^2

The dimensionless hanger stiffness, χ^2 relates the stiffness of suspenders to the total weight of the bridge.

2.3.5. *Sag Ratio,* h_s/f

The parameter, h_s/f affects the stiffness distribution in hangers. h_s is the length of the longest hanger. Also the cable sag f is dependent on the horizontal component of dead load tension in cable.

2.3.6. *Side span Ratio,* α

The ratio of side span to the main span, α will come into play for three span suspension bridge alone, as a suspension bridge with unloaded backstay has its equation of motion of the center span uncoupled from the side span.

2.4. *Non-linear Analysis*

The concept of tangent stiffness matrix, used in conjunction with the standard modal superposition method, provides a systematic approach to the nonlinear dynamic analysis of suspension bridges. Tangent stiffness matrix of a bar at any loaded configuration is given by

$$[k]t = [k]o + [k]g \qquad \text{Eq. (12)}$$

$$[k]g = Q/L * \begin{bmatrix} l-l^2 & -lm & -ln & l-l^2 & lm & ln \\ -lm & l-m^2 & -mn & lm & m^2-l & mn \\ -ln & -mn & l-n^2 & ln & mn & n^2-l \\ l-l^2 & lm & ln & l-l^2 & -lm & -ln \\ lm & m^2-l & mn & -lm & l-m^2 & -mn \\ ln & mn & n^2-l & -ln & -mn & l-n^2 \end{bmatrix} \qquad \text{Eq. (13)}$$

in which, [k]o = ordinary stiffness matrix in which Lo = the unloaded length of the member, L = the deformed length of the member, [k]g = geometric stiffness matrix, Q = Axial force, positive if tensile, l, m , n = direction cosines.

3. FINITE ELEMENT MODEL

MIDAS/Civil 2015 is used in the present study to model the suspension bridge. This widely used commercial Finite element package is superior in modelling and analysis of suspension bridges compared to its competent softwares like SAP2000, Sofistik, Lusas etc. The software offers unique modelling and analysis features. This software is used for the present study is due to the ease of modeling with suspension bridge wizard function and its ability to perform Eigen value analysis considering the geometric stiffness induced due to initial dead load cable tension.

Table 1. Element Material Properties

Element	Material	Young's Modulus (kN/m^2)	Density (kN/m^3)
Cable	High Strength Steel Wire	$2x10^8$	82.67
Hanger	High Strength Steel Wire	$1.4x10^8$	78.5
Deck	Structural Steel	$2.1x10^8$	78.5
Pylon	Structural Steel	$2.1x10^8$	78.5

Table 2. Element Sectional Properties

Element	Area, A (m^2)	Moment of Inertia, I_{zz} (m^4)
Cable	0.283	0^*
Hanger	$5.7x10^{-4}/m$	0^*
Deck	0.5395	3.2667
Pylon	0.169	0.1143
Pylon Transverse Beam	0.105	0.0913

* perfectly flexible

4. RESULTS

Natural frequencies for different modes in linear vibration and non-linear vibration are obtained by solving equations using MATLAB and tabulated below. Based on these results, a parametric study is conducted.

Table 3. Natural frequencies for linear motion (Symmetric Modes)

Mode	Symmetric modes			
	I	II	III	IV
	(a) $\mu^2 = 1 \times 10^{-3}$			
1	2.169	2.162	2.161	2.149
2	3.135	3.119	3.117	3.093
3	4.504	4.461	4.456	4.391
4	5.645	5.583	5.573	5.478
5	7.072	7.013	7.010	6.928
6	8.534	8.410	8.391	8.204
7	12.070	11.898	11.873	11.588
8	12.273	12.154	12.147	11.991
	(a) $\mu^2 = 10 \times 10^{-3}$			
1	2.512	2.508	2.507	2.500
2	4.131	4.128	4.127	4.122
3	5.163	5.139	5.135	5.100
4	9.332	9.331	9.330	9.328
5	13.034	13.032	13.032	13.029
6	16.915	16.898	16.894	16.863
7	26.986	26.733	26.670	26.051
8	27.509	27.164	27.126	26.361

Table 4. Natural frequencies for linear motion (Antisymmetric Modes)

Mode	Antisymmetric Modes			
	I	II	III	IV
	(a) $\mu^2 = 1 \times 10^{-3}$			
1	2.039	2.034	2.033	2.024
2	3.164	3.155	3.154	3.140
3	4.303	4.268	4.262	4.207
4	6.982	6.889	6.875	6.735
5	7.072	7.014	7.010	6.929
6	10.213	10.054	10.035	9.806
7	12.245	12.130	12.123	11.971
8	14.089	13.844	13.877	13.529
	(b) $\mu^2 = 10 \times 10^{-3}$			
1	2.362	2.361	2.360	2.358
2	4.184	4.180	4.180	4.174
3	6.424	6.421	6.421	6.416
4	12.803	12.802	12.801	12.799
5	13.034	13.032	13.032	13.029
6	21.637	21.556	21.536	21.372
7	27.498	27.148	27.110	26.327
8	32.950	32.270	32.105	29.853

Table 5. Natural frequencies for non-linear motion

Mode	Symmetric	Asymmetric
1	0.77	0.60
2	0.92	1.14
3	1.60	1.28
4	1.80	2.11
5	2.61	2.62
6	2.62	3.15
7	3.75	4.41
8	4.66	4.65

4.1. *Parametric Study*

The parametric variation did not show any particular trend for each natural frequency and hence cannot be used directly to predict the natural frequency by using curve fitting technique.

4.1.1. *Effect of χ^2 on Natural Frequencies*

The decrease in the hanger flexibility has less pronounced variation in the lower modes of vibration for both three span suspension bridge and suspension bridge with unloaded backstays.

4.1.2. *Effect of μ^2 on Natural Frequencies*

The increase in the relative girder stiffness increases the natural frequency. But this increment is more in case of unloaded backstay when compared to three span suspension bridge at higher modes of vibration and become more pronounced for higher values of μ^2.

4.1.3. *Effect of h_s/f on Natural Frequencies*

The variation of natural frequency to the parameter h_s/f is obtained by comparing Case II and Case III in Table 3 and Table 4. In fact it is possible to comment on the effect of pre-stressing and tower height. Pre-stressing will decrease the sag and hence increase the value of h_s/f . h_s is in fact the length of the hanger at tower location. Hence the increase in the value of h_s/f indicates an increase in tower height. Thus an increased value h_s/f indicates an additional pre-stressing or increased tower height.

4.1.4. *Effect of $\alpha_{1,3}$ on Natural Frequencies*

The general trend of increasing α value is to decrease the natural frequency and is considerable at higher mode shapes. As the girder stiffness parameter value increases, the natural frequency was also found to increase and is highly pronounced at μ^2 of the order 1×10^{-1} at higher modes. The effect of λ^2 is to increase the natural frequency, at its higher values, for symmetric mode shapes.

4.1.5. *Effect of $\overline{m}_d$ on Natural Frequencies*

Relative mass distribution parameter, $\overline{m}_d$ has a very less effect in controlling the dynamic behavior of both the suspension bridge models for lower modes even when non-dimensional girder stiffness values is high.

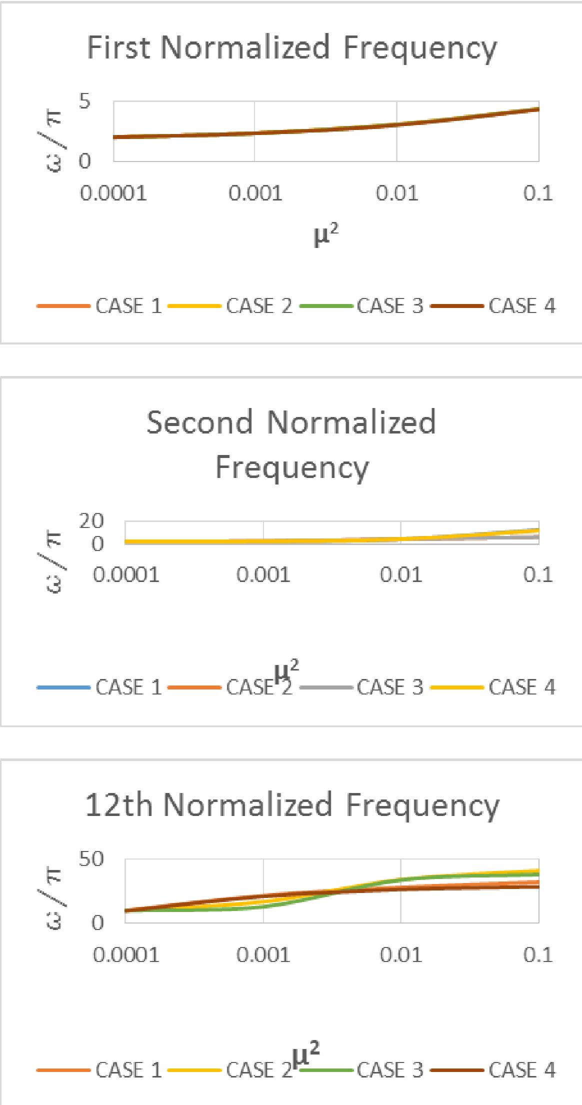

Fig. 2. Natural frequency v/s Girder stiffness

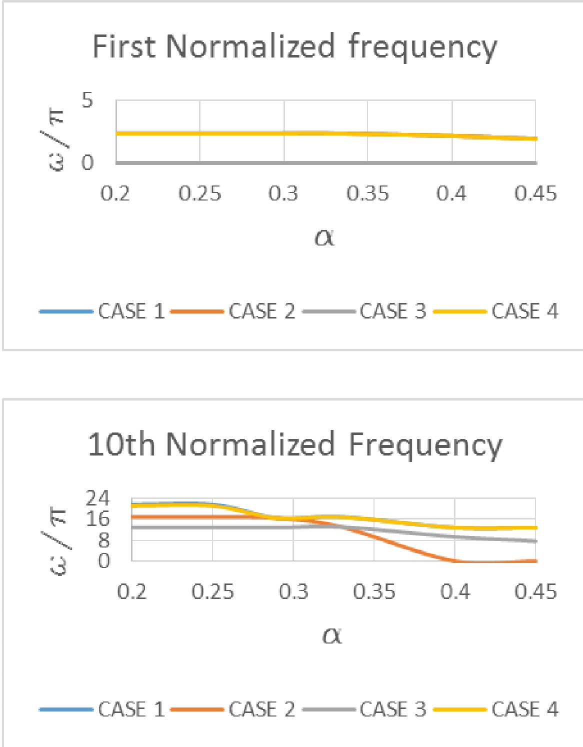

Fig. 3. Natural frequency v/s Side span ratio

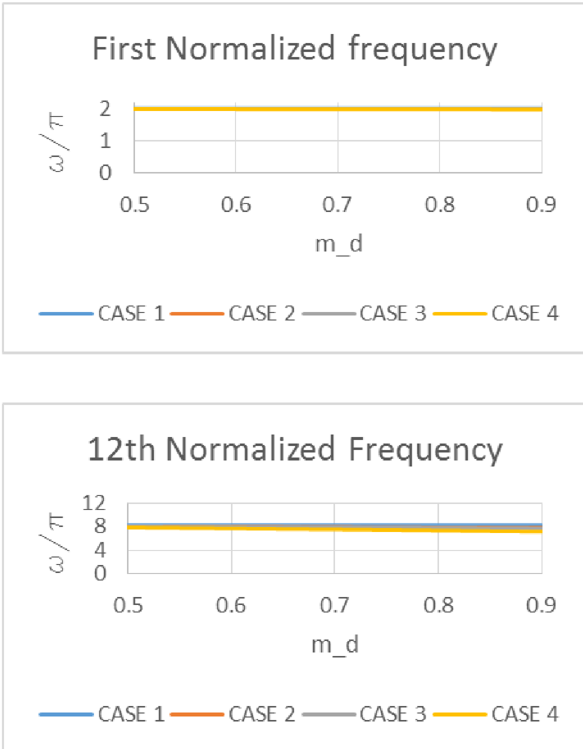

Fig. 4. Natural Frequency v/s relative mass

Table 6. Natural frequency for various hanger spacing

Mode	Hanger Spacing (m)					
	30	25	20	15	10	5
1	0.499	0.502	0.502	0.512	0.532	0.503
2	0.939	0.943	0.943	0.962	1.045	0.895
3	0.973	0.978	0.978	0.997	1.179	0.902
4	1.173	1.182	1.188	1.195	1.231	1.176
5	1.454	1.461	1.464	1.488	1.591	1.313
6	1.537	1.551	1.566	1.597	1.692	1.462
7	1.573	1.579	1.592	1.603	1.899	1.576
8	2.025	2.039	2.059	2.054	2.130	1.712
9	2.095	2.096	2.101	2.128	2.234	2.038
10	2.122	2.125	2.128	2.173	2.355	2.088
11	2.207	2.218	2.247	2.225	2.567	2.130
12	2.430	2.433	2.445	2.469	2.895	2.134

5. CONCLUSIONS

A simplified model used for the free vertical vibration analysis of Suspension Bridges is reviewed. The hanger flexibility has a very less effect on the dynamic behaviour of suspension bridges and can be disregarded for models requiring less number of modes. The increase in tower height and additional pre-stressing of main cable is found to increase the natural frequency. Some of the natural frequencies were found immune to cable stiffness and are identified as those corresponding to anti-symmetric mode shapes. The natural frequencies found to reduce with increase in side span length. The relative mass distribution has very less effect in the observed range of values adopted.

For the first vertical or torsional mode of symmetric or anti symmetric vibrations, the result of the nonlinear calculation shows only a slight deviation from that of the linear one in the domain of practical vibrational displacement. For higher modes, the deviation between linear and nonlinear calculations grows significantly. Also, when two modes (one torsional and one vertical) exhibit very closely spaced frequencies, the coupling is very strong and the energy initially imparted to one of these modes can, in general, be continuously exchanged between the two during the nonlinear motion. This contrasts with the linear solution, which predicts that the two modes are uncoupled.

Hanger spacing have a very less effect in the vibrational characteristics. The assumption of massless hanger is justified by the finding that their natural frequencies lies much above the lower natural frequencies of the structure.

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REFERENCES

[1] Ahmed M. Abdel-Ghaffar; Lawrence I. Rubin (1983): Nonlinear Free Vibrations of Suspension Bridges: Theory. J. Eng. Mech. @ ASCE .**109**:pp,313-329.

[2] Ahmed M. Abdel-Ghaffar; Lawrence I. Rubin (1983): Nonlinear Free Vibrations of Suspension Bridges: Application. J. Eng. Mech. @ ASCE .**109**:pp,330-345.

[3] Bleich F.; McCullough C. B; Rosecrans R.; Vincent G. S. (1950): The mathematical theory of vibration in suspension bridges. Washington: the US Government Printing Office.

[4] Chatterjee. P. K; Datta T. K; Surana C. S (1994): Vibration of Suspension Bridges Under Vehicular Movement. J. Struct. Eng. @ ASCE **120**:pp,681-703.

[5] Choi D; Gwon S; Yoo H; Na H (2013): Non-linear Static Analysis of Continuos Multi span Suspension Bridges. International Journal of Steel structures **13**,pp 103-115.

[6] Dickey. R. W (1968): The Suspension Bridge Deflection Equations. Journal of Mathematical Analysls And Applications, **24**, pp202-211.

[7] Domenic. A. Coletti (2002): Analytical and Field Investigation of Roma Suspension Bridge. J. Bridge Eng. @ ASCE .**7**,pp156-165.

[8] Gregor.P.Wollmann (2001): Preliminary Analysis of Suspension Bridges. J. Bridge Eng. @ ASCE.**6**,pp227-233.

[9] Jewel Sarker; Dr. Tanvir (2013): Optimum Dimensions of Suspension Bridges considering Natural period. IOSR journal of Mech & Civil Engg :vol **6**:pp 67-76.

[10] Josef malik (2006): Non-Linear Models of Suspension Bridges. J.Math.Anal.Appl. 320

[11] Jose Turmo; J. Enrique Luco (2010): Effect of Hanger Flexibility on Dynamic Response of Suspension Bridges. J. Eng. Mech. @ ASCE **136**:pp1444-1459.

[12] Kim M. Y;Kwon S. D; Kim N.I (2000): Analytical and numerical study on free vertical vibration of shear-flexible suspension bridges. Journal of Sound and Vibration, **238**(1), pp 65–84.

[13] Luca Sgambi (2012): Genetic Algorithms for the Dependability Assurance in the Design of a Long-span Suspension Bridge. Computer-

Aided Civil and Infrastructure Engineering **27,** pp 655–675.

[14] Luco J. E; Turmo J. (2010): Linear vertical vibrations of suspension bridges: A review of continuum models and some new results. Soil Dynamics and Earthquake Engineering, **30**, pp 769–781.

[15] MATLAB® Primer © COPYRIGHT 1984–2014 by The MathWorks, Inc.

[16] Mehmet Çevika; Mehmet Pakdemirlib (2005): Non-linear vibrations of suspension bridges with external excitation. International Journal of Non-Linear Mechanics **40**:pp901–923.

[17] MIDAS IT, MIDAS/Civil – Analysis Reference, (2012)

[18] Naif B Almutairi,; Hassan M. F; Terro. M (2006): Control of Suspensioon Bridge Non-linear vibrations due to moving loads. J. Eng. Mech. @ ASCE **132**,pp659-670.

[19] Serap Altın; Kubilay Kaptan; Semih S. Tezcan (2012): Dynamic Analysis of Suspension Bridges. Open Journal of Civil Engineering, **2**,pp 58-67.

[20] Wei-Xin Ren; George E Blandford (2004): Roebling Suspension Bridge I : Finite Element model and Free vibration Response. J. Bridge Eng.@ ASCE **9**,pp110-118.

[21] Zhang. J; Prader. J; Moon. F; Aktan A. E (2013): Experimental Vibration Analysis for Structural Identification of a Long-Span Suspension Bridge. J. Eng. Mech. @ ASCE **139**,pp748-759.